

Second term Mathematics E-Lesson Note

SUBJECT: MATHEMATICS

CLASS: SS 3

SCHEME OF WORK

WEEK	TOPIC
1.	Calculation on interest on bonds and debentures using logarithm table and problems on taxes and value added tax.
2.	Coordinate Geometry of straight line: Cartesian coordinate graphs, distance between two points, midpoint of the line joining two points.
3.	Coordinate Geometry of straight lines: Gradient and Intercepts of a line, angle between two intersecting straight lines and application.
4.	Differentiation of algebraic functions: meaning of differentiation, differentiation from first principle and standard derivatives of some basic functions.
5.	Differentiation of algebraic functions: Basic rules of differentiation such as sum and difference, product rule, quotient rule and maximal and minimum application.
6.	Integration and evaluation of simple algebraic functions: Definition, method of integration: substitution, partial fraction and integration by parts, area under the curve and use of Simpson's rule.
7– 12.	Revision and mock examination.

REFERENCE TEXT

- New General Mathematics for SS book 3 by J.B Channon
- Essential Mathematics for SS book 3
- Mathematics Exam Focus
- Waec and Jamb past Questions

WEEK ONE

- Calculation on interest on bonds and debentures using logarithm table

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- Problems on taxes and value added tax.

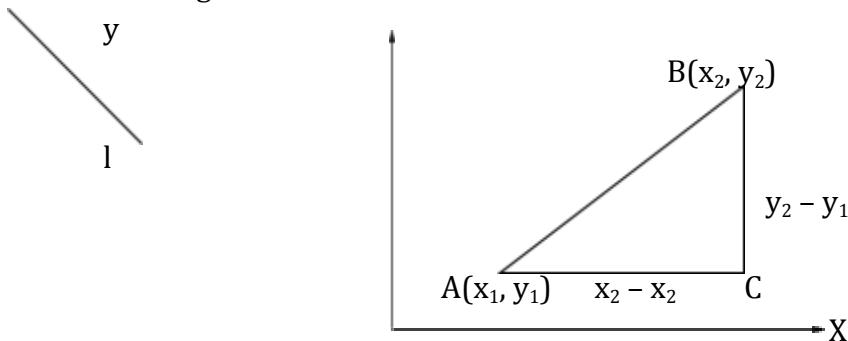
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WEEK TWO

- Coordinate Geometry of straight line: Cartesian coordinate graphs
- distance between two points
- midpoint of the line joining two points
- Coordinate Geometry of Straight line:
- Cartesian coordinate graph:

Distance between two lines:

In the figure below, the coordinates of the points A and B are (x_1, y_1) and (x_2, y_2) , respectively. Let the length of AB be l .



Using Pythagoras theorem:

$$AB^2 = AC^2 + BC^2$$

$$l^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

Find the distance between the each pair of points: a. (3, 4) and (1, 2) b. (3, - 3) and (- 2, 5)

Solution:

