ELMWOOD PARK PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

ALGEBRA II

Grades 10, 11, & 12

Prerequisites: Algebra I, Geometry

5 credits

ABSTRACT

This full-year course builds upon the concepts learned in Algebra I and Geometry. It develops advanced mathematical abilities and strengthens higher order thinking skills. The course emphasizes applications in business, science and engineering. Topics such as logarithms and radical equations are explored; basic concepts of trigonometry, probability and statistics are introduced. The major topics include: modeling functions with linear relationships; examining and solving quadratic equations and functions, polynomial functions, exponential functions, logarithmic functions, trigonometric and rational functions; calculating probability and statistics, and evaluating arithmetic and geometric sequences and series.

This course incorporates technology and other tools to pursue mathematical investigations, and to enhance, deepen, and extend understanding of mathematics. Students learn how to use technology as a tool for processing information, visualizing and solving problems, exploring and testing conjectures, accessing data, and verifying their solutions. The use of graphing calculators is mandatory and computer laboratory time is assigned. This course is a prerequisite for *Pre-Calculus*. In preparation for the state assessments, students will be given formative and summative assessments throughout the course.

UNIT #: *Unit Title* Number of Days	**Unit 1:** *Linear Functions and Systems* 17 Days	**Unit 2:** *Quadratic Functions and Equations* 20 Days	**Unit 3:** *Polynomial Functions* 18 Days
STAGE 1: DESIRED RESULTS ***What will students understand as a result of the unit? What are the BIG ideas?***			
ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Algebra** A-CED.A.1 A-CED.A.2 A-CED.A.3 A-REI.C.6 A-REI.D.11 **Functions** F-BF.A.2 F-BF.B.3 F-IF.A.3 F-IF.B.5 F-IF.B.4 F-IF.C.7 F-IF.C.7b F-IF.B.6 F-LE.A.2 **Statistics and Probability** S-ID.B.6.a **Mathematical Practices** MP.1 MP.2 MP.3	**Algebra** A-APR.B.3 A-CED.A.2 A-REI.B.4 (a-b) A-REI.C.7 A-REI.D.11 A-SSE.A.2 A-SSE.B.3.a **Function** F-BF.B.3 F-BF.A.1a F-IF.B.4 **Number and Quantity** N-CN.A.1 N-CN.A.2 N-CN.A.3(+) N-CN.C.7 **Statistics and Probability** S-ID.B.6a **Mathematical Practices**	**Algebra** A-APR.A.1 A-APR.B.2 A-APR.B.3 A-APR.C.4 A-APR.C.5(+) A-APR.D.6 A-SSE.A.2 **Functions** F-BF.A.1.b F-BF.B.3 F-IF.B.4 F-IF.B.6 F-IF.C.7.C F-IF.C.9 **Number and Quantity** N-CN.C.8(+) N-CN.C.9(+) **Mathematical Practices** MP.1 MP.2 MP.3

	MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3 9.2.12.CAP.4 9.2.12.CAP.6 9.2.12.CAP.7 9.4.12.CT.2	MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3 9.2.12.CAP.4 9.2.12.CAP.6 9.2.12.CAP.7 9.4.12.CI.2 9.4.12.CI.3	MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3 9.2.12.CAP.4 9.2.12.CAP.6 9.2.12.CAP.7 9.4.12.CI.3 9.4.12.CT.2
ENDURING UNDERSTANDINGS: *(Students will understand that . . .)*	•The key features of a graph - including the domain, range and intercepts - reveal the relationship between two quantities. •A function of the form f(x)=a(f[b(x-h)]+k is transformed by changing the values of a, b, h, or k. •A piecewise-defined function is used to	•All quadratic functions are transformations of the parent function f(x)=x^2. The vertex form of a quadratic function highlights the key features of the function's graph and shows how the graph of the parent function can be transformed. •A quadratic function in vertex form	•A polynomial function is a function whose rule is either a monomial or a sum of monomials. The key features of the graph of a polynomial function - such as its end behavior, intercepts, and turning points - can be used to sketch a graph of the function. •Just as with real numbers, the

	model situations in which there are different rules for different parts of the domain of the function. •An arithmetic sequence is a sequence of numbers in which the terms have a common difference. An arithmetic series is the sum of the terms in a finite arithmetic sequence and can be found using an explicit definition for the sum. •To solve an equation or inequality by graphing, set each expression equal to y and graph the two equations on the same grid. Their intersection represents the solution. •The solution of a system of linear equations or inequalities is the set of ordered pairs that satisfy all the equations or inequalities in the system. Systems of equations or inequalities can also be represented by a matrix. •Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. •A matrix can be used to represent a system of linear equations. Row operations can be applied to the matrix to convert it to the identity matrix with an additional column that indicates the solution of the original system of equations	can be rewritten in standard form to highlight different features of the function's graph. The key features are used to interpret values in context. •The factored form of a quadratic function is used to find zeros of the function by identifying the values that make one or both of the factors equal to zero. •The complex numbers contain both real and imaginary parts. The four basic operations can be applied to complex numbers •Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. •A quadratic equation can be solved by completing the square by completing the square to transform the equation to an equivalent equation, (x-p)^2=q. •The Quadratic Formula can be used to solve any quadratic equation, including those with complex solutions. •A linear-quadratic system consists of a linear equation and a quadratic	properties of operations can be used to add, subtract, and multiply polynomials. polynomial functions can be used to represent and compare real-world situations. •Polynomial identities and the Binomial Theorem are helpful tools for efficiently rewriting expressions and describing mathematical relationships. •Polynomial expressions can be divided by linear functions using long division or synthetic division. The Remainder Theorem is used to determine the remainder of a division problem. •The zeros of a polynomial function can be determined using factoring or synthetic division. The zeros of a function can be used to sketch its graph. •Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. •Theorems such as the Rational Root Theorem, the Fundamental Theorem of Algebra, and the Conjugate Root Theorems are helpful tools for determining the roots of a polynomial

		equation. The points of intersection are the solutions.	function. •Polynomial functions are categorized as even, odd, or neigher. Even functions are symmetric about the y-axis and for all x in the domain f(x)=f(-x). Odd functions are symmetric about the origin, and for all x in the domain, f(-x)=-f(x).
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	•How do graphs and equations reveal information about a relationship between two quantities? •What do the differences between the equation of a function and the equation of its parent function tell you about the differences in the graphs of the two functions? •How do you model a situation in which a function behaves differently over different parts of its domain? •What is an arithmetic sequence, and how do you represent and find its terms and their sums? •How can you solve an equation or inequality by graphing? •How can you find and represent solutions of systems of linear equations and inequalities? •How can matrix row operations be used to solve a system of linear equations?	•How does the equation of a quadratic function in vertex form highlight key features of the function's graph? •What key features can you determine about a quadratic function from an equation in standard form? •How is the factored form helpful in solving quadratic equations? •How can you represent and operate on numbers that are not on the real number line? •How can you solve a quadratic equation by completing the square? •How can you use the Quadratic Formula to solve quadratic equations or to predict the nature of their solutions? •How can you solve a system of two equations or inequalities in which one is linear and one is quadratic?	•How do the key features of polynomial functions help you sketch its graph? •How do you add, subtract, and multiply polynomials? •How can you use polynomial identities to rewrite expressions efficiently? •How can you divide polynomials? •How are the zeros of a polynomial function related to the equation and graph of a function? •How are the roots of polynomial equations related to the coefficients and degree of the polynomial? •How are symmetry and transformation represented in the graph and equation of a polynomial function?

