CSE 457 HW 1

Due Tuesday January 23, 11pm
Turn in a pdf on Gradescope.

- Please **type** your answers. Make sure the questions and parts are clearly labeled. You can make a copy of this document and fill in the answers.
- For diagram questions, you can hand-draw them and insert a photo, or you can draw them digitally. Make sure the labels on the diagrams are legible and nicely written.
- Please format the document so that the answer to each question is on the same page (i.e. please avoid having your answer to one question span two pages unless it needs to)

Question 1 (15 pts)

This matrix performs a rotation by angle θ and translation by vector $[a \;\; b]^T$.

$$\begin{bmatrix} cos\theta & -sin\theta & a \\ sin\theta & cos\theta & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

(a) (5 points) Specify the product of two 3x3 matrices, one rotation and one translation, that produces the same matrix as above.

(b) (5 points) In which order do these transformations act on the point on the right? I.e., does the rotation act first and then the translation, or vice versa?

(c) (5 points) Give a single 3 x 3 matrix that applies the same rotation and translation but in the opposite order (show your work).

Question 2 (15 pts)

(a) (5 points) A 2D 180 degree rotation can be performed with two 2D reflections. Prove this statement by giving the 2x2 180 degree rotation matrix and showing that it's the same as the product of the two reflection matrices.

(For part (b) and (c)) Here's a matrix that performs a 3D reflection in **X**

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The equivalent transformation can instead be achieved via a 3D reflection in **Y** and a 3D rotation matrix **R**.

(b) (5 points) Specify the axis of rotation (X, Y, or Z), and the angle (in degrees) for **R**

(c) (5 points) Specify the two matrices (**R** and **Y**-reflection), and show how they produce the X reflection when multiplied together.

Question 3 (10 pts)

Consider a triangle with normal pointing in the direction $[1\ \ 1\ \ 0]^T$ and vertices:

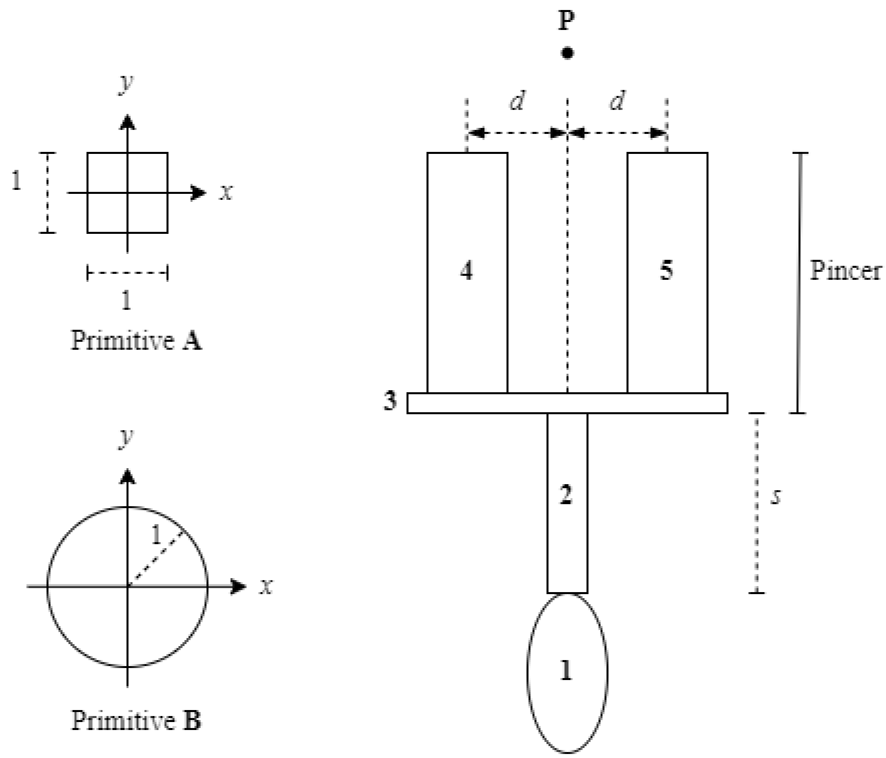
$$\mathbf{A} = [0\ \ 0\ \ 0]^T$$
$$\mathbf{B} = [-1\ \ 1\ \ 0]^T$$
$$\mathbf{C} = [x\ \ y\ \ 1]^T$$

(a) (5 points) What are all possible values for the elements of C? Use the cross product equation to derive your answer.

(b) (5 points) Now derive the same answer using only the dot product equation (no cross-products). Hint: $\mathbf{V} \bullet \mathbf{W} = 0$ when $\mathbf{V}$ and $\mathbf{W}$ are perpendicular.

Question 4 (35 pts)

Suppose you want to model a pincer robot arm illustrated below. The model is composed of 5 parts, using primitives **A** and **B**. The model is controlled by parameters *s* and *d*. The pincer operates by translating primitives **4** and **5** towards or away from each other according to d. The illustration on the right also shows a point P that the robot is reaching towards.



Assume that *s* takes values in range [1, 10]. As part of your solution to this problem, you will need to define the instance transformations that should be applied to a given primitive on the left so that it is the same shape as a desired primitive on the right. Object **1** is 2 units long on the major axis and 1 unit long on the minor axis. Object **2** has width 1/2 units and height *s*. Object **3** has width 4 and height 1/4. Objects 4 and 5 have width 1 and height 3.

The following transformations are available to you (use it as a shorthand for the transformation matrix it represents):

- $R(\theta)$ – rotate by θ degrees (counter clockwise)
- $T(a, b)$ – translate by $[\, a \; b \,]$
- $S(s_x, s_y)$ – scale the x-component by s_x and scale the y-component by s_y

(a) (10 points) Construct a tree to describe this hierarchical model using part **1** as the root. The child of each node can either be a part number (**1**. . . **5**) or a reference to the canonical geometry that will be drawn (**A** or **B**). A node can have one or more children. Two parts connected physically should be connected in the tree unless it is already connected to another part (i.e., if there are multiple parents, choose one).

(b) (15 points) Along each of the edges of the tree, write an expression for the transformations that are applied along that edge to connect parent to child. Write all transformations using the notation above; you do not need to write out the matrices, just the symbolic references to them and their arguments. Remember that the order of transformations is important! Show your work wherever the transformations are not "obvious." Assume that the center of part **1** sits on the origin in world coordinates. You may submit it part (a) and (b) as a single image.

(c) (5 points) Write out the full sequence of transformations for drawing the geometric primitive for part 4. Again, use the symbolic matrix notation above that appears in your tree.

(d) (5 points) What values of s and d would have the model extend out and close the pincer just enough to precisely grasp the point $P = [0\ 6.5]^T$? You can assume point P is infinitesimally small. Show your work.