Using $l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

a. $l = \sqrt{(3 - 1)^2 + (4 - 2)^2}$

$$l = \sqrt{2^2 + 2^2}$$

$$l = \sqrt{8} = 2\sqrt{2} \text{ units}$$

b. $l = \sqrt{(3 - (-2))^2 + (-3 - 5)^2}$

$$= \sqrt{5^2 + (-8)^2}$$

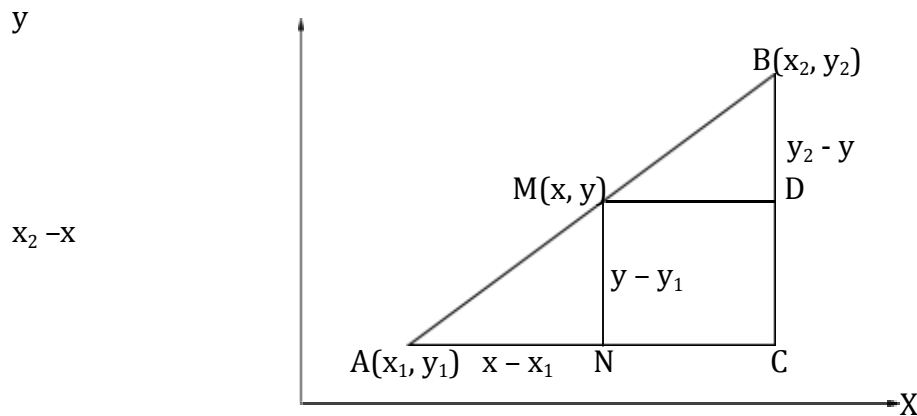
$$= \sqrt{25 + 64} = \sqrt{89} = 9.43 \text{ units}$$

Evaluation: Find the distance between the points in each of the following pairs leaving your answers in surd form: 1. (-2, - 5) and (3, - 6) 2. (- 3, 4) and (- 1, 2)

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Mid-point of a line:

The mid-point of the line joining two points:



Triangle MAN and BMD are congruent, so $AM = MD$ and $BD = MN$

$$x - x_1 = x_2 - x$$

$$y - y_1 = y_2 - y$$

$$x + x = x_2 + x_1$$

$$y + y = y_2 + y_1$$

$$2x = x_2 + x_1$$

$$2y = y_2 + y_1$$

$$x = \frac{x_2 + x_1}{2}$$

$$y = \frac{y_2 + y_1}{2}$$

Hence, the **mid-point** of a straight line joining two is $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$

Example: Find the coordinates of the mid-point of the line joining the following pairs of points.

- a. (3, 4) and (1, 2) b. (2, 5) and (-3, 6)

Solution:

$$\text{Mid-point} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

$$\text{a. Mid-point} = \left(\frac{1+3}{2}, \frac{4+2}{2} \right) = (2, 3)$$

$$\text{b. Mid-point} = \left(\frac{-3+2}{2}, \frac{6+5}{2} \right) = \left(-\frac{1}{2}, \frac{11}{2} \right)$$

Evaluation: Find the coordinates of the mid-point of the line joining the following pairs of points.

- a. (-2, -5) and (3, -6) b. (3, 4) and (-1, -2)

General Evaluation

- Find the distance between the points in each of the following pairs leaving your answers in surd form: 1. (7, 2) and (1, 6)
- What is the value of r if the distance between the points (4, 2) and (1, r) is 3 units?
- Find the coordinates of the mid-point (-3, -2) and (-7, -4)

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Reading Assignment: NGM for SS 3 Chapter 9 page 77 – 78,

Weekend Assignment:

1. Find the value of $\alpha^2 + \beta^2$ if $\alpha + \beta = 2$ and the distance between the points $(1, \alpha)$ and $(\beta, 1)$ is 3 units.
2. The vertices of the triangle ABC are A (7, 7), B (- 4, 3) and C (2, - 5). Calculate the length of the longest side of triangle ABC.
3. Using the information in '2' above, calculate the line AM, where M is the mid-point of the side opposite A.

WEEK THREE

- Coordinate Geometry of straight lines:
- Gradient and Intercepts of a line
- Angle between two intersecting straight lines and application

Gradient and Intercepts of a line

Gradient of a line of the form $y = mx + c$, is the coefficient of x, which is represented by m and c is the intercept on the y axis.

Example

1. Find the equation of the line with gradient 4 and y-intercept -7.

Solution

$$m = 4, c = - 7,$$

Hence, the equation is; $y = 4x - 7$.

Evaluation:

1. What is the gradient and y intercept of the line equation $3x - 5y + 10 = 0$?
2. Find the equation of the line with gradient - 9 and y-intercept 4.

Gradient and One Point Form

The equation of the line can be calculated given one point (x, y) and gradient (m) by using the formula; $y - y_1 = m(x - x_1)$

Example

Find the equation of the line with gradient -8 and point(3, 7).