STAGE 2: ASSESSMENT EVIDENCE ***What evidence will be collected to determine whether or not the understandings have been developed, the knowledge and skills attained, and the state standards met? [Anchor the work in performance tasks that involve application, supplemented as needed by prompted work, quizzes, observations, etc.]***			
PERFORMANCE TASKS: *(Through what authentic performance tasks will students demonstrate the desired understandings?)* *(By what criteria will performances of understanding be judged?)*	•Make sense of problems and persevere in solving them. •Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities •Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments	•Make sense of problems and persevere in solving them. •Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities •Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments	•Make sense of problems and persevere in solving them. •Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities •Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments

	•Written Responses on real-world problems and non-routine problems. •STEM Project: Students will analyze historical data on fuel efficiency for cars. Interested students could research other types of vehicles, such as minivans or trucks. •3 ACT Math: "Current Events" Students analyze the current used by pairs of power tools. They need to determine whether the circuit can handle all three power tools.	•Written Responses on real-world problems and non-routine problems. •STEM Project: Students will learn how the design of a ballpark influences the number and frequency of home runs. •3 ACT Math: "Swift Kick" Students analyze partial views of soccer shots to determine whether the attempts are successful. They need to determine whether the ball's path will pass through the goal. •Felix wants to make a graphic design as decoration for a new website. His initial idea for the design involves using the shapes of three parabolas graphed on a coordinate plane. Students will need to graph and write an equation in vertex form for each parabola.	•Written Responses on real-world problems and non-routine problems. •STEM Project: Students learn how to use a fitted curve to predict population growth. •3 ACT Math: "What Are the Rules?" Students will explore and apply concepts related to polynomial functions and equations. Students will analyze a situation in which a polynomial function affects numbers differently, and they will be tasked with determining why. •Arthur wants to make a raised rectangular frame to grow basil plants. The basil plants are transplanted, with their soil, from 2-in. wide pots into the frame. The diagram will show a top-down view of the frame and each circle will represent a transplanted basil plant with its soil. Arthur will add more soil to the frame until the soil is $3x$ in. deep. Students will write a polynomial function V to represent the volume of the soil in the frame in terms of x. Students will also predict the end behavior of the graph.
OTHER EVIDENCE: *(Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the*	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Class Participation/Discussions •Teacher Observations	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Caass Participation/Discussions •Teacher Observations	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Caass Participation/Discussions •Teacher Observations

desired results?) *(How will students self-assess their learning?)*	•Error Analysis •Scoring Rubrics •Peer and Self Evaluation •Benchmark Assessments	•Error Analysis •Scoring Rubrics •Peer and Self Evaluation •Benchmark Assessments	•Error Analysis •Scoring Rubrics •Peer and Self Evaluation •Benchmark Assessments
RESOURCES:	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets
STAGE 3: LEARNING PLAN ***What learning experiences and instruction will enable students to achieve the desired results? Utilize the WHERETO* acronym to consider key design elements.***			
SKILLS AND TOPICS: *(What specific activities will students do and what skills will students know as a result of the unit?)*	•Interpret slope and construct equations using slope. •Describe and apply properties of rational and real numbers. •Describe and apply properties of exponents. •Describe the function and apply effects of transformations, including $f(x-h)$ and $f(x) + k$. •Determine the composition of functions	•Find the maximum and minimum value of a function by algebraic and graphical means. •Students investigate the characteristics of quadratic functions. •Use quadratic equations and mathematical models to solve word problems. •Translate word phrases into algebraic expressions, equations and inequalities. •Solve linear-quadratic systems of	•Write a polynomial function given its real and/or complex zeros. •Analyze characteristics of polynomials including end behavior, symmetry, zeros, multiplicity, increasing and decreasing, domain and range. •State the domain and range using compound inequalities and by interval notation. •Classify, add and subtract polynomials.

	•Students investigate the characteristics of linear functions. •Use equations and mathematical models to solve word problems. •Translate word phrases into Algebraic expressions, equations and inequalities. •Solve compound inequalities using algebraic and graphing means •Create and evaluate arithmetic sequences and series. •Solve linear systems of equations algebraically, graphically, and through matrices.. •Use matrices to model scenarios and to evaluate problems using matrix methods.	equations algebraically and graphically. •Factor a quadratic and solve quadratic equations by graphing. •Translate between vertex form, factored form and standard form of quadratic functions. •Rewrite and solve quadratic equations by completing the square. •Complete operations such as add, subtract, multiply, and divide complex and imaginary numbers. •Simplify expressions over the complex plane. •Model any given quadratic function on the coordinate plane in factored form, standard form, or vertex form. •Identify transformations of quadratic functions and model them on the coordinate plane.	•Factor polynomials and solve them. •Model polynomial functions on the coordinate plane from standard form and factored form. •Create equations given characteristics of the graph of a polynomial function. •Prove polynomial identities using algebraic methods. •Divide polynomial function using polynomial long division and synthetic division. •Determine the zeros of a polynomial function when given the graph, characteristics, standard form, or factored form of the function. •Determine zeros of a polynomial function using the Rational Root/Zero Theorem.
CROSS-CURRICULAR / DIFFERENTIATION: *(What cross-curricular (e.g. writing, literacy, math, science, history, technology) learning activities are included in this unit that will help achieve the desired*	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)**	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)**	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)**

results?) *(What type of differentiated instruction will be used for ELL, SP.ED. and G&T students?)*	Current event problems Exploratory Activities Project-Based Learning Integrate Technology: • Textbook online resources at www.savvasrealize.com (assessments, learning tools) •Chromebook (for above, and use of other apps such as Socrative and YouTube, as appropriate) Special Education/504: •Some students may struggle with writing a rule for an absolute value function. Have students practice using a graph to write a rule for an absolute value function. •Provide modifications and accommodations as listed in the student's IEP/504 plan • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulatives • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies	Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) •Chromebook (for above, and use of other apps such as Socrative and YouTube, as appropriate) Special Education/504: • Students may have difficulty factoring quadratic expressions that have a leading coefficient other than 1. Have students practice multiplying binomial expressions. This allows them to work through the princess backward and see the connection to the factored form. • Provide modifications and accommodations as listed in the student's IEP/504 plan • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulatives • Use graphic organizers • Use interactive math journals • Use online resources for skill building	Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) •Chromebook (for above, and use of other apps such as Socrative and YouTube, as appropriate) Special Education/504: • Some students may struggle with interpreting verbal descriptions, as well as with interval notation. Teacher will use descriptions to take a closer look at how to interpret the actual shape of the graph. • Provide modifications and accommodations as listed in the student's IEP • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulates • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies

