AP Calculus Curriculum

AP Calculus AB and BC

Full Year

This is a comprehensive calculus course that follows a College Board approved syllabus. Students are expected to have completed Pre Calculus Honors before taking AP Calculus

According to the College Board: In Calculus AB students will see that Calculus helps make sense of change by grappling with questions that inspire thinkers from around the globe, across time, and in many disciplines. Can change occur in an instant? When is the next solar eclipse or the turning point for an economy? In AP Calculus AB, you'll develop a deeper understanding of mathematical principles that can help you answer questions such as these.

In AP Calculus BC, you'll cover many of the mathematical principles in AP Calculus AB and build upon them. AP Calculus BC helps you prepare for further study in mathematics and other disciplines, such as engineering, computer science, or economics.

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Unit 1: Limits

Unit Plan Explanatory Document

Grade Level: 12	**AP Calculus AB and BC** **Unit 1 Topic: Limits** **Duration AB: 3 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	According to the College Board description: Limits introduce the subtle distinctio a function at a point and considering what value the function is approachin approaches a point. This distinction allows us to extend understanding of asy graphs with formal definitions of continuity. Consider reviewing rational funct limits, rather than beginning the year with a full review of precalculus topics. foundation for differentiation (Unit 2), integration (Unit 6), and infinite series (U the basis for important definitions and for theorems that are used to solve rea change and to justify conclusions.
Essential Questions	• *What is a limit and how can it be evaluated?* • *What does it mean for a function to be continuous?* • *In what way do limits connect to the algebraic concept of slope and rate* • *How is continuity used to identify the existence of important values of a fur*
Misconceptions	*Students may think that…* • *The function must be defined at a point for the limit to exist.* • *Infinite limits are actual function values.* • *Difference between does not exist and indeterminate* • *0/0 is an acceptable answer.*
Transfer	*Students will be able to use what they learn to…* • *Analyze and interpret the behavior of functions and evaluate the limit from or by using the Squeeze Theorem.* • *Explain the conditions that make a function continuous and use the interm to explore continuity.* • *Recognize and classify different types of discontinuities.*

	• *Use limits to support real-world modeling such as using Instantaneous Spee* •
Important Facts and Key Concepts	*Students will know…* • *the difference between average and instantaneous velocity.* • *Limits can be expressed in multiple ways, including graphically, numerically* • *One-sided limits describe behavior from the left or right and helps to deter* • *A function is continuous at a point if the limit as x approaches a exists, the is defined, and f(a) is equal to its limit.* • *The Intermediate Value Theorem guarantees values between two endpoi value between f(a) and f(b) if the function is continuous.*

Standards

NJSL Content Specific Student Learning Standards

F-IF.B.4: Interpret functions that arise in applications in context.

F-IF.C.7: Graph functions expressed symbolically and show key features of the graph.

N-Q.A.3 : Choose a level of accuracy appropriate to limitations on measurement when reporting

WIDA Standards for Multilingual Learners

Multilingual learners will..

Interpret mathematical explanations by

- Identifying concept or entity
- Analyzing data and problem-solving approaches
- Evaluating rationales, models, and interpretations based on evidence and math principle

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions.

Computer Science and Design Thinking

8.1.12.DA.5 : Create data visualizations from large data sets to summarize, communicate, and su interpretations of real-world phenomena.

Career Readiness, Life Literacies, and Key Skills

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills to solve authentic pr

Assessment Evidence

Students will demonstrate understanding by...

Formative Assessment *: Classwork and homework assignments evaluating limits*

Benchmark Assessment *:Quizzes and WebAssign Assignments.*

Summative Assessment : *Unit test at the end of the unit*
Alternative Assessment*: Students may have the option to give a presentation.*

Interdisciplinary Connections

• In physics: Students will connect limits to motion using instantaneous and average velocity to mo
objects.
• In Biology: Limits support the knowledge of population growth models and rate of change in bio
• In Business: Students can use limits to look at how marginal cost and marginal revenue change
get closer to a certain value.

Standards for Cross-Content Areas:
Science Standards:
HS-PS1-1: Use the periodic table as a model to predict the relative properties of elements.

Accommodations and Modifications

Special Education	• Accommodations consistent with IEP • *Repeat/confirm directions* • Extended time • Guided notes • Preferential seating • Chunking material

	• Modified assignments • NHS Tutor (Mu Alpha Theta) • Provide a calculator
504	• Accommodations consistent with 504 • Extended time • Preferential seating • Tutoring Center • *Check-ins for understanding* • *Break tasks into smaller chunks*
At Risk/ I&RS	• *Scaffolded instruction with peer support* • *Frequent feedback and review sessions* • Tutoring Center • Parent Communication • MTSS (Multi-Tiered System of Supports)
ML	• *Visual word walls* • *Sentence frames for math explanations* • *Support with bilingual dictionaries or glossaries* • *Diagrams and drawings* •*Working with partners or small group*
Gifted	• *Challenge problems and free response questions with later topics* • *Early exposure to AP Calculus BC concepts (L'Hospital's rule for indetermin* • *Differentiated formative assessments to develop learning experiences that gifts and talents.* • *Standardized (adaptive, above-grade-level) and classroom assessments th academic progress of students with gifts and talents.*

Resources

• *AP Classroom and College Board materials.*
• *WebAssign.*
• *Khan Academy and Desmos.*

Unit 2: Differentiation

Calculus AB and BC
it 2 Topic: Differentiation: Definition and Fundamental Properties
ration AB: 2 weeks Duration BC: 2 weeks BC

cording to the College Board description: Derivatives allow us to determine instantaneous es of change. To develop understanding of how the definition of the derivative applies limits average rates of change, create opportunities for students to explore average rates of ange over increasingly small intervals. Graphing calculator explorations of how various erations affect slopes of tangent lines help students to make sense of basic rules and perties of differentiation. Encourage students to apply the order of operations as they select erentiation rules. Developing differentiation skills will allow students to model realistic antaneous rates of change in Unit 4 and to analyze graphs in Unit 5.

hat are the different methods I can use to find a derivative?

ow can derivatives help us analyze and describe the behavior of a function?

ow are continuity and differentiability connected?

hy does continuity matter when finding maximums and minimums?

ow can I use derivatives to solve real-life problems or model real situations?

ow can the product and quotient rules be applied to differentiate trigonometric functions?

dents may think that…

ll continuous functions are differentiable.

ey can use the power rule to find the derivative of every function.

udents think they can just divide the derivatives: $(\frac{f}{g})' = \frac{f'}{g'}$ instead of using the quotient .

ultiplying functions and composing functions are the same, for example $x^2\sin(x)$ and $\sin(x^2)$.

ey need to take the derivative of every function, regardless or whether they are mposed:

$'(g'(x))$ rather than $f'(g(x)) \cdot g'(x)$

dents will be able to use what they learn to…

- Use derivatives in real life application of derivatives problems, like rectilinear motion in Unit 5 and

related rates in Unit 4.

- Use derivatives when analyzing curves to find a minimum, maximum, or critical point in Unit 5.
- Apply the chain rule to take derivatives implicitly in Unit 4.
- Compare rates of change for indeterminate limits by taking derivatives to apply L'Hospital's rule in Unit 4.
- Find antiderivatives in Unit 6 by knowing which functions have a derivative as the given function.
- use u substitution for finding antiderivatives of composed functions in Unit 6 by understanding the chain rule for finding derivatives of composed functions.

dents will know…

e difference quotients $\frac{f(a+h)-f(a)}{h}$ *and* $\frac{f(x)-f(a)}{x-a}$ *express the average rate of*

nge of a function over an interval.

e instantaneous rate of change of a function at x a = can be expressed by $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ or $\frac{f(x)-f(a)}{x-a}$ provided the limit exists. These are equivalent forms of the definition of the derivative and denoted f'(a).

e derivative of f is the function whose value at x is $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, provided this limit exists.

r $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, $and\ y'$.

e derivative can be represented graphically, numerically, analytically, and verbally.

e derivative of a function at a point is the slope of the line tangent to a graph of the ction at that point.

e derivative at a point can be estimated from information given in tables or graphs.

chnology can be used to calculate or estimate the value of a derivative of a function at a nt.

a function is differentiable at a point, then it is continuous at that point. In particular, if a nt is not in the domain of f, then it is not in the domain of f'.

continuous function may fail to be differentiable at a point in its domain.

rect application of the definition of the derivative and specific rules can be used to culate the derivative for functions of the form $f(x) = x^r$.

ms, differences, and constant multiples of functions can be differentiated using derivative s.

e power rule combined with sum, difference, and constant multiple properties can be used ind the derivatives for polynomial functions.

pecific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic

ctions.
some cases, recognizing an expression for the definition of the derivative of a function
ose derivative is known offers a strategy for determining a limit.
erivatives of products of differentiable functions can be found using the product rule.
erivatives of quotients of differentiable functions can be found using the quotient rule.
earranging tangent, cotangent, secant, and cosecant functions using identities allows
erentiation using derivative rules.