Solution

$$m = - 8, (x_1, y_1) = (3, 7)$$

$$\text{Equation: } y - 7 = - 8(x - 3)$$

$$y = -8x + 24 + 7$$

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$$y = -8x + 31$$

Evaluation:

1. Find the equation of the line with gradient 5 and point(-2, -7).
2. Find the equation of the line with gradient -12 and point (3, -5).

Two Point Form:

Given two points (x_1, y_1) and (x_2, y_2) , the equation can be obtained using the formula:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Example: Find the equation of the line passing through (2,-5) and (3,6).

Solution

$$\frac{y - (-5)}{6 - (-5)} = \frac{x - 2}{3 - 2}$$

$$11 = y + 5/x - 2$$

$$11(x - 2) = y + 5$$

$$11x - 22 = y + 5$$

$$y - 11x + 27 = 0$$

Evaluation:

1. Find the equation of the line passing through (3, 4) and (-1, -2).
2. Find the equation of the line passing through (-8, 5) and (-6, 2).

Angles between Lines

Parallel lines:

The angle between parallel lines is 0° because they have the same gradient

Perpendicular Lines:

Angle between two perpendicular lines is 90° and the product of their gradients is -1 . Hence, $m_1 m_2 = -1$

Examples:

1. Show that the lines $y = -3x + 2$ and $y + 3x = 7$ are parallel.

solution:

$$\text{Equation 1: } y = -3x + 2, \quad m_1 = -3$$

$$\text{Equation 2: } y + 3x = 7,$$

$$y = -3x + 7, \quad m_2 = -3$$

since; $m_1 = m_2 = -3$, then the lines are parallel

2. Given the line equations $x = 3y + 5$ and $y + 3x = 2$, show that the lines are perpendicular.

solutions:

$$\text{Equation 1: } x = 3y + 5, \text{ make } y \text{ the subject of the equation.}$$

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$$3y = x + 5$$

$$y = x/3 + 5/3$$

$$m_1 = 1/3$$

Equation 2: $y + 3x = 2,$

$$y = -3x + 2, \quad m_2 = -3$$

hence: $m_1 \times m_2 = 1/3 \times -3 = -1$

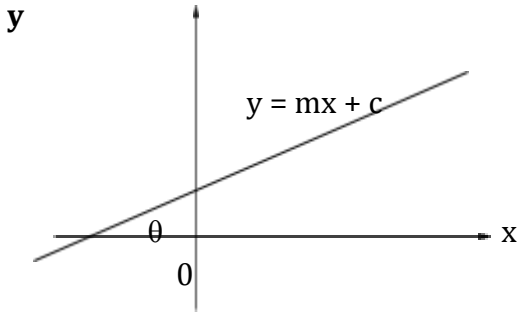
since: $m_1 m_2 = -1$, then the lines are perpendicular.

Evaluation: State which of the following pairs of lines are: (i) perpendicular (ii) parallel

(1) $y = x + 5$ and $y = -x + 5$ (2). $2y - 6 = 5x$ and $3 - 5y = 2x$ (3) $y = 2x - 1$ and $2y - 4x = 8$

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Angles between Intersecting Lines:



The gradient of $y = mx + c$ is $\tan \theta$. Hence **$m = \tan \theta$** can be used to calculate angles between two intersecting lines. Generally the angle between two lines can be obtained using: $\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

Example: Calculate the acute angle between the lines $y = 4x - 7$ and $y = x/2 + 0.5$.

Solution:

$$Y = 4x - 7, m_1 = 4, y = x/2 + 0.2, m_2 = 1/2.$$

$$\tan \theta = \frac{0.5 - 4}{1 + (0.5 \cdot 4)} = -3.5/3$$

$$\tan \theta = -1.1667$$

$$\theta = \tan^{-1}(-1.1667) = 49.4$$

Evaluation: Calculate the acute angle between the lines $y = 3x - 4$ and $x - 4y + 8 = 0$.