	such as small groups • Provide pictorials for types of functions and equations • Provide pictorials for transformations, with visible examples of how each transformation affects a function <u>English Language Learners</u>: • Ask students to tell what a reflection is. Then give each student a mirror and have them draw the reflections of different lines and shapes on a coordinate grid - some across the *x*-axis and some across the *y*-axis. Have them describe their reflections. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Provide a resource page in different colors to show how different types of functions correspond to different graphs • NJDOE resources • Adapt a strategy adjusting strategy for ELL: <u>http://www.teachersfirst.com/content/esl/adaptstrat.cfm</u> • Instruction will be based on language proficiency. <u>At Risk Students</u>:	• Provide teacher notes • Use collaborative grouping strategies such as small groups • Use online resources • NJDOE resources • Solving by square roots will be simpler problems, incorporating 1 extra step • Solving with quadratic formula will reinforce solving with real solutions, and slowly incorporate complex solutions <u>English Language Learners</u>: • Have students discuss the meanings of the terms *stretch* and *compress*. Ask half the students to do a morning stretch, spreading their limbs out as wide as possible. Ask the other to bend down and wrap their hands around their knees, compressing their bodies as much as possible. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Notes will be translated, and will include visual examples • NJDOE resources • Adapt a strategy adjusting strategy for ELL: <u>http://www.teachersfirst.com/content/esl/adaptstrat.cfm</u>	such as small groups • When adding/subtracting, polynomials will be given in degree order so it is easier for students to identify common terms • When multiplying, polynomials will be no more than (binomial)(trinomial), but will focus on (binomial)(binomial) • The first 3 rows of Binomial Theorem will be given so students can identify the pattern • Expanding through Binomial Theorem will incorporate more variables and less constants • Finding rational/complex solutions, problems will be limited to three factors <u>English Language Learners</u>: • Have students underline phrases with negatives, such as *not*, *no*, and *neither*. Have students explain those phrases in positive terms. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Modification plan • Notes will be translated using simpler, more direct wording to support the notes in the lesson

	• Students should practice writing series in sigma notation. •Students may need help to recognize and use the graph of an inequality containing an absolute value symbol to find the solutions. • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources • Color-coordinated notes showing the connection between graphs and function, as well as for transformations • Matching worksheets to assess understanding of different functions and the graphs they produce • NJDOE resources • Utilize online resources such as www.khanacademy.org Gifted and Talented Students: • Students use what they know about writing an equation from a graph to practice writing an equation from context. Another train ride is available that lasts 45 minutes and travels a total distance of 25 km. • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified:	• Instruction will be based on language proficiency. At Risk Students: • Some students may struggle with reflecting points across the *x*-axis. Students will practice reflecting points across the *x*- and *y*-axes. • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources • Multiple practice problems will be used to reinforce the fact that the square root of an answer can be positive or negative • Solving quadratic problems will focus on identifying the a, b, and c values in the function, and correctly plugging them into the quadratic formula • NJDOE resources • Utilize online resources such as www.khanacademy.org Gifted and Talented Students: • Have students identify the key features of quadratic functions when the coordinates of the vertex are fractions. • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified:	• An example problem will be given so students can see what the correct process is to solve their own problems • NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm • Instruction will be based on language proficiency. At Risk Students: • Students may need to practice the step-by-step process of expanding expressions using the Binomial Theorem and Pascal's Triangle. • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources • One- and two-step problems will be given to reinforce basic concepts in addition, subtraction, and multiplication • Rational functions will focus on identifying the factors given zeros • NJDOE resources • Utilize online resources such as www.khanacademy.org Gifted and Talented Students: • Students may find it interesting that the leading coefficient and its degree have such a significant effect on the graph of functions. Use polynomial functions and a table to determine how the leading coefficient and its degree control the end

	abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Absolute Value functions will involve more transformations as well as factoring methods to identify transformations • Use of web based resources such as www.khanacademy.org • NJDOE resources	abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Students will be given more difficult problems involving complex numbers, including powers above 4 • Complex problems to solve using square roots (division, multiplication, etc.) • Solving quadratics will mainly be focused on those with complex solutions only • Use of web based resources such as www.khanacademy.org • NJDOE resources	behavior of the graphs and why their effects are important. • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified: abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Polynomial problems will be given in the form of word problems, assessing the student's ability to interpret information from the word problem itself. • Additional worksheets will be given to those who finish the notes/practice early, and they will be able to work on these in their own group • Use of web based resources such as www.khanacademy.org • NJDOE resources

***WHERETO**

W = Help the students know **WHERE** the unit is going and **WHAT** is expected. Help the teacher know **WHERE** the students are coming from (prior knowledge, interests).

H = **HOOK** all students and **HOLD** their interest.

E = **EQUIP** students, help them **EXPERIENCE** the key ideas and **EXPLORE** the issue.
R = Provide opportunities to **RETHINK** and **REVISE** their understanding and work.
E = Allow students to **EVALUATE** their work and its implications.
T = **TAILORED** to the different needs, interests, and abilities of learners.
O = **ORGANIZE** to maximize initial and sustained engagement as well as effective learning.

UNIT #: *Unit Title* Number of Days	**Unit 4:** *Rational and Radical Functions and Equations* 30 Days	**Unit 5:** *Exponential and Logarithmic Functions* 15 Days	**Unit 6:** *Trigonometric Functions* 25 Days
STAGE 1: DESIRED RESULTS ***What will students understand as a result of the unit? What are the BIG ideas?***			
ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Algebra** A-APR.D.6 A-APR.D.7(+) A-CED.A.1 A-CED.A.2 A-CED.A.4 A-REI.A.1 A-REI.A.2 A-REI.B.3 A-REI.D.11 A-SSE.A.1 A-SSE.A.2 **Functions** F-BF.A.1.b F-BF.A.1.c(+) F-BF.B.3	**Algebra** A-CED.A.1 A-REI.A.1 A-SSE.A.1.b A-SSE.A.2 A-SSE.B.3.c A-SSE.B.4 **Functions** F-BF.A.1 F-BF.A.1.a F-BF.A.2 F-BF.B.3 F-BF.B.4 F-BF.B.4.a F-BF.4c(+) F-BF.B.5(+)	**Functions** F-BF.B.3 F-IF.B4 F-IF.B.6 F-IF.C.7.E F-IF.C.9 F-TF.A.1 F-TF.A.2 F-TF.A.3(+) F-TF.B.5 F-TF.C.8 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4