Standards

tudent Learning Standards

expressed symbolically and show key features of the graph.
defined by an expression in different but equivalent forms to reveal and explain different
n.
es of two functions each represented in a different way (algebraically, graphically, numerically in

ions in which a quantity grows or decays proportionally.
nd exponential functions, including arithmetic and geometric sequences, given a graph, a
o, or two input-output pairs.
phs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing
s a polynomial function.

rpretive

explanations by
ot or entity

d owning problem-solving approaches

les, models, and/or interpretations based on evidence and mathematical principles
pressive:
explanations that
natical concept or entity

n others

d/or approach used to solve a problem

ed to generate own or alternate solutions

essive

arguments that

njecture, using definitions, previously established results, and stated assumptions

relationships across cases

conclusions with evidence and mathematical principles

nd others' arguments

esign Thinking

visualizations from large data sets to summarize, communicate, and support different

rld phenomena.

ways computing impacts personal, ethical, social, economic, and cultural practices.

eral models of the same type of product and make recommendations.

iteracies, and Key Skills

ues associated with student loan debt, requirements for repayment, and consequences of

oan debt. Analyze and calculate rates (financial rates of change, growth models).

the ability to reflect, analyze, and use creative skills and ideas(choosing appropriate

.

Assessment Evidence

e following list of assessments

sswork and Homework assignments evaluating derivatives of basic functions

izzes on basic derivatives, and limits of a difference quotients

Style test covering Multiple Choice and Open Ended AP questions covering basic derivatives and the

roup presentations of classwork or homework. Students teach others about how to evaluate a derivative and

Interdisciplinary Connections

ea may connect to the core content area in this unit.

l apply derivatives to cost and revenue functions. Derivatives show how much cost and
g or selling one more item.
relate derivatives to velocity and acceleration as measures of positional change.
learn how to use derivatives to calculate how fast a population grows.

Accommodations and Modifications

eferential seating
Give step-by-step examples of derivatives in the guided notes.
ovide assignments in chunks that concentrate on a single differentiation rule at a time.
llow for extra time.
ccommodations consistent with IEP
odified assignments
HS Tutor (Mu Alpha Theta)
ovide a calculator

ssign less work while maintaining rigor.
xtended time.
llow use of calculator
HS Tutor (Mu Alpha Theta)

ffer structured practice problems transitioning from the basic power rule to the product and otient rules
equent check-ins to assess progress and quickly clear up any confusion.
air students with peers who can help them.

se graphs of tangent lines and slopes to help explain concepts.
llow the use of translation tools or bilingual glossaries for key terms .
ovide vocabulary support for derivative notations $\frac{dy}{dx}$, $f'(x)$, $and\ y'$.
Vorking with partners or small group

troduce advanced concepts like implicit differentiation at an earlier stage.
ive real-world problems to solve and let students try projects using derivatives such as using
ivatives to model speed, fuel use, and cost.

Resources

lus Course and Exam Description (CED)
-84)

alculus AB Differentiation modules and Desmos.
graphing.
problem.

Unit 3: Advanced Differentiation

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 3 Topic: Differentiation: Composite, Implicit, and Inver** **Duration AB: 3.5 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	In this unit, students learn how to differentiate composite functions using the c apply that understanding to determine derivatives of implicit and inverse func need to understand that for composite functions, y is a function of u while u is of x. Leibniz notation for the chain rule, $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$, accounts for these rela Units analysis can strengthen the connection, as in $\frac{psi}{min} = \frac{psi}{m} \cdot \frac{m}{min}$. Saying, what's inside," every time we apply the chain rule reminds students to avoid c Mastering the chain rule is essential to success in all future units.
Essential Questions	• *How can implicit differentiation be used to find derivatives when a fu solved for y?* • *How is the derivative of an inverse function related to the derivative o function?* • In what situations is it appropriate (or not appropriate) to apply L'Hôp
Misconceptions	*Students may think that…* • Forgetting the chain rule in composite functions. • Any limit can be evaluated with L'Hospital's rule • Students may stop half way leaving $\frac{dy}{dx}$ in multiple terms without solvin • Students may assume $(f^{-1})'(x) = \frac{1}{f'(x)}$ instead of the correct formula (
Transfer	*Students will be able to use what they learn to…* • *Differentiate with respect to a* *ny variable, which can help calculus transfer "rate of change" to many contexts* • *Prepare for differentiating parametric curves in Unit 9.* • *Understand how to deal with functions that are not explicitly defined, differential equations are in Unit 8.*

Important Facts and Key Concepts	*Students will know…* • *The chain rule provides a way to differentiate composite functions.* • The chain rule is the basis for implicit differentiation. • The chain rule and definition of an inverse function can be used to fin inverse function, provided the derivative exists. •The chain rule applied with the definition of an inverse function, or the derivative of an inverse function, can be used to find the derivatives of functions. • Differentiating f ′ produces the second derivative f ″, provided the de repeating this process produces higher-order derivatives of f. • Higher-order derivatives are represented with a variety of notations. F for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y ″. Higher-order derivat $\frac{d^ny}{dx^n}$, $f^n(x)$.

Standards

NJSL Content Specific Student Learning Standards

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph.

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal a properties of the function.

***F.IF.C.9*:** Compare properties of two functions each represented in a different way (algebraically, grap tables)

F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.A.2: Write arithmetic and geometric sequences both recursively and explicitly, use them to model between the two forms.

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) and f (both positive and negative); find the value of k given the graphs. Experiment with cases and i explanation of the effects on the graph using technology. Include recognizing even and odd graphs and algebraic expressions for them.

F.LE.A.1.c: Recognize situations in which a quantity grows or decays by a constant percent rate per uni another.

WIDA

ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity
- Analyzing data and owning problem-solving approaches
- Evaluating rationales, models, and/or interpretations based on evidence and mathema

ELD-MA 9-12 Explain Expressive:

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated as
- Generalize logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments

Computer Science and Design Thinking

***8.1.12.DA.5*:** Create data visualizations from large data sets to summarize, communicate, and interpretations of real-world phenomena.

8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cu

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendation

8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a sys investigation, and propose an innovative sustainable solution.

Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.4: Identify issues associated with student loan debt, requirements for repayment, and con failure to repay student loan debt.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and use problem solving p

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem sol

9.4.12.CT.3: Collaborate with individuals to analyze a variety of potential solutions.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: Daily warm-ups on chain rule practice and exit tickets for implicit differentiation
Benchmark Assessment *: Quizzes on chain rule, implicit, and inverse differentiation.*
Summative Assessment : *Unit test including multiple-choice and free-response questions modeled acc*
Alternative Assessment: Group presentations that show how the chain rule or implicit differentiation is

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.
In physics: Students will apply differentiation techniques to solve related rates problems involving veloc
In technology: Students will use technology to verify and explore derivative relationships.