General Evaluation:

1. Calculate the acute angle between the lines $y = 2x - 1$ and $2y + x = 2$.
2. If the lines $3y = 4x - 1$ and $qy = x + 3$ are parallel to each other, find the value of q .
3. Find the equation of the line passing through $(2, -1)$ and gradient 3.

Reading Assignment: NGM for SS 3 Chapter 9 page 75 – 81

Weekend Assignment

1. Find the equation of the line passing through $(5, 0)$ and gradient 3.
2. Find the equation of the line passing through $(2, -1)$ and $(1, -2)$.
3. Two lines $y = 3x - 4$ and $x - 4y + 8 = 0$ are drawn on the same axes.
 - (a) Find the gradients and intercepts on the axes of each line.
 - (b) Find the equation parallel to $x - 4y + 8 = 0$ at the point $(3, -5)$

WEEK FOUR

- Differentiation of algebraic functions: meaning of differentiation

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- Differentiation from first principle
- Standard derivatives of some basic functions.

Consider the curve whose equation is given by $y = f(x)$ Recall that $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+x) - f(x)}{dx}$

As point B moves close to A, dx becomes smaller and tends to zero.

The limiting value is written on $\lim_{dx \rightarrow 0} \frac{f(x+x) - f(x)}{dx}$ and is denoted by $f'(x)$

$f'(x)$ is called the **derivative of $f(x)$** and the **gradient function of the curve**

The process of finding the derivative of $f(x)$ is called differentiation. The notations which are commonly used for the derivative of a function are $f'(x)$ read as f - prime of x, df/dx read as dee x of f, dy/dx read dee - f dee - x, dy/dx read dee - y dee - x

If $y = f(x)$, this $dy/dx = f'(x)$ (it is called the differential coefficient of y with respect to x.

Differentiation from first principle: The process of finding the derivative of a function from the consideration of the limiting value is called differentiation from first principle.

Example 1

Find from first principle, the derivative of $y = x^2$

Solution

$$y = x^2$$

$$y + y = (x + x)^2$$

$$y + y = x^2 + 2xx + (x)^2$$

$$y = x^2 + 2xx + (x)^2 - y$$

$$y = x^2 + 2xx + (x)^2 - x^2$$

$$y = 2xx + (x)^2$$

$$y = (2x + x)x$$

$$y = 2x + x$$

$$x$$

$$\lim_{x \rightarrow 0} x = 0$$

$$\frac{dy}{dx} = 2x$$

$$dx$$

Example 2:

Find from first principle, the derivative of $1/x$

Solution

$$\text{Let } y = \frac{1}{x}$$

$$x$$

$$y + y = \frac{1}{x + x}$$

$$y = \frac{1}{x + x} - y$$

$$y = \frac{1}{x + x} - \frac{1}{x}$$

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$$y = x - \frac{(x + x)}{(x + x)x}$$

$$y = x - \frac{x - x}{x^2 + xx}$$

$$\frac{dy}{dx} = \frac{-x}{x^2 + x}$$

$$y = \frac{-1}{x^2 + x}$$

$$\lim_{x \rightarrow 0} x = 0$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$

Evaluation: Find from first principle, the derivatives of y with respect to x:

$$1. Y = 3x^3 \quad 2. Y = 7x^2 \quad 3. Y = 3x^2 - 5x$$

Rules of Differentiation: Let $y = x^n$

$$y + dy = (x + dx)^n \\ = x^n + nx^{n-1}dx + \frac{n(n-1)}{2!}x^{n-2}(dx)^2 + \dots (dx)^n$$

$$= x^n + nx^{n-1}dx + \frac{n(n-1)}{2!}x^{n-2}(dx)^2 + \dots + (dx)^n - x^n$$

$$= nx^{n-1}dx + \frac{n(n-1)}{2!}x^{n-2}(dx)^2$$

$$\frac{dy}{dx} = nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}dx$$

$$\lim_{dx \rightarrow 0} \frac{dy}{dx} = nx^{n-1}$$

$$dx = 0$$

$$\text{Hence; } \frac{dy}{dx} = nx^{n-1} \text{ if } y = x^n$$

Example 3:

