4.9.2 Timmy & Jenna's Jeep Tours

4.9.2 Timmy Jimmy & Jenna Doe's Jeep Tours Name:

Timmy Jimmy's sister Jenna Doe, who lives near a tourist area *in another part of the country*, is impressed by the success of Timmy Jimmy's business. She decides to join the business, running tours on the same days as Timmy Jimmy in her slightly smaller vehicle, under the name Jenna Doe's adventures. After a year of steady bookings, Jenna Doe discovers that the number of passengers E on her half-day tours has the following probability distribution shown below.

Jenna				
# Passengers e_i:	2	3	4	5
Probability p_i:	0.3	0.4	0.2	0.1

Timmy					
# Passengers g_i:	2	3	4	5	6
Probability p_i:	0.15	0.25	0.35	0.20	0.05

Let T = How many **total passengers** Timmy Jimmy and Jenna Doe expect to have on their tours on a randomly selected day. Fill out the probability model below for their combined number of passengers.

Value of t_i:								
Probability p_i:								

Interpretation

1. Find the **expected value**: $\mu_T = E(T) =$

2. Find the **standard deviation**: $\sigma_T = SD(T) =$

3. Find the **variance**: $\sigma^2_T = Var(T) =$

4. How did you transform the random variables?

5. How does this transformation affect the
   a. New Mean
   b. New Standard Deviation
   c. New Variance

For comparison

	Timmy Jimmy	Jenna Doe
Expected Value	3.75	3.10
Standard Deviation	1.089724736	0.943
Variance	1.1875	0.889249

Let D = How many **more** (or fewer) passengers Timmy Jimmy can have compared to Jenna Doe on a randomly selected day (TJ's Passengers - JD's Passengers).

Value of d_i:								
Probability p_i:	0.015	0.055	0.145	0.235	0.260	0.195	0.08	0.015

INTERPRETATION

6. Find the **expected value**: $\mu_D = E(D) =$

7. Find the **standard deviation**: $\sigma_D = SD(D) =$

8. Find the **variance**: $\sigma^2_D = Var(D) =$

9. How did you transform the random variables?

10. How does this transformation affect the
    a. New Mean
    b. New Standard Deviation
    c. New Variance

For comparison

	Timmy Jimmy	Jenna Doe
Expected Value	3.75	3.10
Variance	1.1875	0.889249
Standard Deviation	1.089724736	0.943

11. What do you notice about the **variance** and **standard deviation** of the random variables when you subtracted compared to when you added?

Check Your Understanding

1. A mathematics competition uses the following scoring procedure to discourage students from guessing (choosing an answer randomly) on the multiple-choice questions. For each correct response, the score is 7. For each question left unanswered, the score is 2. For each incorrect response, the score is 0. If there are 5 choices for each question, what is the minimum number of choices that the student must eliminate before it is advantageous to guess among the rest?
   A. 0
   B. 1
   C. 2
   D. 3
   E. 4

2. A company that makes fleece clothing uses fleece produced from two farms, Northern Farm and Western Farm. Let the random variable X represent the weight of fleece produced by a sheep from Northern Farm. The distribution of X has mean 14.1 pounds and standard deviation 1.3 pounds. Let the random variable Y represent the weight of fleece produced by a sheep from Western Farm. The distribution of Y has mean 6.7 pounds and standard deviation 0.5 pounds. Assume X and Y are independent. Let W equal the total weight of fleece from 10 randomly selected sheep from Northern Farm and 15 randomly selected sheep from Western Farm. Which of the following is the standard deviation, in pounds, of W?

   First, explain why the answer isn't D. Then show and explain how to get the correct answer.
   A. $1.3 + 0.5$
   B. $\sqrt{1.3^2 + 0.5^2}$
   C. $\sqrt{10(1.3)^2 + 15(0.5)^2}$
   D. $\sqrt{10^2(1.3)^2 + 15^2(0.5)^2}$
   E. $\sqrt{\frac{1.3^2}{10} + \frac{0.5^2}{15}}$

3. A shopping mall has three automated teller machines (ATMs). Because the machines receive heavy use, they sometimes stop working and need to be repaired. Let the random variable *X* represent the number of ATMs that are working when the mall opens on a randomly selected day. The table shows the probability distribution of *X*.

Number of ATMs working when the mall opens	**0**	**1**	**2**	**3**
Probability	**0.15**	**0.21**	**0.40**	**0.24**

   a. What is the probability that at least one ATM is working when the mall opens?

   b. What is the expected value of the number of ATMs that are working when the mall opens?

   c. What is the probability that all three ATMs are working when the mall opens, given that at least one ATM is working?

   d. Given that at least one ATM is working when the mall opens, would the expected value of the number of ATMs that are working be less than, equal to, or greater than the expected value from part (b)? Explain.