Accommodations and Modifications

Special Education	• Preferential seating • Provide step-by-step guided notes for chain, product, and quotient r • Allow additional time for multi-step implicit differentiation problems. • Allow for extra time. • Accommodations consistent with IEP • Modified assignments • NHS Tutor (Mu Alpha Theta) • Provide a calculator
504	• *Assign less work while maintaining rigor.* • *Extended time.* • *Allow use of calculator* • NHS Tutor (Mu Alpha Theta)
At Risk/ I&RS	• Use small group teaching for challenging concepts like the chain rule • More practice before moving to AP-level free response questions. • Frequent check-ins to correct misunderstandings at an early stage. • Use peer tutoring.
ML	• Allow peer discussion to enhance mathematics communications. • Allow the use of translation tools or bilingual glossaries for key terms . • Include notations for higher-order derivatives for example $\frac{d^2y}{dx^2}$, $f''(x)$

	• Provide vocabulary support for derivative notations $\frac{dy}{dx}$, $f'(x)$, $and\ y'$.
Gifted	• Assign students enrichment tasks with implicit differentiation and inve previous AP Calculus BC free-response questions. • Assign students more difficult and higher level applications such as pa vector functions earlier than the rest of class. • Have students explore proofs of the chain rule.
Resources	
• College Board AP Calculus Course and Exam Description (CED) • AP Classroom problem sets • WebAssign. • Khan Academy • Graphing calculators (TI-84)	

Unit 4: Applications of Differentiation

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 4 Topic: Contextual Applications of Differentiation** **Duration AB: 3 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	Unit 4 begins by developing understanding of average and instantaneous rate problems involving motion. The unit then identifies differentiation as a common on which to build understanding of change in a variety of contexts. Students' of measure often reinforces their understanding of contextual applications of problems involving related rates, identifying the independent variable comma may help students to correctly apply the chain rule. When applying differentia of certain indeterminate forms using L'Hospital's rule, students must show that
Essential Questions	• How can derivatives be used to describe and predict motion? • How can derivatives help determine whether a quantity is increasing, down, or Speeding up? • In what ways do related rates connect one quantity's change to ano How can we use derivatives to explain and justify the behavior of a func
Misconceptions	Students may think that… • Students may think the derivative has no units. • Students may think that if $v(t) = 0$, the particle is not moving at all. In always mean the object stops moving, it often marks a moment where the object ch • Students forget to add $\frac{dx}{dt}$ or $\frac{dy}{dt}$ In related rates, they must add it be changes with time. We can have a quick reminder for them about it in advance.
Transfer	*Students will be able to use what they learn to…* • *This will appear again in Unit 6 (Integration & Accumulation): Understa foundation* *for calculating the accumulation of change.* • *This will come up again in Unit 7 (Integration Applications): Understan*

	students *Apply calculus to area, volume, and motion.*
Important Facts and Key Concepts	*Students will know…* • *The derivative of a function can be interpreted as the instantaneous r* *respect to* *its independent variable.* • The derivative can be used to express information about rates of chan contexts. • The unit for $f'(x)$ is the unit for f divided by the unit for x. • The derivative can be used to solve rectilinear motion problems involv speed,velocity, and acceleration. • The derivative can be used to solve problems involving rates of chang • The chain rule is the basis for differentiating variables in a related rates to the same independent variable. • Other differentiation rules, such as the product rule and the quotient r necessary to differentiate all variables with respect to the same independent variab • The derivative can be used to solve related rates problems; that is, fin one quantity is changing by relating it to other quantities whose rates of change are •The tangent line is the graph of a locally linear approximation of the fu of tangency. • For a tangent line approximation, the function's behavior near the po determine whether a tangent line value is an underestimate or an overestimate o function value. • When the ratio of two functions tends to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ in the limit, such f indeterminate. • Limits of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ may be evaluated using
Standards	

NJSL Content Specific Student Learning Standards

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph.

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal c
of the
function.

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, gra
tables)

F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.A.2: Write arithmetic and geometric sequences both recursively and explicitly, use them to mode
between the
two forms.

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) and
(both positive
and negative); find the value of k given the graphs. Experiment with cases and illustrate an e
effects on the graph
using technology. Include recognizing even and odd functions from their graphs and algebra
them.

F.LE.A.1.c: Recognize situations in which a quantity grows or decays by a constant percent rate per un
another.

WIDA

ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity (Velocity, acceleration, and rate in/ out)
- Analyzing data and owning problem-solving approaches
- Evaluating rationales, models, and/or interpretations based on evidence and mathema

ELD-MA 9-12 Explain Expressive:

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem

- State reasoning used to generate own or alternate solutions

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated a
- Generalize logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments

Computer Science and Design Thinking

***8.1.12.DA.5*:** Create data visualizations from large data sets to summarize, communicate, and interpretations of real-world phenomena.

8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and c

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendatio

8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a sy investigation, and propose an innovative sustainable solution.

Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.4: Identify issues associated with student loan debt, requirements for repayment, and co failure to repay student loan debt.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and use problem solving

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem so

9.4.12.CT.3: Collaborate with individuals to analyze a variety of potential solutions.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: Daily warm-ups on rate-in/rate-out and related rates setups, interpretation of

Benchmark Assessment *: Quizzes on related rates, velocity/ acceleration, and interpreting derivative*

Summative Assessment : Unit test including AP-style multiple-choice and free-response questions on c (motion, rates in/out, related rates).

Alternative Assessment: Group presentations that show how to model rate of change scenario such

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

Students will

- Connect calculus to several real world situations.
- Find the rate at which the area impacted by a wave radiating outwards is increasing if we know how quic
- Solve how quickly an ice cube vs ice frozen in a cylindrical pellet shape is melting and what volume it wi
- Solve physics problems about rectilinear motion, for example, finding when an object is speeding up or when an

object is moving to the left or right.

Accommodations and Modifications

Special Education	• Guided notes with step-by-step examples of rate problems • Allow use of calculators for derivatives computation. • Use diagrams and graphic organizers.
504	• Offer reduced problem sets without lowering rigor. • Allow oral explanation if needed.
At Risk/ I&RS	• Frequent check-ins to monitor progress. • Peer tutoring.
ML	• Include notations for higher-order derivatives for example $\frac{d^2y}{dx^2}$, $f''($ context Explanations (acceleration as rate of velocity). • Provide vocabulary lists for derivative notations $\frac{dy}{dx}$, $f'(x)$, $and\ y'$ with definition.
Gifted	• Explore BC level materials such as parametric motion and polar coord • Ask students to create their own related rates problems with full solutio

Resources

• College Board AP Calculus Course and Exam Description (CED)

• AP Classroom problem sets • WebAssign. • Khan Academy: Applied derivatives and related rates modules • Graphing calculators (TI-84)

Unit 5: Analytical Applications of Differentiation

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 5 Topic: Analytical Applications of Differentiation** **Duration AB: 4 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	*In this unit, the superficial details of contextual applications of differentiati* *away* *to focus on abstract structures and formal conclusions. Reasoning with de* *theorems* *establishes that answers and conclusions are more than conjectures; they* *analytically* *determined. As when students showed supporting work for answers in prev* *will learn to present justifications for their conclusions about the behavior o* *certain intervals or the locations of extreme values or points of inflection. T* *this study of differentiation by applying abstract reasoning skills to justify so* *optimization problems.*
Essential Questions	• In what ways do the first and second derivatives provide information ab concavity, and inflection points? • Why are the Extreme Value Theorem and the Mean Value Theorem so in study of Calculus? • *How are a function, its first derivative, and its second derivative connec* *another?* • *How do derivatives help us analyze and interpret the behavior of functi*
Misconceptions	*Students may think that…* •*"The derivative is increasing" means the same thing as "the function is in* • *Local extrema are always global extrema* • *If a graph of the derivative is given, we do not have any information ab* *derivative.* • *Looking for whether there is a "smile" or "frown" on any graph indicates*

	function is *concave up or concave down.* *• The graph of a derivative having a positive slope means the function is i* *• Seeing a local extrema on the graph of the derivative can help find the* *original function.* *• Assuming the graph of f , f', f'' are unrelated* *• Confusing points of inflection with local minima/maxima* *Thinking and believing that the Mean Value Theorem always means there*
Transfer	*• Relationships between a function and its derivative are examined in this* *explored* *once the Fundamental Theorem of Calculus and the antiderivative is intro* *Then,* *additional questions about a function, its antiderivative, and its derivative* *investigated. The skills* *used in justifying these connections through equations and written explan* *to* *mathematics.* *• Knowledge of the first and second derivative can assist with solving opt* *involving* *concrete economic and business applications by using the extrema of fu* *• Understanding the relationship between a function and its first and seco* *transitions nicely* *to using these concepts in kinetics in physics.*
Important Facts and Key Concepts	*Students will know…* *•If a function f is continuous over the interval [a, b] and differentiable ove* *then the* *Mean Value Theorem guarantees a point within that open interval where* *rate of* *change equals the average rate of change over the interval.* *•If a function f is continuous over the interval (a, b), then the Extreme Valu*