Find the derivative of the following with respect to x: (a) x^7 (b) $x^{1/2}$ (c) $5x^2 - 3x$ (d) $-3x^2$ (e) $y = 2x^3 - 3x + 8$

Solution

a. Let $y = x^7$
 $\frac{dy}{dx} = 7x^{7-1} = 7x^6$

b. Let $y = x^{1/2}$
 $\frac{dy}{dx} = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

c. Let $y = 5x^2 - 3x$
 $\frac{dy}{dx} = 10x - 3$

d. Let $y = -3x^2$
 $\frac{dy}{dx} = 2 \times -3x^{2-1} = -6x$

e. Let $y = 2x^3 - 3x + 8$
 $\frac{dy}{dx} = 3 \times 2x^{3-1} - 3 + 0$

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$$= 6x^2 - 3$$

Evaluation:

1. If $y=5x^4$, find dy/dx
2. Given that $y=4x^{-1}$ find y^1

General Evaluation

1. Find, from first principles, the derivative of $4x^2 - 2$ with respect to x .
2. Find the derivative of the following a. $3x^3 - 7x^2 - 9x + 4$ b. $2x^3$ c. $3/x$
3. Using idea of difference of two square; simplify $243x^2 - 48y^2$
4. Expand $(2x-5)(3x-4)$
5. If the gradient of $y=2x^2-5$ is -12 find the value of y .

Reading Assignment: NGM for SS 3 Chapter 10 page 82 -88,

Weekend Assignment

Objective

1. Find the derivative of $5x^3$ (a) $10x^2$ (b) $15x^2$ (c) $10x$ (d) $15x^3$
2. Find dy/dx , if $y = 1/x^3$ (a) $-3/x^4$ (b) $3/x^4$ (c) $4/x^3$ (e) $-4/x^3$
3. Find $f^1(x)$, if $f(x) = x^3$ (a) $3x$ (b) $3x^2$ (c) $\frac{1}{2}x^3$ (d) $2x^3$
4. Find the derivative of $1/x$ (a) $1/x^2$ (b) $-1/x^2$ (c) $-x$ (d) $-x^2$
5. If $y = -2/3 x^3$. Find dy/dx (a) $4/3 x^2$ (b) $2x^2$ (c) $-2x^2$ (d) $-2x$

Theory

1. Find from first principle, the derivative of $y = x + 1/x$
2. Find the derivative of $2x^2 - 2/x^3$

WEEK FIVE

- Differentiation of algebraic functions:
- Basic rules of differentiation such as sum and difference, product rule, quotient rule
- Maximal and minimum application.

Derivative of algebraic functions

Let f, u, v be functions such that

$$f(x) = u(x) + v(x)$$

$$f(x+x) = u(x+x) + v(x+x)$$

$$f(x+x) - f(x) = \{u(x+x) + v(x+x) - v(x+x) - u(x) - v(x)\}$$
$$= u(x+x) - u(x) + v(x+x) - v(x)$$

$$f(x+x) - f(x) = u(x+x) - u(x) + v(x+x) - v(x)$$

$$\lim_{x \rightarrow 0} \frac{f(x+x) - f(x)}{x} = U^1(x) + V^1(x)$$

if $y = u + v$ and u and v are functions of x , then **$dy/dx = du/dx + dv/dx$**

Examples: Find the derivative of the following

- 1) $2x^3 - 5x^2 + 2$
- 2) $3x^2 + 1/x$
- 3) $2x^3 + 2x^2 + 1$

Solution

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- Let $y = 2x^3 - 5x^2 + 2$
 $dy/dx = 6x^2 - 10x$
- Let $y = 3x^2 + 1/x = 3x^2 + x^{-1}$
 $dy/dx = 6x - x^{-2} = 6x - \frac{1}{x^2}$
- Let $y = 2x^3 + 2x^2 + 1$
 $dy/dx = 6x^2 + 4x$

Evaluation: 1. If $y = 3x^4 - 2x^3 - 7x + 5$. Find dy/dx
 2. Find $d(8x^3 - 5x^2 + 6)/dx$

Function of a function (chain rule)

Suppose that we know that y is a function of u and that u is a function of x , how do we find the derivative of y with respect to x ?