	F-BF.B.4.a F-BF.B.4.b(+) F-BF.B.4.c(+) F-BF.B.4.d(+) F-IF.B.4 F-IF.C.7.B F-IF.C.7.D(+) **Number and Quantity** N-RN.A.1 N-RN.A.2 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3 9.2.12.CAP.4 9.2.12.CAP.6	F-IF.B.3 F-IF.B.4 F-IF.B.5 F-IF.B.6 F-IF.C.7.E F-IF.C.8.B F-IF.C.9 F-LE.A.2 F-LE.A.4 F-LE.B.5 **Statistics and Probability** S-ID.B.6.a **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3	MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3 9.2.12.CAP.4 9.2.12.CAP.7 9.4.12.CT.2

	9.2.12.CAP.7 9.4.12.CT.2 9.4.12.CI.3	9.2.12.CAP.4 9.2.12.CAP.6 9.2.12.CAP.7 9.4.12.CT.1 9.4.12.GCA.1	
ENDURING UNDERSTANDINGS: *(Students will understand that . . .)*	•The reciprocal function is used to model inverse variation, which is a proportional relationship between two variables such that when one variable increases, the other decreases. •A rational function is any function R(x)=P(x)/Q(x) where P(x) and Q(x) are polynomial functions. The domain of a rational function is all real numbers except any x-values for which Q(x) equals zero. The graph of a rational function has one or more asymptotes, which guide the end behavior of the graph. •Rational expressions form a system similar to the system or rational numbers and can be multiplied and divided by applying the properties of operations as they apply to the rational expressions. •The properties of operations used to add and subtract rational numbers can be applied to adding and subtracting rational expressions. •Rational equations contain a rational	•The rate of exponential growth or decay is the ratio between two consecutive output values in an exponential function. •Exponential models are useful for representing situations in which the rate increases by the same percent for each period of time and for interpreting problems that involve compound interest. Exponential regression can be used to generate exponential models for real-world contexts. •A logarithmic function is the inverse is the inverse of an exponential function. Logarithms are found by determining the exponent that must be applied to a base to yield a given result. •The inverse relationship between exponential and logarithmic functions reveals key features of the graphs of both functions. Logarithmic functions can be used to model several real-world situations. •Properties of Logarithms can be	•For any right triangle, the ratios of the side are always the same for a given angle theta. These ratios define the six basic trigonometric functions: since, cosine, secant, cosecant, tangent, and cotangent. •An angle in standard position has a vertex at the origin and an initial side along the positive x-axis. Reference angles on the unit circle are used to extend trigonometric ratios to angles greater than 90 degrees. •Reference angles are reference triangles used to find and evaluate the six trigonometric functions on the unit circle. •Periodic functions are functions that repeat a pattern of y-values at regular intervals. The sine function y=asinbx and the cosine function y-acosbx are periodic functions that have an amplitude of a and a frequency of b/2pi. •Many real-world problem situations can be represented with a mathematical model, but that model

	expression and can be solved by multiplying each side of the equation by a common denominator to eliminate the fractions. Any solution that is excluded from the domain of the original equation is extraneous. •Many real-world problem situations can be represented with a mathematical model, but the model might represent the real-world situation exactly. •Rational exponents and radicals represent the number of roots a polynomial has. The roots of a polynomial are used to simplify expressions and solve equations. •The properties of integer exponents can be applied to terms with rational exponents, as well as to radicals. The properties of exponents and radicals can be used to rewrite radical expressions. When rewriting radical expressions, like radicals, which have the same index, can be added and subtracted. •The function g(x)=anthroot(x-h)+k represents the transformation of the parent radical function f(x)=nthroot(x), where a stretches or compresses the graph vertically, h translates the graph horizontally, and k translates the graph vertically.	used to rewrite logarithmic expressions and to evaluate logarithms by changing the base. •Some exponential equations can be solved by rewriting both sides with a common base. For others, rewriting the equation using logarithms and applying properties of logarithms, is a more efficient method. •A geometric sequence is a sequence of numbers in which terms are related to the previous term by a common ratio, r. A geometric series is a sum of a certain number of terms in a geometric sequence.	might not represent the real-world situation exactly. •The tangent, cotangent, secant, and cosecant functions can be defined using ratios or reciprocals of the sine and cosine functions. The graphs of these functions are periodic, have no amplitude, and have vertical asymptotes. •The parameters a, b, c and d represent values that transform the functions y=asinb(x-c)+d and y=acosb(x-c)+d. Absolute value of a represents the amplitude of the function. The period of the function is 2pi/b. c represents a horizontal shift, called a phase shift; and d represents a vertical shift, called a midline shift.

	•Solving equations that include radicals or rational exponents is similar to solving rational equations. •Functions can be combined by operations (+,-,x, dividing) and by composition. The result of the operation or composition can be described as a single function. The domain of the result may be different from the domains of the original functions. •The inverse of a function is found by exchanging the roles of the independent and dependent variables. Composition can be used to verify that two functions are inverses.		
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	•How are inverse variation and reciprocal functions related and represented? •How can you graph a rational function? •How does understanding operations with fractions help you multiply and divide rational expressions? •How do you rewrite rational expressions to find sums and differences? •How can you solve rational equations and identify extraneous	•How do graphs and equations reveal key features of exponential growth and decay functions? •How can you develop exponential models to represent and interpret situations? •What are logarithms and how are they evaluated? •How is the relationship between logarithmic and exponential functions revealed in the key features of their graphs? •How are the properties of logarithms	•How can ratios of lengths of sides within right triangles help determine other lengths and angle measures in the triangles? •How can we extend the trigonometric ratios to angles greater than 90 degrees? •How is the unit circle related to trigonometric functions? •How can you identify key features of sine and cosine functions? •How do key features of one trigonometric function relate to key

	solutions? •How are exponents and radicals used to represent roots of real numbers? •How can properties of exponents and radicals be used to rewrite radical expressions? •How can you use what you know about transformations of function to graph radical functions? •How can you solve equations that include radicals or rational exponents? •How do you combine, multiply, divide, and compose functions, and how do you find the domain of the resulting function? •How can you find the inverse of a function and verify the two functions are inverses?	used to simplify expressions and solve logarithmic equations? •How do properties of exponents and logarithms help you solve equations? •How can you represent and use geometric sequences and series? • How is climate change affecting the planet right now? •How does the availability of natural resources, occurrence of natural hazards, and climate change influence human activity?	features of other trigonometric functions? •How can you find and use translations of graphs of trigonometric functions?
STAGE 2: ASSESSMENT EVIDENCE ***What evidence will be collected to determine whether or not the understandings have been developed, the knowledge and skills attained, and the state standards met? [Anchor the work in performance tasks that involve application, supplemented as needed by prompted work, quizzes, observations, etc.]***			
PERFORMANCE TASKS: *(Through what authentic*	•Make sense of problems and persevere in solving them.	•Make sense of problems and persevere in solving them.	•Make sense of problems and persevere in solving them.