	guarantees that *f has at least one minimum value and at least one maximum value on [a,* *• A point on a function where the first derivative equals zero or fails to exis* *the function.* • All local (relative) extrema occur at critical points of a function, though n are local extrema. • *The first derivative of a function can provide information about the func* *including* *intervals where the function is increasing or decreasing.* • The first derivative of a function can determine the location of relative (l the function. *•Absolute (global) extrema of a function on a closed interval can only oc* *or at* *Endpoints.* • The graph of a function is concave up (down) on an open interval if the derivative is increasing (decreasing) on that interval. *•The second derivative of a function provides information about the funct* *including* *intervals of upward or downward concavity.* • The second derivative of a function may be used to locate points of infle of the original function. *•The second derivative of a function may determine whether a critical po* *a relative* *(local) maximum or minimum.* *•When a continuous function has only one critical point on an interval on* *critical point* *corresponds to a relative (local) extremum of the function on the interval,* *point also*

	corresponds to the absolute (global) extremum of the function on the inte • Key features of functions and their derivatives can be identified and relc graphical, numerical, and analytical representations. •*Graphical, numerical, and analytical information from f' and f'' can be us* *explain the* *behavior of f.* • Key features of the graphs of f, f', and f'' are related to one another. •*The derivative can be used to solve optimization problems; that is, findin* *maximum* *value of a function on a given interval.* •*Minimum and maximum values of a function take on specific meanings i* •*A point on an implicit relation where the first derivative equals zero or do* *critical point of* *the function.* •*Applications of derivatives can be extended to implicitly defined functio* •*Second derivatives involving implicit differentiation may be relations* x, y,
Standards	
NJSL Content Specific Student Learning Standards **F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key featu intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; s behavior; and periodicity. **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in using technology for more complicated cases. * **F.LE.A.1:** **A.APR.B.3:** Identify zeros of polynomials when suitable factorizations are available, and use the rough graph of the function	

defined by the polynomial.

- **MP.2:** *Reason abstractly and quantitatively.*
- **MP.4:** *Model with mathematics.*
- **MP.6:** *Attend to precision.*

WIDA

ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity (Velocity, acceleration, and rate in/ out)
- Analyzing data and owning problem-solving approaches
- Evaluating rationales, models, and/or interpretations by comparing different justifications
  extrema (first vs
  Second derivative test) and critique reasoning that confuses inflection points with maxima

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated assu
  logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments by proposing the relationships between first and s
  and the
  The shape of graph and prove whether a critical point is a local extrema.

Computer Science and Design Thinking

***8.1.12.AP.5*:** Decompose problems into smaller components through systematic analysis, using c
procedures,
modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and use problem solving pro
authentics
problems.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: Students will receive teacher feedback in groups or individually as problems are
as practice.
In this way, the teacher can assess class-wide competency as students develop and demonstrate their u
skills.
Benchmark Assessment *:* Students will complete questions in WebAssign and show work. Students are aw
Free
Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all
to reach
their solution. Answers without supporting work will usually not receive credit. Thus, students can receive f
submission of
written work for problems to better understand what is expected of them on exams and the AP Exam.

Summative Assessment : Students will complete a unit exam that has both multiple choice and free resp
There will be
a calculator portion and a non-calculator portion to the Unit 6 exam. This follows the format of the AP Co
Sometimes the exam
questions will combine content in surprising ways requiring students to show synthesis of ideas and conce
different
modalities. Students may not have seen problems identical in format during class to those on the exam,
presented
with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 6 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at th
problems will be randomly assigned at the beginning of class to provide a representative sample of stud
previously taught concepts. Misunderstandings can be corrected immediately, before covering addition
depending on prior content. Being able to verbalize understanding of previously assigned problems allow
the mathematics at play in each question and appreciate the logical progression of steps that lead to a
justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

Economics: Knowledge of the first and second derivative can assist with solving optimization pro
concrete economic
and business applications by using the extrema of functions. Maximization/ minimization (

Science: Understanding the relationship between a function and its first and second derivatives using these concepts in kinetics in physics.

Art/ Design: Sketching curve and inflection in graphic design

Accommodations and Modifications

Special Education	Dynamic graphs generated on Desmos can help students to visualize and unders between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its $f''(x)$. Labeling the overlapping graphs can help, but Desmos especially aids stu show or hide graphs so as not to overwhelm students with too much information National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
504	Dynamic graphs generated on Desmos can help students to visualize and unders between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its $f''(x)$. Labeling the overlapping graphs can help, but Desmos especially aids stu show or hide graphs so as not to overwhelm students with too much information National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
At Risk/ I&RS	Dynamic graphs generated on Desmos can help students to visualize and unders between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its $f''(x)$. Labeling the overlapping graphs can provide vocabulary support for the ty wanted, but Desmos especially aids students as you can show or hide graphs so students with too much information at once. Encourage students to label graphs. If given a graph of the original function, draw graph. Label with ++ for large positive slopes, + for positive slopes, 0 for horizont negative slopes, and - - for negative slopes with large magnitudes. Identify what the graph is labeled + (increasing) and - (decreasing). National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
ML	Dynamic graphs generated on Desmos can help students to visualize and unders between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its $f''(x)$. Labeling the overlapping graphs can provide vocabulary support for the ty wanted, but Desmos especially aids students as you can show or hide graphs so students with too much information at once. Encourage students to label graphs with symbols. If given a graph of the original tangent lines on the graph. Label with ++ for large positive slopes, + for positive s horizontal slope, - for negative slopes, and - - for negative slopes with large magn

	is happening when the graph is labeled + (increasing) and - (decreasing). Giving many examples in complete sentences of the explanation and connection that teachers are looking for are helpful for multilingual learners. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
Gifted	Interested students may pursue these connections between all combinations of g on a first derivative graph, what would an extremum mean on the original graph? derivative graph only has one x intercept on an interval, what does that prove abo function on that same interval? If the first derivative is increasing, what does that concavity of the original function? Students can also consider points of inflection derivative is undefined but changes signs. How would that be represented on the derivative? On the original function? Interested students can rigorously pursue additional optimization problems, whic about any subject when one wants to minimize or maximize different quantities g constraints. Interesting problems exist about optimizing the ratio of the height of a radius of a soda can (or any cylinder) given construction materials, laying optima pipelines across a river, or maximizing profit.

Resources

College Board AP Calculus Course and Exam Description (CED)

- AP Classroom problem sets
- WebAssign.
- Khan Academy
- Graphing calculators (TI-84)

Unit 6: Integration and Accumulation

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 6 Topic: Integration and Accumulation** **Duration AB: 4 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	The College board delineates that this unit establishes the relationship between differentiat Fundamental Theorem of Calculus. Students begin by exploring the contextual meaning of bounded by rate functions. Integration determines accumulation of change over an interval determines instantaneous rate of change at a point. Students should understand that integ sum of products (areas) in the same way that differentiation is a limiting case of a quotient Future units will apply the idea of accumulation of change to a variety of realistic and geom
Essential Questions	*What strategies can I use to break down or simplify an integral?* *How are derivatives and integrals related?*
Misconceptions	*Students may think that...* ● Antiderivatives and integrals are exactly the same ● Definite and indefinite integrals are the same ● The chain rule does not impact antiderivatives ● The power rule can be used to find every antiderivative function ● Every integral can be solved
Transfer	*Students will be able to use what they learn to…* ● Students should be able to clearly explain how integrals represent net change and t their solutions both graphically and algebraically. This supports later units in this cou Applications of Integration) and other STEM and STEM adjacent fields (engineering communication of quantitative reasoning is essential. ● Recognizing the integral as the area under a curve, students can analyze cost funct energy usage—transferring the skill into economics, business, and sustainability stu ● In more advanced STEM courses (including AP Physics), students will need to choo graphical, or analytical methods based on given information. This will also be useful 7 and 8 of AP Calculus.
Important Facts and Key Concepts	*Students will know…* ● *The area of the region between the graph of a rate of change function and the x axi change.* ● *In some cases, accumulation of change can be evaluated by using geometry.* ● *If a rate of change is positive (negative) over an interval, then the accumulated char* ● *The unit for the area of a region defined by rate of change is the unit for the rate of c for the independent variable.* ● *Definite integrals can be approximated for functions that are represented graphically and verbally.* ● *Definite integrals can be approximated using a left Riemann sum, a right Riemann s*

