AP Calculus AB and BC

Full Year

This is a comprehensive calculus course that follows a College Board approved syllabus. Students are expected to have completed Pre Calculus Honors before taking AP Calculus

According to the College Board: In Calculus AB students will see that Calculus helps make sense of change by grappling with questions that inspire thinkers from around the globe, across time, and in many disciplines. Can change occur in an instant? When is the next solar eclipse or the turning point for an economy? In AP Calculus AB, you'll develop a deeper understanding of mathematical principles that can help you answer questions such as these.

In AP Calculus BC, you'll cover many of the mathematical principles in AP Calculus AB and build upon them. AP Calculus BC helps you prepare for further study in mathematics and other disciplines, such as engineering, computer science, or economics.

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Unit Plan Explanatory Document

Grade Level: 12	**AP Calculus AB and BC** **Unit 1 Topic: Limits** **Duration AB: 3 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	According to the College Board description: Limits introduce the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as x approaches a point. This distinction allows us to extend understanding of asymptotes and holes in graphs with formal definitions of continuity. Consider reviewing rational functions when introducing limits, rather than beginning the year with a full review of precalculus topics. Limits are the foundation for differentiation (Unit 2), integration (Unit 6), and infinite series (Unit 10) bc only. They are the basis for important definitions and for theorems that are used to solve realistic problems involving change and to justify conclusions.
Essential Questions	• *What is a limit and how can it be evaluated?* • *What does it mean for a function to be continuous?* • *In what way do limits connect to the algebraic concept of slope and rate of change?* • *How is continuity used to identify the existence of important values of a function?*
Misconceptions	*Students may think that…* • *The function must be defined at a point for the limit to exist.* • *Infinite limits are actual function values.* • *Difference between does not exist and indeterminate*

	• *0/0 is an acceptable answer.* .
Transfer	*Students will be able to use what they learn to…* • *Analyze and interpret the behavior of functions and evaluate the limit from graphs and equations or by using the Squeeze Theorem.* • *Explain the conditions that make a function continuous and use the intermediate value theorem to explore continuity.* • *Recognize and classify different types of discontinuities.* • *Use limits to support real-world modeling such as using Instantaneous Speed in Physics & Motion* •
Important Facts and Key Concepts	*Students will know…* • *the difference between average and instantaneous velocity.* • *Limits can be expressed in multiple ways, including graphically, numerically, and analytically.* • *One-sided limits describe behavior from the left or right and helps to determine if a limit exists.* • *A function is continuous at a point if the limit as x approaches a exists, the function is defined (f(a) is defined, and f(a) is equal to its limit.* • *The Intermediate Value Theorem guarantees values between two endpoints or takes on every value between f(a) and f(b) if the function is continuous.*
Standards	
NJSL Content Specific Student Learning Standards	

F-IF.B.4: Interpret functions that arise in applications in context.

F-IF.C.7: Graph functions expressed symbolically and show key features of the graph.

N-Q.A.3 : Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

WIDA Standards for Multilingual Learners

Multilingual learners will..

Interpret mathematical explanations by

- Identifying concept or entity
- Analyzing data and problem-solving approaches
- Evaluating rationales, models, and interpretations based on evidence and math principles

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions.

Computer Science and Design Thinking

8.1.12.DA.5 : Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Career Readiness, Life Literacies, and Key Skills

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills to solve authentic problems.

Assessment Evidence

Students will demonstrate understanding by...

Formative Assessment *: Classwork and homework assignments evaluating limits*

Benchmark Assessment *:Quizzes and WebAssign Assignments.*

Summative Assessment : *Unit test at the end of the unit*
Alternative Assessment*: Students may have the option to give a presentation.*

Interdisciplinary Connections

• In physics: Students will connect limits to motion using instantaneous and average velocity to model real world motion of objects.
• In Biology: Limits support the knowledge of population growth models and rate of change in biological processes.
• In Business: Students can use limits to look at how marginal cost and marginal revenue change as production quantities get closer to a certain value.

Standards for Cross-Content Areas:
Science Standards:
HS-PS1-1: Use the periodic table as a model to predict the relative properties of elements.