performance tasks will students demonstrate the desired understandings?) *(By what criteria will performances of understanding be judged?)*	•Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities •Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments •Written Responses on real-world problems and non-routine problems. •STEM Project: For businesses to calculate whether producing an item is profitable, they must determine how much each item costs to produce and how much it will sell for. In this project, students will learn about fixed and variable costs and how they relate to the	•Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities •Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments •Written Responses on real-world problems and non-routine problems. •STEM Project: Students will learn when and how to use Benford's Law to analyze data. •3 ACT Math: "The Crazy Conditioning" Students will explore and apply concepts related to exponential equations and	•Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities •Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments •Written Responses on real-world problems and non-routine problems. •STEM Project: Students will learn that visible light is only part of the electromagnetic spectrum and how invisible rays affect humans. •3 ACT Math: "What Note Was That?" Students will explore and apply concepts

	profitability of a business. •3 ACT Math: "Real Cool Waters" Students will explore and apply concepts related to rational functions and equations. Students will analyze a situation in which two hoses fill up a pool at different rates. They will be tasked with determining a third, combined rate. •STEM Project: Students will investigate different ways to tune a piano, select a musical piece, and decide what tuning sounds best for it. •3 ACT Math: "The Snack Shack" Students will explore and apply concepts related to radical expressions. Students will analyze a situation that involves finding the quickest way to a destination. They will be tasked with writing a mathematical model for each person and then simplifying radical expressions to answer the question posed. •Students readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow	functions. Students will watch an athlete performing a running drill. They will be tasked with extrapolating both the time and distance for a certain round of the drill.	related to trigonometric functions. Students will see and hear a musician playing a short piece of music. When the final note sounds, a wave representing that note appears. Students will be tasked with using information about this representation of the musical note to determine which note it is. •The first Ferris wheel was designed and built by an American engineer named George W. G. Ferris in 1893. The radius of the wheel was 125 feet. A Ferris wheel modeled on the original design takes 20 seconds to complete one revolution counterclockwise. Suppose a Ferris wheel is tested for functionality. Two engineers are in non-adjacent careers where the central angle between the cars is $\frac{x}{6}$ radians. Students will find the arc length between the cars of engineers.

	through to ensure the problem is solved, whether through their own actions or the actions of others.		
OTHER EVIDENCE: *(Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired results?) (How will students self-assess their learning?)*	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Class Participation/Discussions •Teacher Observations •Error Analysis •Scoring Rubrics •Peer and Self Evaluations	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Class Participation/Discussions •Teacher Observations •Error Analysis •Scoring Rubrics •Peer and Self Evaluations	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Class Participation/Discussions •Teacher Observations •Error Analysis •Scoring Rubrics •Peer and Self Evaluations
RESOURCES:	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets
STAGE 3: LEARNING PLAN ***What learning experiences and instruction will enable students to achieve the desired results? Utilize the WHERETO* acronym to consider key design elements.***			
SKILLS AND TOPICS: *(What specific activities will students do and what skills will*	•Simplify an expression written with fractional exponents. •Simplify rational expressions using	•Create and model growth and decay functions on the coordinate plane. •Identify key features of exponential	•Prove the Pythagoren Identity and use it to confirm angles. •Given the equation $y = asinb\theta$ of a

students know as a result of the unit?)	common factors to reduce. •Students will be graphing square root and cube root functions graphically and with the use of computer software and/or graphing calculators. •Students will recognize rational exponents and radicals are both ways to represent roots of quantities. •The same properties of exponents can be used for both rational exponents and integer exponents. •Solve simple rational and radical equations in one variable. •Graph Rational Functions on the coordinate plane with all characteristics. •Identify characteristics of rational functions and models on the coordinate plane. •Differentiate between solutions and extraneous solutions when solving radicals and rational functions.	functions from the coordinate plane and identify their transformations from the function. •Solve application questions involving real-world scenarios using compound interest, exponential growth and exponential decay. •Solve exponential equations using algebraic and graphical means. •Write and simplify logarithmic equations and expressions using the laws of logarithms including the change-of-base formula. •Understand what a natural log and common log is and when to use them. •Prove expressions are equivalent using properties of logarithmic equations. •Prove extraneous solutions exist for logarithmic equations. •Logarithmic equations are the inverse of exponential functions which can be used to solve both logarithmic and exponential equations.	trig function, e.g. determine the amplitude and period. •Re-create the unit circle with the correct ordered pairs in every quadrant. •Use special right triangles to explore the creation of the Unit Circle. •Graph multiple periods of any given basic trigonometric function on the coordinate plane. •Identify positive and negative coterminal angles for any given measure in radians or degrees. •Convert between radian measure and degree measure. •Evaluate trigonometric functions using a calculator for approximate answers and the unit circle when possible for exact answers.
CROSS-CURRICULAR: *(What cross-curricular (e.g.*	**English Language Arts (RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate	**English Language Arts (RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate	**English Language Arts (RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate

writing, literacy, math, science, history, technology) learning activities are included in this unit that will help achieve the desired results?)	and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) •Chromebook (for above, and use of other apps such as Socrative and YouTube, as appropriate) Special Education/504: • Students may need practice simplifying expressions with *n*th roots. Have students practice simplifying expressions with square roots. • Students may have trouble rewriting a radical expression in reduced radical form. Examples will be provided for students. • Provide modifications and accommodations as listed in the student's IEP • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery	and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Science** **(HS-ESS3-6)** Use computational representation to illustrate the relationships among Earth systems and how those relationships are modified due to human activity (i.e, Climate Change). **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) •Chromebook (for above, and use of other apps such as Socrative and YouTube, as appropriate) Special Education/504: • Some students may have difficulty understanding how thinking of a logarithmic expression in terms of an exponential equation can help them evaluate logarithms. Speak with students about what *evaluate* means. • Provide modifications and accommodations as listed in the student's	and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) • Chromebook (for above, and use of other apps such as Socrative and YouTube, as appropriate) Special Education/504: • Students may struggle with finding the average rate of change of a sine or cosine function over an interval. Students will practice finding the average rate of change of other functions over a given interval. • Provide modifications and accommodations as listed in the student's IEP • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you

	• Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulates • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies such as small groups • When graphing, students will be guided to identifying x- and y- values. • When simplifying, any problems involving fractions will have fractions for all terms, even if the denominator is 1 • Solving equations will include common denominators, or a common denominator will be easily distinguishable • Exponents on radical expressions will be placed outside parentheses so it is easily identifiable • Students will be given a table and guided on how to complete the table to find new points for graphing • Students will see a basic example of addition, subtraction, and multiplication, and use these examples to add, subtract, multiply rational functions • Use online resources • NJDOE resources <u>English Language Learners</u> • Cycle means any complete *round*, as in a series of occurrences that repeats; or as in a recurring period of time, such as the	IEP • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulates • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies such as small groups • When graphing, students will be guided to identifying x- and y- values. • Students will be given a table and guided on how to complete the table to find new points for graphing • Students will see a basic example of each logarithmic property used in solving and simplifying expressions and equations. • Use online resources • NJDOE resources <u>English Language Learners</u> • Students may need to use a glossary or dictionary to read the definitions of *transformation, reflection, translation* and *shift*. • Place student next to same-language speaker, if possible	and student at risk • Prioritize tasks • Provide manipulates • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies such as small groups • Students will be provided with the acronym SOH CAH TOA to help them understand the different trig ratios • The unit circle will be completed in different colors so students understand the relationship of each point and how they relate to each other around the circle • Use online resources • NJDOE resources <u>English Language Learners</u> • Encourage students to use vocabulary including *terminal side, quadrant, acute, supplementary, x-axis*, etc. in their conversations. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Opposite, Adjacent, and hypotenuse will be translated and used as the main terminology for the trig evaluating.

	cycle of months in a year. Have students draw a *round* wheel with spokes, or lines drawn from the center of the wheel to the circumference. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Students will receive simplified notes, with key words being highlighted and use of arrows to direct attention from the word to the meaning of the word • Notes will be color coded to show common terms • NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm • Instruction will be based on language proficiency. At Risk Students • Students may struggle to understand the concept of an asymptote and how a function can continually approach a line but never reach it. • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources	• Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Students will receive simplified notes, with key words being highlighted and use of arrows to direct attention from the word to the meaning of the word • Notes will be color coded to show common terms • NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm • Instruction will be based on language proficiency. At Risk Students • Some students may have difficulty finding the average rate of change for an exponential function given its equation. • Have students practice finding the growth factor of an exponential model using pairs of points. 1. (4, 8) and (5, 12) 2. (7, 32) and (8, 40) 3. (12, 20) and (13, 26) • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources •Graphic organizers will be provided for	• NJDOE resources • Adapt a strategy adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm • Instruction will be based on language proficiency. At Risk Students • Help students understand how to calculate the tangent using sine and cosine, and the importance of the quadrant of the terminal point in the calculation. • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources • Diagrams will be labeled with O, A, H to identify opposite, adjacent, and hypotenuse sides of the triangle • The hypotenuse will be identified by drawing an arrow from the right angle to the hypotenuse •Complete Unit Circle will be provided for reference. • NJDOE resources • Utilize online resources such as www.khanacademy.org Gifted and Talented Students • Have students explore finding negative angle measures less than -360 degrees that are coterminal with a given angle. • Process should be modified: higher-order thinking skills, open-ended

	• Students will follow a standard form for writing radical expressions and rational exponents to convert one to the other • Students will indicate which terms can be simplified. •A common denominator will be given for all solving, and students will follow examples for solving the equation • NJDOE resources • Utilize online resources such as www.khanacademy.org Gifted and Talented Students • Students that grasp the concept of translations can take their understanding a step further by identifying translations from one function to another. • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified: abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied. • Problems will include multiple exponents.	students to utilize that contain the properties of logarithmic expressions and identities. •Vocabulary terms and context clues will be provided to remind students of the steps and properties that are available for them to use when solving and simplifying expressions. • NJDOE resources • Utilize online resources such as www.khanacademy.org Gifted and Talented Students • Students will explore writing exponential growth and decay models, given the initial value and the rate of growth or the rate of decay. 1. initial value: 45; rate of growth: 4.5% 2. initial value: 240; rate of decay: 7% 3. initial value: 13; rate of decay: 25% • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified: abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations	thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified: abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Students will be asked to complete the table of values for each special right triangle, and ensure that all denominators are rationalized as necessary • Practice with finding side and angle measures using defined trig ratios • Use of web based resources such as www.khanacademy.orggeogebra.org • NJDOE resources

	•Simplifying will include multiple complex terms, not limited to exponents and complex radicals, all with varying denominators. • Solving problems will include increasingly difficult denominators to identify the common denominator. • Use of web based resources such as www.khanacademy.org • NJDOE resources	• Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Complex graphs with multiple transformations, including reflections •Students will be given higher order thinking questions including rational root theorem to solve for a logarithmic solution. • Use of web based resources such as www.khanacademy.org • NJDOE resources	

***WHERETO**

W = Help the students know **WHERE** the unit is going and **WHAT** is expected. Help the teacher know **WHERE** the students are coming from (prior knowledge, interests).

H = **HOOK** all students and **HOLD** their interest.

E = **EQUIP** students, help them **EXPERIENCE** the key ideas and **EXPLORE** the issue.

R = Provide opportunities to **RETHINK** and **REVISE** their understanding and work.

E = Allow students to **EVALUATE** their work and its implications.

T = **TAILORED** to the different needs, interests, and abilities of learners.

O = **ORGANIZE** to maximize initial and sustained engagement as well as effective learning.

UNIT #: *Unit Title* Number of Days	**Unit 7:** *Conic Sections* 15 Days	**Unit 8:** *Probability and Statistics* 30 Days	

STAGE 1: DESIRED RESULTS

What will students understand as a result of the unit? What are the BIG ideas?

ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Algebra** A-REI.C.7 A-SSE.A.2 A-SSE.B.3 **Geometry** G-GPE.A.1 G-GPE.A.2 G-GPE.A.3(+) **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.2.12.CAP.1 9.2.12.CAP.3	**Numbers and Quantity** N-Q.A.2 **Statistics and Probability** S-CP.A.1 S-CP.A.2 S-CP.A.3 S-CP.A.4 S-CP.A.5 S-CP.B.6 S-CP.B.7 S-CP.B.8(+) S-CP.B.9(+) S-IC.A.1 S-IC.A.2 S-IC.B.3 S-IC.B.4 S-IC.B.5 S-IC.B.6 S-ID.A.4 S-MD.A.1(+) S-MD.A.2(+) S-MD.A.3(+) S-MD.A.4(+) S-MD.B.5.a(+) S-MD.B.5.b(+) S-MD.B.6(+) S-MD.B.7(+) **Mathematical Practices** MP.1 MP.2	

	9.2.12.CAP.4 9.2.12.CAP.6 9.2.12.CAP.7 9.4.12.CT.2 9.4.12.IML.3	MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.CFR.2 9.2.12.CAP.1 9.2.12.CAP.3 9.2.12.CAP.4 9.2.12.CAP.6 9.2.12.CAP.7 9.4.12.CI.1 9.4.12.CT.2 9.4.12.IML.2	
ENDURING UNDERSTANDINGS: *(Students will understand that . . .)*	•A parabola is the set of all points on a plane that are equidistant from the focus and the directrix. This definition is used to derive the equation of a parabola and apply the geometric properties of a parabola. •A circle is a set of points at a fixed distance from the center. The standard	•A statistical question is a question that can be answered by collecting many pieces of information, or data. The data can be categorical (qualitative) or statistical (quantitative). The data are measured by parameters, which describe the population, and statistics, which describe a sample of the population.	