Given that $y = f(x)$ and $u = h(x)$, what is dy/dx ?

$dy/dx =$, this is called the chain rule

Examples

Find the derivative of the following. (a) $y = (3x^2 - 2)^3$ (b) $y = (1 - 2x^3)^{1/2}$ (c) $5/(6-x^2)^3$

Solution

- $y = (3x^2 - 2)^3$
 Let $u = 3x^2 - 2$
 $y = (3x^2 - 2)^3 \Rightarrow y = u^3$
 $y = u^3$
 $dy/du = 3u^2$
 $du/dx = 6x$

$$dy/dx = 3u^2 \times 6x$$

$$= 18xu^2 = 18x(3x^2 - 2)^2$$

- $y = (1 - 2x^3)^{1/2} \Rightarrow (1 - 2x^3)^{1/2}$
 Let $u = 1 - 2x^3$, hence $y = u^{1/2}$
 $dy/dx = \frac{1}{2} u^{-1/2} \times (-6x^2)$
 $= -3x^2 u^{-1/2} = \underline{-3x^2}$

$u^{1/2}$

$$\frac{-3x^2}{\sqrt{u}} = \frac{-3x^2}{\sqrt{1 - 2x^3}}$$

- $y = \frac{5}{(6 - x^2)^3} = 5(6 - x^2)^{-3}$
 Let $u = 6 - x^2$
 $y = 5(u)^{-3}$
 $dy/du = -15u^{-4}$
 $du/dx = -2x$
 $dy/dx = dy/du \times du/dx = -15u^{-4} \times (-2x) = 30x u^{-4} = 30x (6 - x^2)^{-4}$
 $= \underline{\frac{30x}{(6 - x^2)^4}}$

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$$(6 - x^2)^4$$

Evaluation:

1. Given that $y = \frac{1}{(2x + 3)^4}$ find dy/dx
2. If $y = (3x^2 + 1)^3$, Find dy/dx

Product Rule

We shall consider the derivative of $y = uv$ where u and v are function of x .

Let $y = uv$

Then $y + \Delta y = (u + \Delta u)(v + \Delta v)$

$$= uv + u\Delta v + v\Delta u + \Delta u\Delta v$$

$$y + \Delta y = uv + u\Delta v + v\Delta u + \Delta u\Delta v$$

$$y + \Delta y = uv + v\Delta u + u\Delta v + \Delta u\Delta v$$

$$\frac{\Delta y}{\Delta x} = u \frac{\Delta v}{\Delta x} + v \frac{\Delta u}{\Delta x} + \frac{\Delta u\Delta v}{\Delta x}$$

As $\Delta x \rightarrow 0$, $\Delta u \rightarrow 0$, $\Delta v \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{uv + u\Delta v + v\Delta u + \Delta u\Delta v}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{uv}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{u\Delta v}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{v\Delta u}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{\Delta u\Delta v}{\Delta x}$$

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Hence $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

Examples

Find the derivatives of the following.

(a) $y = (3 + 2x)(1 - x)$ (b) $y = (1 - 2x + 3x^2)(4 - 5x^2)$

Solution

1. $y = (3 + 2x)(1 - x)$
 Let $u = 3 + 2x$ and $v = (1 - x)$
 $\frac{du}{dx} = 2$ and $\frac{dv}{dx} = -1$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= (1 - x) 2 + (3 + 2x) (-1) = 2 - 2x - 3 - 2x \\ \frac{dy}{dx} &= -1 - 4x \end{aligned}$$