	sum, or a trapezoidal sum; approximations can be computed using either uniform or • *Definite integrals can be approximated using numerical methods, with or without tec* • *Depending on the behavior of a function, it may be possible to determine whether a integral is an underestimate or overestimate for the value of the definite integral.* • *The limit of an approximating Riemann sum can be interpreted as a definite integral* • *A Riemann sum, which requires a partition of an interval I, is the sum of products, e the function at a point in a subinterval multiplied by the length of that subinterval of t* • *The definite integral of a continuous function f over the interval [a, b], denoted by* $\int_a^b f$ *Riemann sums as the widths of the subintervals approach 0. That is,* $\int_a^b f(x)dx = max$ *n is the number of subintervals, xΔ i is the width of the ith subinterval, and xi * is a v* • *A definite integral can be translated into the limit of a related Riemann sum, and the be written as a definite integral.* • *The definite integral can be used to define new functions.* • *If f is a continuous function on an interval containing a, then* $\frac{d}{dx}\int_a^x f(t)dt = f(x)$*, whe* • *Graphical, numerical, analytical, and verbal representations of a function f provide in* *g defined as* $g(x) = \int_a^x f(t)dt$ • *In some cases, a definite integral can be evaluated by using geometry and the conn integral and area.* • *Properties of definite integrals include the integral of a constant times a function, the functions, reversal of limits of integration, and the integral of a function over adjacen* • *The definition of the definite integral may be extended to functions with removable d* • *An antiderivative of a function f is a function g whose derivative is f.* • *If a function f is continuous on an interval containing a, the function defined by* $F(x)$ *antiderivative of f for x in the interval.* • *If f is continuous on the interval [a, b] and F is an antiderivative of f, then* $\int_a^x f(t)dt =$ • $\int f(x)$ *is an indefinite integral of the function f and can be expressed as* $\int f(x) = F($ *and C is any constant.* • *Differentiation rules provide the foundation for finding antiderivatives.* • *Many functions do not have closed-form antiderivatives.* • *Substitution of variables is a technique for finding antiderivatives.* • *For a definite integral, substitution of variables requires corresponding changes to th*
Standards	
NJSL Content Specific Student Learning Standards *G.MG.A.2*	

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per

F.IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notatio

F.IF.C.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using techn cases.

A.REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.SSE.A.1

Interpret expressions that represent a quantity in terms of its context.

A.SSE.A.1.a

Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.A.1.b

Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+ a factor not depending on P.

N.Q.A.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret uni choose and interpret the scale and the origin in graphs and data displays.

WIDA

ELD-MA 9-12 Explain Interpretive: Interpret mathematical explanations by analyzing data and owning probl

ELD-MA 9-12 Explain Expressive: Construct mathematical explanations that introduce a mathematical conc solutions with others, and describe data and/or approach used to solve a problem

Computer Science and Design Thinking

- *8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using cons*

modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- *9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas*

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In
assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response
Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answer
usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to be
expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to
justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new m
last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a ca
non-calculator portion to the Unit 6 exam. This follows the format of the AP Calculus exam. Sometimes the exam que
in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Stud
problems identical in format during class to those on the exam, but will have been presented with all the concepts, sk
solve the problems presented on the Unit 6 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board.
randomly assigned at the beginning of class to provide a representative sample of student understanding of previous
Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Bei
understanding of previously assigned problems allows students to process the mathematics at play in each question
progression of steps that lead to an answer with justification. This also gives students who are not presenting the opp
questions.

Interdisciplinary Connections

By recognizing the integral as the area under a curve, students can analyze cost functions, profit over time usage—transferring the skill into economics, business, and sustainability studies. This can connect calculus areas, including physics, biology, climate studies, economics, and business.

Accommodations and Modifications

Special Education	Dynamic graphs generated on Desmos can help students to visualize and understa between definite integrals and the area under a curve. Students can be supported with guided questioning to help students use strategies evaluating integrals either analytically or graphically. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
504	Dynamic graphs generated on Desmos can help students to visualize and understa between definite integrals and the area under a curve. Students can be supported with guided questioning to help students use strategies evaluating integrals either analytically or graphically. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
At Risk / I&RS	Video lectures on AP Classroom are available for students who miss course conten to rewind and pause videos to support note taking can benefit students at risk of fal curriculum. Complex tasks can be chunked into manageable steps. Acknowledge effort and improvement. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
ML	Dynamic graphs generated on Desmos can help students to visualize and understa between definite integrals and the area under a curve. Encourage the use of bilingual dictionaries as needed. Vocabulary building activities can be incorporated.

	Complex tasks can be chunked into manageable steps.
Gifted	Interested students can extend this unit by studying additional types of antiderivativ Calculus: trig substitution, partial fraction decomposition, improper integration, and/ Additionally, students can find data where there is not a known function for the area collected data can be analyzed with Riemann sums.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.o and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multi previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via phy students to bring home) or electronic access through WebAssign.

WebAssign.com (assignments from textbook)

*Note that this is Unit 5 for BC, but Unit 6 for AB

Unit 7: Applications of Integration

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 7 Topic: Applications of Integration** **Duration AB: 4 weeks Duration BC: 2 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students will learn how to find the average va particle motion and net change, and determine areas, volumes, and lengths bc only defined Emphasis will be placed on developing an understanding of integration that can be transfer other applications. Understanding that the area, volume, and length problems studied in thi Riemann sums of rectangle areas, prism volumes, or segment lengths saves students from seemingly unrelated formulas and develops meaningful understanding of integration.
Essential Questions	*How can I relate the position, distance, and displacement to the velocity of an objec* *How can I use integrals to represent accumulation in many different situations and* *How can I calculate the volume of a three dimensional object using two dimensiona*
Misconceptions	*Students may think that...* • Only one function is needed to calculate the volume of any solid of revolution • Volumes of solids of revolution and volumes of perpendicular cross sections same formula • The axis of revolution does not factor into the integral setup • Only one function is needed to calculate the volume of any solid of revolution • The base of a solid of perpendicular cross sections and radius of a solid of re since each quantity is squared in solving for volume • They do not need to visualize the three dimensional shapes but that they can use for every volume problem • they can just memorize integrand expressions rather than understand the dif how the functions relate to the volume • The axis of revolution does not factor into the integral setup for volumes of so
Transfer	*Students will be able to use what they learn to…* • Break down three dimensional objects into two dimensional shapes that are 3D printing objects. • Calculate averages in complex situations, like the average temperature of the • Calculate the position, displacement, and distance traveled over a time interv of an object, as in physics and in unit 9 of AP Calculus BC.
Important Facts and Key Concepts	*Students will know…* • *The average value of a continuous function f over an interval [a, b] is* $\frac{1}{b-a}\int_a^b f(x)dx.$ • *For a particle in rectilinear motion over an interval of time, the definite integral of vel displacement over the interval of time, and the definite integral of speed represents traveled over the interval of time.*

	• *A function defined as an integral represents an accumulation of a rate of change.* • *The definite integral of the rate of change of a quantity over an interval gives the ne that interval.* • *The definite integral can be used to express information about accumulation and ne contexts.* • *Areas of regions in the plane can be calculated with definite integrals.* • *Areas of regions in the plane can be calculated using functions of either x or y.* • *Areas of certain regions in the plane may be calculated using a sum of two or more evaluating a definite integral of the absolute value of the difference of two functions.* • *Volumes of solids with square and rectangular cross sections can be found using de formulas for these shapes.* • *Volumes of solids with triangular cross sections can be found using definite integrals these shapes.* • *Volumes of solids with semicircular and other geometrically defined cross sections c integrals and the area formulas for these shapes.* • *Volumes of solids of revolution around the x- or y-axis may be found by using defini method.* • *Volumes of solids of revolution around any horizontal or vertical line in the plane ma integrals with the disc method.* • *Volumes of solids of revolution around the x- or y-axis whose cross sections are ring definite integrals with the washer method.* • *Volumes of solids of revolution around any horizontal or vertical line whose cross se be found using definite integrals with the washer method.*

Standards

NJSL Content Specific Student Learning Standards

G.SRT.C.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.MG.A.1
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk cylinder).
G.MG.A.2
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile,
G.GMD.B.4
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dime by rotations of two-dimensional objects.
F.IF.B.6
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over Estimate the rate of change from a graph.*
A.CED.A.3
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and int or nonviable options in a modeling context. For example, represent inequalities describing nutritional and co combinations of different foods.
A.CED.A.4
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Fo

Ohm's law V=IR to highlight resistance R.
A.SSE.A.1.b
Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, inte
product of P and a factor not depending on P.
N.Q.A.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and in
formulas; choose and interpret the scale and the origin in graphs and data displays.