	form of an equation of a circle with center (h,k) and radius r is (x-h)^2+(y-k)^2=r^2. This definition is used to derive the equation of a circle and apply the geometric properties. •An ellipse is the set of points P in a plane such that the sum of the distances from P to two fixed points F1 and F2 is a constant. •The standard form of an equation of an ellipse centered at the origin is x^2/b^2+y^2/a^2=1 when the major axis is vertical and x^2/a^2+y^2/b^2=1 when the major axis is horizontal. •A hyperbola is the set of all points P such that the difference of the distances from any point P to two fixed points, or foci, of the hyperbola is constant. This definition is used to derive the equation of a hyperbola and apply the geometric properties of a hyperbola.	•There are three types of statistical studies. The way in which samples are chosen for a study affects how well they represent the population. To avoid bias, samples should be random. •A data distribution can be normal, skewed left, or skewed right. Normal distributions are described using the mean and standard deviation. For skewed distributions, median and quartiles are used to describe the data. •The normal distribution is used to explain where data values fall within a population. The standard normal distribution allows for a comparison of values across different population distributions. •Sample statistics tend to be normally distributed and therefore can be used to estimate population parameters. The margin of error gives the maximum expected difference between the sample result and the population parameter. •A hypothesis and a null hypothesis can be written and tested for a statistical question. Statistics are used to compare two data groups and determine which hypothesis the data supports. Graphs and simulations can be used to determine whether differences between parameters are significant.	

		•Two events that cannot both occur are mutually exclusive. Two events are independent if the occurrence of one does not affect the probability of the other. The probability that two independent events both occur is the product of their probabilities •The conditional probability that event A will occur, given that another event B can be calculated using set notation. •A permutation is an arrangement of items in which the order of the items matters, while a combination is an arrangement in which order does not matter. •You can define a theoretical probability distribution by calculating the probability of each outcome in an experiment or an experimental probability distribution by using the real-world relative frequency of each outcome. •Expected value is the probability-weighted average of all possible values of a variable. It can be interpreted as the average outcome for many trials of an experiment. Use expected value to find expected payoffs or to compare options with differing costs and benefits. •To determine whether a procedure is	

		fair, compare the probabilities of possible outcomes. To choose among options, compare expected values. In situations with two possible outcomes for each trial, use binomial probabilities.	
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	•What are the geometric properties of a parabola, and how do they relate to algebraic representations of a parabola? •What are the geometric properties of a circle, and how do they relate to algebraic representations of a circle? •How does the equation of an ellipse relate to the features of its graph? •How does the equation of a hyperbola relate to the features of its graph? •How can we determine if an ellipse/hyperbola is horizontal or vertical? •How can we use the properties of the conic functions to create an equation in standard form to best represent them?	•What kinds of questions about quantities and relationships among quantities can be answered with statistics? •How can you choose the best type of study to answer given statistica questions and choose a reasonable sample? •How can you interpret the distribution of data in a data set? •How can you use the normal distribution to explain where data values fall within a population? •How can you determine how far a statistic is likely to be from a parameter? •How do you formulate and test a hypothesis using statistics? •How does describing events as independent or mutually exclusive affect how you find probabilities? •How are conditional probability and independence related in real-world experiments? •How are permutations and combinations	

		useful when finding probabilities? •What does a probability distribution tell you about an experiment? •What does expected value tell you about situations involving probability? •How can you use probability to make decisions?	
STAGE 2: ASSESSMENT EVIDENCE ***What evidence will be collected to determine whether or not the understandings have been developed, the knowledge and skills attained, and the state standards met? [Anchor the work in performance tasks that involve application, supplemented as needed by prompted work, quizzes, observations, etc.]***			
PERFORMANCE TASKS: *(Through what authentic performance tasks will students demonstrate the desired understandings?)* *(By what criteria will performances of understanding be judged?)*	•Make sense of problems and persevere in solving them. •Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities	•Make sense of problems and persevere in solving them. •Reason abstractly and quantitatively. •Construct viable arguments and critique the reasoning of others. •Model with mathematics. •Use appropriate tools strategically. •Attend to precision. •Look for and make use of structure. •Look for and express regularity in repeated reasoning. •Group Activities	

	•Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments •Written Responses on real-world problems and non-routine problems. •STEM Project: Students will explore how conic sections can be used to design rooms that direct sound to specific locations. They will then design their own such room. •3 ACT Math: "Watering the Lawn" Students will explore and apply concepts related to conic sections. A landscaper must place three sprinklers that each water a circular region of a lawn. Students are tasked with determining the size and location of each circular region in a coordinate place.	•Projects •District and State Standardized Testing •Computer-Technology Based Projects •Independent assignments •Written Responses on real-world problems and non-routine problems. •STEM Project: Students incorporate probability as they simulate a plausible forecast of February's weather in a specific location. February is generally the coldest month of the year and challenging for accurate weather predictions. •3 ACT Math: "Place Your Guess" Students will explore and apply concepts related to both theoretical and experimental compound probability. Students will analyze a situation in which two separate probability experiments are performed at the same time. They will be tasked with determining which desired outcome will occur most often for a set number of trials. •Students find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in	

		acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.	
OTHER EVIDENCE: *(Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired results?)* *(How will students self-assess their learning?)*	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Class Participation/Discussions •Teacher Observations •Error Analysis •Scoring Rubrics •Peer and Self Evaluation •Benchmark and Standards Assessments	•Formative Assessments •Summative Assessments •Homework Assignments •Do Now Questions •Class Assignments •Class Participation/Discussions •Teacher Observations •Error Analysis •Scoring Rubrics •Peer and Self Evaluation •Benchmark Assessments	
RESOURCES:	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks •Tiered Worksheets	• enVision Mathematics textbook • my.savvasrealize.com o Adaptive Practice o MathXL o Lesson Quiz •Kuta Software •Graphing Calculators •Khan Academy •Chromebooks/Ipads •Tiered Worksheets	

STAGE 3: LEARNING PLAN

What learning experiences and instruction will enable students to achieve the desired results? Utilize the WHERETO* acronym to consider key design elements.