2. $y = (1 - 2x + 3x^2)(4 - 5x^2)$
 Let $u = (1 - 2x + 3x^2)$ and $v = (4 - 5x^2)$
 $\frac{du}{dx} = -2 + 6x$ and $\frac{dv}{dx} = -10x$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= (1 - 2x + 3x^2)(-10x) + (4 - 5x^2)(-2 + 6x) \\ &= -10x + 20x^2 - 30x^3 + (-8 + 10x^2 + 24x - 30x^3) \\ &= -10x + 20x^2 - 30x^3 - 8 + 10x^2 + 24x - 30x^3 \\ &= 14x + 30x^2 - 60x^3 - 8 \end{aligned}$$

Evaluation

Given that (i) $y = (5+3x)(2-x)$ (ii) $y = (1+x)(x+2)^{3/2}$, find $\frac{dy}{dx}$

Quotient Rule:

If $y = \frac{u}{v}$

v

then; $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Examples:

Differentiate the following with respect to x. (a) $\frac{x^2 + 1}{1 - x^2}$ (b) $\frac{(x-1)^2}{\sqrt{x}}$

Solution:

1. $y = \frac{x^2 + 1}{1 - x^2}$
 Let $u = x^2 + 1$ $\frac{du}{dx} = 2x$
 $v = 1 - x^2$ $\frac{dv}{dx} = -2x$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

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$$\frac{dx dx dx}{v^2}$$

$$dy/dx = \frac{(1-x^2)(2x) - (x^2+1)(-2x)}{(1-x^2)^2}$$

$$= \frac{2x - 2x^3 + 2x^3 + 2x}{(1-x^2)^2}$$

$$dy/dx = \frac{4x}{(1-x^2)^2}$$

2. $y = \frac{(x-1)^2}{\sqrt{x}}$

Let $u = (x-1)^2$ $du/dx = 2(x-1)$
 $v = \sqrt{x}$ $dv/dx = 1/2\sqrt{x}$

$$dy/dx = \frac{\sqrt{x} \cdot 2(x-1) - (x-1)^2 \cdot 1/2\sqrt{x}}{(\sqrt{x})^2}$$

$$dy/dx = \frac{\sqrt{x} \cdot 2(x-1) - (x-1)^2 \cdot 1/2\sqrt{x}}{x}$$

Evaluation: Differentiate with respect to x: (1) $\frac{(2x+3)^3}{(x^3-4)^2}$ (2) $\frac{\sqrt{x}}{\sqrt{x+1}}$

Applications of differentiation:

There are many applications of differential calculus.

Examples:

1. Find the gradient of the curve $y = x^3 - 5x^2 + 6x - 3$ at the point where $x = 3$.

Solution:

$$Y = x^3 - 5x^2 + 6x - 3$$

$$dy/dx = 3x^2 - 10x + 6$$

$$\text{where } x = 3; dy/dx = 3(3^2) - 10(3) + 6$$

$$= 27 - 30 + 6$$

$$= 3.$$

2. Find the coordinates of the point on the graph of $y = 5x^2 + 8x - 1$ at which the gradient is -2

Solution:

$$Y = 5x^2 + 8x - 1$$

$$dy/dx = 10x + 8$$

replace; dy/dx by -2

$$10x + 8 = -2$$

$$10x = -2 - 8$$

$$x = -10/10 = -1$$

3. Find the point at which the tangent to the curve $y = x^2 - 4x + 1$ at the point $(2, -3)$

Solution:

$$Y = x^2 - 4x + 1$$

$$dy/dx = 2x - 4$$

$$\text{at point } (2, -3): dy/dx = 2(2) - 4$$

$$dy/dx = 0$$

$$\text{tangent to the curve: } y - y_1 = dy/dx(x - x_1)$$

Second term Mathematics E-Lesson Note

$$\begin{aligned}y - (-3) &= 0 \quad (x - 2) \\y + 3 &= 0\end{aligned}$$

Evaluation:

1. Find the coordinates of the point on the graph of $y = x^2 + 2x - 10$ at which the gradient is 8.
2. Find the point on the curve $y = x^3 + 3x^2 - 9x + 3$ at which the gradient is 15.