WIDA
ELD-MA 9-12 Argue Interpretive: Evaluating relationships among evidence and mathematical principles to c

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using cons
  modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw
  conclusions about the data.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In
assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response
Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers
usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to be
expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to
justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new m
last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a ca
non-calculator portion to the Unit 7 exam. This follows the format of the AP Calculus exam. Sometimes the exam que
in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Stud
problems identical in format during class to those on the exam, but will have been presented with all the concepts, sk
solve the problems presented on the Unit 7 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. C
randomly assigned at the beginning of class to provide a representative sample of student understanding of previous
Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Bei
understanding of previously assigned problems allows students to process the mathematics at play in each question
progression of steps that lead to an answer with justification. This also gives students who are not presenting the opp
questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

Students will

Break down three dimensional objects into two dimensional shapes that are stacked, as we do when

Calculate averages in complex situations, like the average temperature of the earth over a year for c

Students will solve simple physics problems, such as calculating the position, displacement, and dist
interval when given the velocity of an object.

Students will generate integral expressions based on a given rate of change to solve for the net char
functions, such as a supplier receiving new inventory and shipping it out at different rates in business

Accommodations and Modifications

Special Education	Provide students with examples of free response question answers about volumes
perpendicular cross sections. Ask them to identify which type of question goes with
answer. Dynamic graphs generated on Desmos can help students to visualize and understa
between the curve being rotated and the solid generated. Physical representations of volumes of solids of revolution and volumes with perper
be presented so that students can better visualize described shapes by seeing the Complex tasks can be chunked into manageable steps. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a
setting and under the guidance of an AP teacher.

504	Dynamic graphs generated on Desmos can help students to visualize and understa between the curve being rotated and the solid generated. Physical representations of volumes of solids of revolution and volumes with perper be presented so that students can better visualize described shapes by seeing the Complex tasks can be chunked into manageable steps. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
At Risk/ I&RS	There are various murals on the 4th floor of Morristown High School that can be vis calculus problems from this unit (spinning top, Remy cooking soup, Totoro eating ac honey, etc) to help engage students. Teachers can create context specific problems that may appeal to at risk students to the content. Complex tasks can be chunked into manageable steps. Acknowledge effort and improvement. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
ML	Encourage the use of bilingual dictionaries as needed, especially to translate the co Focus on vocabulary for motion, including position, velocity, acceleration, speeding left, moving right, standing still, etc. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
Gifted	Interested students can extend this unit to calculating volumes of solids of revolutio Students can also create problems with an appropriate real world context of “rate in
Resources	

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.o and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multi previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via phy students to bring home) or electronic access through WebAssign.

WebAssign.com (assignments from textbook)

*Note that this is unit 7 for AB, but Unit 6 for BC

Unit 8: Differential Equations

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 8 Topic: Differential Equations** **Duration AB: 2 weeks Duration BC: 2 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students will learn to set up and solve separa Slope fields can be used to represent solution curves to a differential equation and build un infinitely many general solutions to a differential equation, varying only by a constant of inte a unique solution relevant to a particular situation, provided they can locate a point on the s solving differential equations leading to models for exponential growth and decay and logis build understanding of topics introduced in Algebra II and other courses.
Essential Questions	*When given a differential equation with two variables, how can we find an antideriva* *If we cannot solve an antiderivative, what methods can be used to approximate an a* *analytically?*
Misconceptions	*Students may think that...* ● All differential equations are only dependent on one variable ● All differential equations can be solved analytically ● Graphing slope fields is the same as finding a solution curve ● Tangent approximations are the same as exact solutions
Transfer	*Students will be able to use what they learn to…* ● Recognize situations, such as population growth, radioactive decay, drug cor bloodstream, compound interest, or Newton's law of cooling as governed by ● Use differential equations to model real life scenarios and find the equilibrium temperature ● Recognize that directed graphs can be used to predict the weather
Important Facts and Key Concepts	*Students will know…* ● *Differential equations relate a function of an independent variable and the function's* ● *Derivatives can be used to verify that a function is a solution to a given differential e* ● *There may be infinitely many general solutions to a differential equation.* ● *A slope field is a graphical representation of a differential equation on a finite set of* ● *Slope fields provide information about the behavior of solutions to first-order differer* ● *Solutions to differential equations are functions or families of functions.* ● *Some differential equations can be solved by separation of variables.* ● *Antidifferentiation can be used to find general solutions to differential equations.* ● *A general solution may describe infinitely many solutions to a differential equation. T* *solution passing through a given point.* ● *The function F defined by* $F(x) = y_0 + \int_a^x f(t)dt$ *is a particular solution to the differer*

	satisfying F(a)=y_0. ● *Solutions to differential equations may be subject to domain restrictions.* ● *Specific applications of finding general and particular solutions to differential equatio line and exponential growth and decay.* ● *The model for exponential growth and decay that arises from the statement "The ra proportional to the size of the quantity" is* $\frac{dy}{dt} = ky$. ● *The exponential growth and decay model,* $\frac{dy}{dt} = ky$, *with initial condition y = y_0. whe form y = $y_0.e^{kt}$.* *For BC Calculus Only:* ● *Euler's method provides a procedure for approximating a solution to a differential eq* ● *solution curve.* ● *The model for logistic growth that arises from the statement "The rate of change of a proportional to the size of the quantity and the difference between the quantity and t* $\frac{dy}{dt} = ky(a-y)$. ● *The logistic differential equation and initial conditions can be interpreted without sol* ● *The limiting value (carrying capacity) of a logistic differential equation as the indepe infinity can be determined using the logistic growth model and initial conditions.* ● *The value of the dependent variable in a logistic differential equation at the point wh be determined using the logistic growth model and initial conditions.*

Standards

NJSL Content Specific Student Learning Standards

F.IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use funct context.

F.IF.C.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and u complicated cases. *

A.CED.A.2

Create equations in two or more variables to represent relationships between quantities; graph equations o labels and scales.

WIDA Standards for Multilingual Learners

- ELD-MA 9-12 Explain Expressive: Share solutions with others

Computer Science and Design Thinking

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In
assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response
Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answer
usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to be
expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to
justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new m
last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a ca
non-calculator portion to the Unit 8 exam. This follows the format of the AP Calculus exam. Sometimes the exam que
in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Stud
problems identical in format during class to those on the exam, but will have been presented with all the concepts, sk
solve the problems presented on the Unit 8 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board.
randomly assigned at the beginning of class to provide a representative sample of student understanding of previous
Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Bei
understanding of previously assigned problems allows students to process the mathematics at play in each question
progression of steps that lead to an answer with justification. This also gives students who are not presenting the opp
questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

- Recognize situations in biology, ecology, chemistry, finance, engineering, and physics, such as popu
  decay, drug concentrations in the bloodstream, compound interest, or Newton's law of cooling as gov
  equations.