SKILLS AND TOPICS: *(What specific activities will students do and what skills will students know as a result of the unit?)*	•Derive the equation of a parabola. •Relate a parabola's focal length to its equation. •Rewrite an equation by completing the square and then use it to find the focus and directrix of a parabola. •Use the center, the radius, and the Pythagorean Theorem to derive the equation of a circle. •Write and graph the equation of a circle and use it to model a real-world situation. •Find the center and radius of a circle by completing the square. •Solve a linear-quadratic system algebraically and verify by graphing. •Derive the equation of an ellipse. •Write and graph the equation of an ellipse and use an ellipse to model a real-world situation. •Graph a transformed ellipse by	•Understand the concept of a random variable. •Describe, in general terms, the normal curve and use its properties to answer questions about sets of data that one assumed to be normally distributed. •Determine the probability of various situations such as tossing a coin 5 times and getting 3 heads or the probability of the number of possible delegations of 3 out of 23 students. •Use tree diagrams to create a systematic list of all possible outcomes. •Review a published experiment and summarize how statistics were used. •Explain independence of events in everyday language and everyday situations. •Use a two-way frequency table to decide if events are independent and to approximate conditional probabilities. •Use mathematical modeling to represent a problem situation and to propose a solution.	

	completing the square to rewrite the equation in an equivalent form. •Use the foci and the Distance Formula to derive an equation of a hyperbola. •Write and graph the equation of a hyperbola and use it to model a real-world situation. •Determine which conic section is represented by a second-degree equation.	•Calculate the number of permutations and combinations in mathematical and real-world contexts. •Use permutations and combinations to compute probabilities of compound events and solve problems. Develop a probability distribution based on theoretical probabilities or empirical data. •Determine the measures of center, shape and spread given data from a table or graph.	
CROSS-CURRICULAR: *(What cross-curricular (e.g. writing, literacy, math, science, history, technology) learning activities are included in this unit that will help achieve the desired results?)*	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) • Chromebook (for above, and use of other apps such as Socrative and	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** •Use reading comprehension to evaluate and analyze real-world problems •Use writing skills to provide reasoning for given problem solutions **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2)** Current event problems Exploratory Activities Project-Based Learning Integrate Technology: •Textbook online resources at www.savvasrealize.com (assessments, learning tools) •Chromebook (for above, and use of other apps such as Socrative and	

	YouTube, as appropriate) Special Education/504: • Students may need assistance with writing the equation of an ellipse given the graph. Help students write the equation of an ellipse by having them answer the following questions. 1. What are the co-vertices of the ellipse? 2. How long is the horizontal axis? 3. How long is the vertical axis? 4. What is the equation representing the ellipse? • Provide modifications and accommodations as listed in the student's IEP • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulates • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies such as small groups •Use a graphic organizer to help identify the different conic sections. •Provide a reference sheet with	YouTube, as appropriate) Special Education/504: • Students will use conditional probability to solve problems. Students can check their work using this familiar method until they are sure they are using the correct formulas. • Provide modifications and accommodations as listed in the student's IEP/504 plan • Position student near helping peer or have quick access to teacher • Modify or reduce assignments/texts • Reduce length of assignment for different mode of delivery • Increase one-to-one time • Utilize working contract between you and student at risk • Prioritize tasks • Provide manipulatives • Use graphic organizers • Use interactive math journals • Use online resources for skill building • Provide teacher notes • Use collaborative grouping strategies such as small groups • Use of relatable examples in discussion of set theory (union, intersection, complement, etc) • Use of keywords to determine if problems are permutations or combinations • Use of relatable experiences and interests of students when teaching conditional, independent, and dependent	

	characteristics and the standard form equations for each of the conic sections. • Use online resources • NJDOE resources <u>English Language Learners</u> • A *grid* is a pattern of straight lines that intersect to form squares. On maps, *grids* are used to help pinpoint specific locations. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Circle, Vertex, Co-Vertex, Ellipse, Hyperbola, Center, and Radius will be translated and used as the main terminology for conic graphing and writing. • NJDOE resources • Adapt a strategy adjusting strategy for ELL: <u>http://www.teachersfirst.com/content/esl/adaptstrat.cfm</u> • Instruction will be based on language proficiency. <u>At Risk Students</u> • Students may need assistance with setting up an equation of a parabola before completing the square. • Have students identify the adjectives	probability • Use online resources • NJDOE resources <u>English Language Learners</u> • Write the phrases *chosen at random, replaced,* and *not replaced* on the board. Have students discuss their meanings. • Place student next to same-language speaker, if possible • Provide text to speech for math problems • Use of translation dictionary or software • Implement strategy groups • Confer frequently • Provide graphic organizers • Modification plan • Translate word problems for students, and use simple terminology for student comprehension • Use relatable experiences to increase student interest in lessons • NJDOE resources • Adapt a strategy adjusting strategy for ELL: <u>http://www.teachersfirst.com/content/esl/adaptstrat.cfm</u> • Instruction will be based on language proficiency. <u>At Risk Students</u> • Guide students with questions as they answer the following questions: 1. 60% of the students in your class are girls. 40% of the girls in your class plan to go to the school play. If you select a	

	used to describe the solar cooker dish, *reflective* and *parabolic*. • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources • Problems will have a list of characteristics to be identified or students will be told which type of function they are working with. • Graphic organizers will be given in the form of lists or diagrams to aid students. • NJDOE resources • Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org Gifted and Talented Students • Have students explore finding the equation of a circle that intersects a line at given points. Write the equation of a circle with center (0, 0) that intersects the line $x - y = 2$ at (-6, -8)s and (8, 6). • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified: abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines,	student in the class at random, what is the probability that the student is a girl who is planning to go to the school play? 2. 70% of the students in the class like chocolate ice cream. 50% of students who like chocolate ice cream also like strawberry ice cream. What is the probability that a student selected at random likes strawberry ice cream and chocolate ice cream? • Tiered interventions following RTI framework • RTI Intervention Bank • Use additional practice and textbook RTI resources • Provide matching activities for set theory and permutations/combinations • Highlight key words in problems to draw attention and assist in identifying the appropriate type of probability • Limit the amount of word problems being given for solving probabilities, and provide step-by-step examples for each type of probability • NJDOE resources • Utilize online resources such as or www.khanacademy.org Gifted and Talented Students • Describe the following experiment involving 5 cards, three that are numbered 1 and two that are numbered 2. One card will be selected at random, then replaced, and then another card will be	

	evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Students will be asked to create a guide to determine the differences between the conic sections. • Use of web based resources such as www.khanacademy.org • NJDOE resources	selected. 1. What is the probability that you will select a 1 card both times? 2. What is the probability that you will select a 2 card both times? 3. What is the probability that the numbers on the two cards will match? • Process should be modified: higher-order thinking skills, open-ended thinking, and discovery • Utilize project-based learning for greater depth of knowledge • Utilize exploratory connections to higher-grade concepts • Contents should be modified: abstraction, complexity, variety, and organization • Products should be modified: real-world problems, audiences, deadlines, evaluation, and transformations • Learning environments should be modified: student-centered learning, independence, openness, and complexity, groups varied • Word problems will be longer and more complex, requiring students to interpret useful information to solve • Combinations and permutations will be more complex, involving more repeating values • Independent, dependent, and conditional probabilities will be more complex, especially playing card problems with overlapping repetitives • Use of web based resources such	

		as www.khanacademy.org • NJDOE resources	