Velocity and Acceleration

Velocity: The velocity after t seconds is the rate of change of displacement with respect to time.

Suppose; s = distance and t = time,

Then; **Velocity** = ds/dt

Acceleration: This is the rate of change of velocity compared with time.

Acceleration = dv/dt

Example:

A moving body goes s metres in t seconds, where $s = 4t^2 - 3t + 5$. Find its velocity after 4 seconds. Show that the acceleration is constant and find its value.

Solution:

$$S = 4t^2 - 3t + 5$$

$$ds/dt = 8t - 3$$

$$\text{velocity} = ds/dt = 8(4) - 3$$

$$= 32 - 3$$

$$= 29$$

$$\text{Acceleration: } dv/dt = 8.$$

Maxima and Minimal

1. Find the maximum and minimum value of y on the curve $6x - x^2$.

Solution:

$$y = 6x - x^2$$

$$dy/dx = 6 - 2x$$

$$\text{equate } dy/dx = 0$$

$$6 - 2x = 0$$

$$6 = 2x$$

$$X = 3$$

The turning point is (3, 9)

2. Find the maximum and minimum of the function $x^3 - 12x + 2$.

Solution:

$$Y = x^3 - 12x + 2$$

$$dy/dx = 3x^2 - 12$$

$$3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 12/3$$

$$x^2 = 4 \quad x = \pm 2$$

minimum point occur when $d^2y/dx^2 > 0$

maximum point occurs when $d^2y/dx^2 < 0$

$$d^2y/dx^2 = 6x$$

Second term Mathematics E-Lesson Note

substitute $x = 2$; $d^2y/dx^2 = 6 \times 2 = 12$

therefore: the function is minimum at point $x = 2$ and $y = -14$

substitute $x = -2$; $d^2y/dx^2 = 6(-2) = -12$

therefore: the function is maximum at point $x = -2$ and $y = 18$

Evaluation:

1. A particle moves in such a way that after t seconds it has gone s metres, where $s = 5t + 15t^2 - t^3$
2. Find the maximum and minimum value of y on the curve $4 - 12x - 3x^2$.

General Evaluation

Use product rule to find the derivative of

1. $y = x^2(1+x)^{1/2}$
2. $y = \sqrt{x}(x^2 + 3x - 2)^2$
3. Find the derivative of $y = (7x^2 - 5)^3$
4. Using completing the square method find t if $s = ut + \frac{1}{2}at^2$
5. If 3 is a root of the equation $x^2 - kx + 42 = 0$ find the value of k and the other root of the equation

READING ASSIGNMENT: NGM for SS 3 Chapter 10 page 90 -101,

WEEKEND ASSIGNMENT

OBJECTIVE

1. Differentiate the function $4x^4 + x^3 - 5$ (a) $4x^3 + 3x^2$ (b) $16x^2 + 3x^2$ (c) $16x^3 + 3x^2$ (d) $16x^4 + 3x^2$
2. Find d^2y/dx^2 of the function $y = 3x^5$ wrt x . (a) $15x^3$ (b) $45x^4$ (c) $60x^3$ (d) $3x^5$ (e) $12x^3$
3. If $f(x) = 3x^2 + 2/x$ find $f'(x)$ (a) $6x + 2$ (b) $6x + 2/x^2$ (c) $6x - 2/x^2$ (d) $6x - 2$
4. Find the derivative of $2x^3 - 6x^2$ (a) $6x^2 - 12x$ (b) $6x^2 - 12x$ (c) $2x^2 - 6x$ (d) $8x^2 - 3x$
5. Find the derivative of $x^3 - 7x^2 + 15x$ (a) $x^2 - 7x + 15$ (b) $3x^2 - 14x + 15$ (c) $3x^2 + 7x + 15$ (d) $3x^2 - 7x + 15$

THEORY

1. Differentiate with respect to x . $y^2 + x^2 - 3xy = 4$
2. Find the derivative of $3x^3(x^2 + 4)^2$