- Use differential equations to model real life scenarios and find the equilibrium of a population or temp

- Recognize that directed graphs can be used to predict the weather

Accommodations and Modifications	
Special Education	Use graphic organizers to identify paths for solving differential equations step-by-ste Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*$ n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the and y_{n+1}.
504	Use structured peer discussions to explain solution steps in solving differential equa learning. Use graphic organizers to identify paths for solving differential equations step-by-ste Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*$ n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the and y_{n+1}.
At Risk/ I&RS	Use graphic organizers to identify paths for solving differential equations step-by-ste Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*$ n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the and y_{n+1}. Implement chunked instruction: break up lessons into short, manageable tasks. Use real-world modeling contexts (e.g., population growth, cooling, interest) to mak tangible. Use structured peer discussions to explain solution steps in solving differential equa learning to engage students with their peers.
ML	Use graphic organizers to identify paths for solving differential equations step-by-ste Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*$ n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the and y_{n+1}. Implement chunked instruction: break up lessons into short, manageable tasks. Use structured peer discussions to explain solution steps in solving differential equa learning to engage students with their peers.
Gifted	For interested students, offer extension problems involving programming computers iterations of Euler's Method. Interested students can also learn more general population modeling with differentia population may have multiple equilibria Interested students may study non-separable differential equations, such as homog equations.
Resources	

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.o and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multi previously released AP Calculus exams)

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WebAssign.com (assignments from textbook)

Unit 9: Parametric and Polar Calculus

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus BC Only** **Unit 9 Topic: Parametric and Polar Calculus** **Duration: 3 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students will build on their understanding of s problems in which particles are moving along curves in the plane. Students will define para vector-valued functions to describe planar motion and apply calculus to solve motion probl polar equations are a special case of parametric equations and will apply calculus to analyz lengths and areas. This unit should be treated as an opportunity to reinforce past learning a skills to new situations, rather than as a new list of facts or strategies to memorize.
Essential Questions	*How do we calculate slope for curves not constructed in the Cartesian plane?* *How do we calculate the area under curves not graphed in the Cartesian plane?* *How can we notate the different rates of change suggested by multiple derivatives?*
Misconceptions	*Students may think that...* ● X and t can be used interchangeably for parametric equations ● Y must always be dependent on x to be a function ● Integrating polar area has area swept from left to right or top to bottom. ● The order of bounds does not matter for polar integration. ● The term “derivative” always means dy/dx instead of dy/dt, dx/dt, dr/dt, dy/dx rate of change.
Transfer	*Students will be able to use what they learn to…* ● Translate between multiple representations of curves. In Multivariable Calcul graph in spherical and cylindrical coordinates. Graphing in polar helps to pre in three dimensional systems other than Cartesian. ● Learning to work with parameters will also help prepare students for paramet multivariable calculus. Parameters are also often used in computer programm animation. ● Model circular or spiral motion in physics and multivariable calculus with para
Important Facts and Key Concepts	*Students will know…* ● *Methods for calculating derivatives of real-valued functions can be extended* ● *For a curve defined parametrically, the value of dy/dx at a point on the curve*

	tangent to the curve at that point. dy/dx, the slope of the line tangent to a cur parametric equations, can be determined by dividing dy/dt by dx/dt , provided zero. ● $\frac{d^2y}{dx^2}$ *can be calculated by dividing* $\frac{d}{dt}$ $(\frac{dy}{dx})$*by dx/dt.* ● *The length of a parametrically defined curve can be calculated using a defini* ● *Methods for calculating derivatives of realvalued functions can be extended t* ● *Methods for calculating integrals of real-valued functions can be extended to vector-valued functions.* ● *Derivatives can be used to determine velocity, speed, and acceleration for a curve in the plane defined using parametric or vector-valued functions.* ● *For a particle in planar motion over an interval of time, the definite integral of represents the particle's displacement (net change in position) over the interv might determine its position. The definite integral of speed represents the pa traveled over the interval of time.* ● *Methods for calculating derivatives of realvalued functions can be extended t coordinates.* ● *For a curve given by a polar equation r = f(θ), derivatives of r, x, and y with re second derivatives of y with respect to x can provide information about the cu* ● *The concept of calculating areas in rectangular coordinates can be extended* ● *Areas of regions bounded by polar curves can be calculated with definite inte*

Standards

NJSL Content Specific Student Learning Standards

G.C.B.5

Find arc lengths and areas of sectors of circles

N.VM.A.1

Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directe appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).

N.VM.A.2.

Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a t

N.VM.A.3.

Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.B.5

Multiply a vector by a scalar.

WIDA Standards for Multilingual Learners

ELD-MA 9-12 Explain Expressive

- Construct mathematical explanations that share solutions with others
- Construct mathematical explanations that describe data and/or approach used to solve a problem

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using cons modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to be expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new m last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a ca non-calculator portion to the Unit 9 exam. This follows the format of the AP Calculus exam. Sometimes the exam que in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Stud problems identical in format during class to those on the exam, but will have been presented with all the concepts, sk solve the problems presented on the Unit 9 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. randomly assigned at the beginning of class to provide a representative sample of student understanding of previous Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Bei understanding of previously assigned problems allows students to process the mathematics at play in each question progression of steps that lead to an answer with justification. This also gives students who are not presenting the opp questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

- Translate between multiple representations of curves. In Multivariable Calculus, students will need to cylindrical coordinates. Graphing in polar helps to prepare students for graphing in three dimensiona Cartesian.
- Learning to work with parameters will also help prepare students for parameterizing a curve in multiv Parameters are also often used in computer programming and basic computer animation.
- Model circular or spiral motion in physics and multivariable calculus with parametric and polar calculu

Accommodations and Modifications

Special Education	Provide color-coded visuals for parametric curves and vector components in Desmo Show area sweeps in Wolfram Demonstrations to illustrate how area is accumulate Allow access to Desmos for graphing when a graph is not provided. Provide extended time as directed by individual education plans. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
504	Provide color-coded visuals for parametric curves and vector components in Desmo Show area sweeps in Wolfram Demonstrations to illustrate how area is accumulate Allow access to Desmos for graphing when a graph is not provided. Provide extended time as directed by 504 plans. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
At Risk/ I&RS	Use chunked instruction in first teaching parametric derivatives and then derivatives National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
ML	Provide examples of common wording of questions for Free Response Questions in calculus. Provide color-coded visuals for parametric curves and vector components in Desmo Show area sweeps in Wolfram Demonstrations to illustrate how area is accumulate National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
Gifted	Interested students can program a simple animation based on parameters. Interest further study limacons and cardioids in polar form.

Resources	
College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.o and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multi previously released AP Calculus exams) *Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)* *Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via phy students to bring home) or electronic access through WebAssign.* *WebAssign.com (assignments from textbook)*	

Unit 10: Power Series for Approximation

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus BC Only** **Unit 10 Topic: Power Series for Approximation** **Duration: 3 weeks**
Unit Summary (Enduring Understandings)	In this unit, students will extend the idea of a tangent line approximating a curve to a polyno curve. Since the first derivatives of the tangent line and the curve match at a point of tange approximation, power series extend the concept to matching second derivatives so that the approximation matches, and so on to third, fourth, and nth derivatives. In fact, an infinite po given function. Students will use sigma notation to describe the patterns of power series w
Essential Questions	*How can a polynomial match a given function?* *How closely does a polynomial approximate a given function?* *How can I notate the pattern for an infinite series?*
Misconceptions	*Students may think that...* ● The number of terms in a series is the same as the order of the series ● A memorized Maclaurin series can be substituted with (x-a) to build a Taylor ● Multiplying a series is the same as substituting a variable ● We cannot intuit any information about a derivative from a given Taylor series ● Error bound is always based on the next degree term, even if it is 0
Transfer	*Students will be able to use what they learn to…* ● Model oscillations or wave behavior, such as sound waves when recording m ● Understand that a signal can be broken down into a series (Fourier series) fo electrical engineering ● Understand Euler's identity $e^{i\pi} + 1 = 0$
Important Facts and Key Concepts	*Students will know…* ● The nth partial sum is defined as the sum of the first n terms of a series ● An infinite series of numbers converges to a real number S (or has sum S), if sequence of partial sums exists and equals S. ● A geometric series is a series with a constant ratio between successive term ● If a is a real number and r is a real number such that |r| <1, then the geometr ● The nth term test is a test for divergence of a series. ● In addition to geometric series, common series of numbers include the harmo harmonic series, and p-series. ● If an alternating series converges by the alternating series test, then the alter can be used to bound how far a partial sum is from the value of the infinite se ● The coefficient of the nth degree term in a Taylor polynomial for a function f c

	• In many cases, as the degree of a Taylor polynomial increases, the nth degre approach the original function over some interval. • Taylor polynomials for a function f centered at x = a can be used to approxim near x = a. • The Lagrange error bound can be used to determine a maximum interval for polynomial approximation to a function. • In some situations, the alternating series error bound can be used to bound t polynomial approximation to the value of a function. • A power series is a series of the form $\sum_{n=0}^{\infty} a_n(x - r)$, where n is a non-negativ sequence of real numbers, and r is a real number. • A Taylor polynomial for f(x) is a partial sum of the Taylor series for f(x). • The Maclaurin series for $\frac{1}{1-x}$ is a geometric series. • The Maclaurin series for sin x, cos x, and e^x provides the foundation for cons series for other functions. • Using a known series, a power series for a given function can be derived usi term-by-term differentiation or term-byterm integration, and by various metho processes, substitutions, or using properties of geometric series).