4. A department supervisor is considering purchasing one of two comparable photocopy machines, *A* or *B.* Machine *A* costs $10,000 and machine *B* costs $10,500. This department replaces photocopy machines every three years. The repair contract for machine *A* costs $50 per month and covers an unlimited number of repairs. The repair contract for machine *B* costs $200 per repair. Based on past performance, the distribution of the number of repairs needed over any one-year period for machine *B* is shown below.

Number of Repairs	**0**	**1**	**2**	**3**
Probability	**0.50**	**0.25**	**0.15**	**0.10**

You are asked to give a recommendation based on overall cost as to which machine, *A* or *B*, along with its repair contract, should be purchased. What would your recommendation be? Give a statistical justification to support your recommendation.

Answers Check Your Understanding

Answers Check Your Understanding

Number One: C

X = # of points as a result of guessing

When there are 5 questions

	Correct	Incorrect	Skip
x	7	0	2
$P(X = x)$	1/5	4/5	

$E(X_5) = 7(\frac{1}{5}) + 0(\frac{4}{5}) = 1.4$ points ⟶ Not worth it. If you guess when there are 5 questions, you have an expected point value of 1.4 which is less than 2 if you skip it.

When there are 4 questions

	Correct	Incorrect	Skip
x	7	0	2
$P(X = x)$	1/4	3/4	

$E(X_5) = 7(\frac{1}{4}) + 0(\frac{3}{4}) = 1.75$ points ⟶ Not worth it. If you guess when there are 4 questions, you have an expected point value of 1.75 which is less than 2 if you skip it.

When there are 3 questions

	Correct	Incorrect	Skip
x	7	0	2
$P(X = x)$	1/3	2/3	

$E(X_5) = 7(\frac{1}{3}) + 0(\frac{2}{3}) = 2.33$ points ⟶ **WORTH IT.** If you guess when there are 3 questions, you have an expected point value of 2.33 which is more than 2 if you skip it. So eliminate 2 answers before you start guessing.

Number 2: C

D is wrong because that would be showing the transformation of multiplying by 10 and 15 to the respective farms. That would be like taking one sheep and making it 10 or 15 times bigger than it actually is, then sheering the fleece.

X = Fleece from Northern Farm $E(X) = 14.1$ lbs $SD(X) = 1.3$ lbs	Y = Fleece from Western Farm $E(Y) = 6.7$ lbs $SD(Y) = 0.5$ lbs

W = 10 sheep from N Farm and 15 sheep from W Farm

This is like a 25 sided triangle that I have to find the hypotenuse of.

$Var(W) = SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2 + SD(X)^2$...

$+ SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2 + SD(Y)^2$

$Var(W) = 10 \cdot SD(X)^2 + 15 \cdot SD(Y)^2$

$\sqrt{Var(W)} = \sqrt{10 \cdot SD(X)^2 + 15 \cdot SD(Y)^2}$

$SD(W) = \sqrt{10 \cdot SD(X)^2 + 15 \cdot SD(Y)^2}$

$SD(W) = \sqrt{10(1.3)^2 + 15(0.5)^2}$

Number 3

INTENT OF QUESTION

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a discrete random variable; (2) calculate the expected value of a discrete random variable; (3) perform a conditional probability calculation from a discrete random variable; and (4) use probabilistic thinking to make a prediction about how an expected value will change given a condition about the random variable.

SOLUTION

Part a.:

The probability that at least one ATM is working when the mall opens is:

$P(X \geq 1) = 0.21 + 0.40 + 0.24 = 0.85$

Part b.:

The expected value of the number of ATMs that are working when the mall opens is:

$E(X) = 0(0.15) + 1(0.21) + 2(0.40) + 3(0.24) = 1.73$ machines.

Part c.:

The probability that all three ATMs are working when the mall opens, given that at least one ATM is working is:

$$P(X = 3 | X \geq 1) = \frac{P(X=3 \text{ and } X \geq 1)}{P(X \geq 1)} = \frac{P(X=3)}{P(X \geq 1)} = \frac{0.24}{0.85} \approx 0.282$$

Part d.:

Given that at least one ATM is working when the mall opens, the expected value of the number of working ATMs should be greater than the expected value calculated in part (b). By eliminating the possibility of 0 working ATMs, the probabilities for 1, 2, and 3 working ATMs all increase proportionally, so the expected value must increase.

Number 4

Machine A Cost = $10,000 Three year contract (36 months) $50 per month = $1800 service fee Total = **$11,800**
Machine B Cost = $10,500 $200 per repair *Expected Repairs* E(B) = 0(0.5)+1(0.25)+2(0.15)+3(0.10) = 0.85 repairs per year Over three years: 3*E(B) = 2.55 repairs over 3 years @ $200 each = $510 Total = 510+10,500 = **$11,010**

Get machine B because it is cheaper.