Roll No…………………….. **Total No. of Printed Pages: 2**

Total No. of Questions: [09]

B. Sc. – M. Sc. (Forensic Science) (Semester – 1st)
DIFFERENTIAL CALCULUS-I
Subject Code: BSNMS1-105
Paper ID: [23480108]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Show that $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

b. Consider the function $f(x)=\begin{cases} \dfrac{x}{1+e^{\frac{1}{x}}} & ; x\neq 0 \\ 0 & ; x=0 \end{cases}$. Is the function derivable at $x=0$?

c. Verify Cauchy's Mean Value theorem for the functions $f(x)=x^4, g(x)=x^2$ in the interval [*a, b*]

d. Evaluate: $\lim_{x\to 0}\frac{x-\sin x}{\tan^3 x}$.

e. Find the equation of tangent and normal at the point (x_1, y_1) for the curve $x^2+y^2=k^2$.

f. Show that the curvature at any point of the curve $y=c\cosh\frac{x}{c}$ varies inversely as the square of the ordinate.

g. If $T=\frac{x^3y^3}{x^3+y^3}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3T$.

h. If $f(x,y)=0, \phi(y,z)=0,$ show that $\frac{\partial f}{\partial y}.\frac{\partial \phi}{\partial x}.\frac{dz}{dx}=\frac{\partial f}{\partial x}.\frac{\partial \phi}{\partial y}$

i. Find the *n*th derivative of $\log\left(bx+x^2\right)$.

j. Show that $e^y \log(1+x) = x + xy - \frac{x^2}{2} + \ldots\ldots\ldots..$

Section – B **(5 marks each)**

Q2. If $y=\left(x^2-1\right)^n$, prove that $\left(x^2-1\right)y_{n+2}+2xy_{n+1}-n\left(n+1\right)y_n=0.$

Q3. Prove that the only singular point on the curve $\left(y-b\right)^2=\left(x-a\right)^3$ is a cusp and find its coordinates.

Q4. If $u^3+v^3+w^3=x+y+z,\ u^2+v^2+w^2=x^3+y^3+z^3,\ u+v+w=x^2+y^2+z^2$, show that $\dfrac{\partial\left(u,v,w\right)}{\partial\left(x,y,z\right)}=\dfrac{\left(y-z\right)\left(z-x\right)\left(x-y\right)}{\left(v-w\right)\left(w-u\right)\left(u-v\right)}.$

Q5. Find the shortest distance from the origin to the hyperbola $x^2+8xy+7y^2=225.$

Q6. Find all the asymptotes of the curve $x^3+2x^2y-xy^2-2y^3+4y^2+2xy+y-1=0.$

Section – C **(10 marks each)**

Q7. (a) Examine the continuity of $f\left(x\right)=\begin{cases}x & ; x \text{ is rational}\\ -x & ; x \text{ is irrational}\end{cases}$ at $x=0$. (5)

(b) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{\frac{1}{x^2}}.$ (5)

Q8. If z is a function of x and y and u and v be two other variables, such that $u=lx+my, v=ly-mx$, show that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=\left(l^2+m^2\right)\left(\dfrac{\partial^2 z}{\partial u^2}+\dfrac{\partial^2 z}{\partial v^2}\right).$

Q9. Trace the curve $x=a\left(\theta-\sin\theta\right), y=a\left(1-\cos\theta\right).$