Standards

NJSL Content Specific Student Learning Standards

F.BF.A.1

Write a function that describes a relationship between two quantities.

F.BF.A.1.c

(+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon

F.BF.A.2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model between the two forms. *

F.IF.A.3

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the i Fibonacci sequence is defined recursively by f(0)=f(1)=1, f(n+1)=f(n)+f(n-1) for n>1.

F.IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numeric descriptions).

A.REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the pre the assumption that the original equation has a solution. Construct a viable argument to justify a solution m

A.SSE.A.1.a

Interpret parts of an expression, such as terms, factors, and coefficients.

WIDA Standards for Multilingual Learners

- ELD-MA.9-12.Argue.Expressive: Construct mathematics arguments that Generalize logical relations
- ELD-MA.-12 Explain Interpretive: Interpret mathematical explanations by identifying concept or entity

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using cons modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem s

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment

Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment

Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to be expected of them on exams and the AP Exam.

Students will complete three quizzes in this unit. Two of the quizzes will consist of open-ended problems for students justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new m last quiz or exam. The other quiz will be a memorization quiz on the four series (sinx, cosx, e^x, 1/(1-x)) mentioned in t description.

Summative Assessment

Students will complete a unit exam that has both multiple choice and free response question types. The unit 10 exam follows the format of the AP Calculus exam questions on sequences, series, and Taylor Series. Sometimes the exam content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalitie seen problems identical in format during class to those on the exam, but will have been presented with all the concep necessary to solve the problems presented on the Unit 10 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. randomly assigned at the beginning of class to provide a representative sample of student understanding of previous Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Bei understanding of previously assigned problems allows students to process the mathematics at play in each question

progression of steps that lead to an answer with justification. This also gives students who are not presenting the opp
questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

- Model oscillations or wave behavior, such as sound waves when recording music in music or audio e
- Understand that a signal can be broken down into a series (Fourier series) for signal processing in e

Accommodations and Modifications

Special Education	Students can develop a graphic organizer for the unit to understand what series mu can be done to manipulate known series into specific functions. Dynamic representations of Taylor polynomials of up to 10 degrees modeling particu exponential, rational) on Desmos can help to visually illustrate the closer and closer the error bound. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
504	Dynamic representations of Taylor polynomials of up to 10 degrees modeling particu exponential, rational) on Desmos can help to visually illustrate the closer and closer the error bound. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
At Risk/ I&RS	Priming with sigma notation, geometric series, and arithmetic series from PreCalcul prepare to succeed in this unit. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one a
ML	Dynamic representations of Taylor polynomials of up to 10 degrees modeling particu exponential, rational) on Desmos can help to visually illustrate the closer and closer the error. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.

	AP Collaboration Hub is available after school for students to collaborate with one a setting and under the guidance of an AP teacher.
Gifted	Interested students can explore the concept of a Fourier transform for modeling wa end of year projects creating models from Taylor polynomials.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.o and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and mult previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

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WebAssign.com (assignments from textbook)

Unit 11: Series Convergence

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus BC Only** **Unit 11 Topic: Series Convergence** **Duration: 2.5 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students need to understand that a sum of in converge to a finite value. They can develop intuition by exploring graphs, tables, and symbo that converge and diverge and for Taylor polynomials that converge to a function. Students past learning, such as how evaluating improper integrals relates to the integral test or how series to represent continuous functions relates to differentiation and integration. Students memorizing a long list of tests and procedures typically find little success achieving a lastin of series.
Essential Questions	*What can cause an infinite series to sum to a finite value?* *How can we know if a series is getting small enough quickly enough to converge?* *How can we find the values of x that will ensure that a power series converges?*
Misconceptions	*Students may think that...* ● All series with fractions converge. ● P series is any series with a polynomial in the denominator of the term. ● The integral test tells us the value that the corresponding series converges to converges or not. ● Comparison tests can be used with any series rather than positive, decreasin ● Getting an infinite limit (or zero) from the ratio test implies an infinite radius o ● They do not need to check endpoints in the interval of convergence
Transfer	*Students will be able to use what they learn to…* ● Test endpoints of an interval of convergence for convergence with the other c in the unit. ● Model probability distributions with series in statistics and probability courses ● Model in physics and engineering with waveforms and electrical circuits ● Select appropriate methods for testing abstract concepts in analysis, quantita analysis, etc as students learn to apply convergence tests to infinite series
Important Facts and Key Concepts	*Students will know…* ● The nth term test is a test for divergence of a series. ● The integral test is a method to determine whether a series converges or diverges. ● In addition to geometric series, common series of numbers include the harmonic se series, and p-series. ● The comparison test is a method to determine whether a series converges or diverg ● The limit comparison test is a method to determine whether a series converges or d ● The alternating series test is a method to determine whether an alternating series c ● The ratio test is a method to determine whether a series of numbers converges or d ● A series may be absolutely convergent, conditionally convergent, or divergent.

	• If a series converges absolutely, then it converges. • If a series converges absolutely, then any series obtained from it by regrouping or re same value. • If a power series converges, it either converges at a single point or has an interval c • The ratio test can be used to determine the radius of convergence of a power series • The radius of convergence of a power series can be used to identify an open interva converges, but it is necessary to test both endpoints of the interval to determine the • If a power series has a positive radius of convergence, then the power series is the to which it converges over the open interval. • The radius of convergence of a power series obtained by term-by-term differentiatio is the same as the radius of convergence of the original power series.

Standards

NJSL Content Specific Student Learning Standards

A.SSE.A.1.a

Interpret parts of an expression, such as terms, factors, and coefficients.

A.REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the pre the assumption that the original equation has a solution. Construct a viable argument to justify a solution m

F.IF.A.3

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the in Fibonacci sequence is defined recursively by f(0)=f(1)=1, f(n+1)=f(n)+f(n-1) for n>1.

WIDA Standards for Multilingual Learners

- ELD-MA 9-12 Explain Expressive: Construct mathematical explanations that state reasoning used to solutions
- ELD-MA.9-12.Argue.Expressive: Construct mathematics arguments that create precise conjecture, u established results, and stated assumptions

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using cons modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment

Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to be expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new m last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. The unit 11 exam follows the format of the AP Calculus exam questions on convergence of series. Sometimes the exam questions will ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and to problems presented on the Unit 11 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. randomly assigned at the beginning of class to provide a representative sample of student understanding of previous Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Bei understanding of previously assigned problems allows students to process the mathematics at play in each question progression of steps that lead to an answer with justification. This also gives students who are not presenting the opp questions.

Interdisciplinary Connections

Students will be able to use what they learn to…

- Model probability distributions with series in statistics and probability courses
- Model in physics and engineering with waveforms and electrical circuits
- Select appropriate methods for testing abstract concepts in analysis, quantitative finance, statistical a learn to apply convergence tests to infinite series

Accommodations and Modifications

Special Education	Physical representations of a p series converging (tearing a piece of paper in half o infinitely many times) can help students to understand that for an infinite series to c each term must be decreasing. Students can create a concept map or flowchart as a graphic organizer to help deci is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections an

	different series convergence tests.
504	Physical representations of a p series converging (tearing a piece of paper in half o infinitely many times) can help students to understand that for an infinite series to c each term must be decreasing. Students can create a concept map or flowchart as a graphic organizer to help deci is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections an different series convergence tests.
At Risk/ I&RS	Physical representations of a p series converging (tearing a piece of paper in half o infinitely many times) can help students to understand that for an infinite series to c each term must be decreasing. Students can create a concept map or flowchart as a graphic organizer to help deci is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections an different series convergence tests.
ML	Physical representations of a p series converging (tearing a piece of paper in half o infinitely many times) can help students to understand that for an infinite series to c each term must be decreasing. The physical representation can help anchor vocabu "term". Students can create a concept map or flowchart as a graphic organizer to help deci is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections an different series convergence tests.
Gifted	Interested students can explore the Riemann rearrangement theorem for conditiona Interested students could also explore different sizes of infinity with cardinality.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.o and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multi previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via phy students to bring home) or electronic access through WebAssign.

WebAssign.com (assignments from textbook)