Accommodations and Modifications

Special Education	• Accommodations consistent with IEP

	• *Repeat/confirm directions* • Extended time • Guided notes • Preferential seating • Chunking material • Modified assignments • NHS Tutor (Mu Alpha Theta) • Provide a calculator
504	• Accommodations consistent with 504 • Extended time • Preferential seating • Tutoring Center • *Check-ins for understanding* • *Break tasks into smaller chunks*
At Risk/ I&RS	• *Scaffolded instruction with peer support* • *Frequent feedback and review sessions* • Tutoring Center • Parent Communication • MTSS (Multi-Tiered System of Supports)
ML	• *Visual word walls* • *Sentence frames for math explanations* • *Support with bilingual dictionaries or glossaries* • *Diagrams and drawings*

	•Working with partners or small group
Gifted	*• Challenge problems and free response questions with later topics* *• Early exposure to AP Calculus BC concepts (L'Hospital's rule for indeterminate forms)* *• Differentiated formative assessments to develop learning experiences that challenge students with gifts and talents.* *• Standardized (adaptive, above-grade-level) and classroom assessments that can measure the academic progress of students with gifts and talents.*
Resources	
• AP Classroom and College Board materials. *• WebAssign.* *• Khan Academy and Desmos.*	

Grade Level: High School	**AP Calculus AB and BC** **Unit 2 Topic: Differentiation: Definition and Fundamental Properties** **Duration AB: 2 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	*According to the College Board description: Derivatives allow us to determine instantaneous rates of change. To develop understanding of how the definition of the derivative applies limits to average rates of change, create opportunities for students to explore average rates of change over increasingly small intervals. Graphing calculator explorations of how various operations affect slopes of tangent lines help students to make sense of basic rules and properties of differentiation. Encourage students to apply the order of operations as they select differentiation rules. Developing differentiation skills will allow students to model realistic instantaneous rates of change in Unit 4 and to analyze graphs in Unit 5.*
Essential Questions	• What are the different methods I can use to find a derivative? • How can derivatives help us analyze and describe the behavior of a function? • How are continuity and differentiability connected? • Why does continuity matter when finding maximums and minimums? • How can I use derivatives to solve real-life problems or model real situations? • How can the product and quotient rules be applied to differentiate trigonometric functions?
Misconceptions	*Students may think that…* • All continuous functions are differentiable. • They can use the power rule to find the derivative of every function. • Students think they can just divide the derivatives: $(\frac{f}{g})' = \frac{f'}{g'}$ instead of using the quotient

	rule. • Multiplying functions and composing functions are the same, for example $x^2\sin(x)$ and $\sin(x^2)$. • They need to take the derivative of every function, regardless or whether they are composed: $f'(g'(x))$ rather than $f'(g(x)) \cdot g'(x)$
Transfer	*Students will be able to use what they learn to…* • Use derivatives in real life application of derivatives problems, like rectilinear motion in Unit 5 and related rates in Unit 4. • Use derivatives when analyzing curves to find a minimum, maximum, or critical point in Unit 5. • Apply the chain rule to take derivatives implicitly in Unit 4. • Compare rates of change for indeterminate limits by taking derivatives to apply L'Hospital's rule in Unit 4. • Find antiderivatives in Unit 6 by knowing which functions have a derivative as the given function. • use u substitution for finding antiderivatives of composed functions in Unit 6 by understanding the chain rule for finding derivatives of composed functions.
Important Facts and Key Concepts	*Students will know…* • *The difference quotients* $\frac{f(a+h)-f(a)}{h}$ *and* $\frac{f(x)-f(a)}{x-a}$ *express the average rate of change of a function over an interval.* • The instantaneous rate of change of a function at x a = can be expressed by $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{h\to 0} \frac{f(x)-f(a)}{x-a}$ provided the limit exists. These are equivalent forms of the definition of the derivative and

	are denoted f'(a). • The derivative of f is the function whose value at x is $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, provided this limit exists. • for $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, $and\ y'$. • The derivative can be represented graphically, numerically, analytically, and verbally. • The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point. • The derivative at a point can be estimated from information given in tables or graphs. *•Technology can be used to calculate or estimate the value of a derivative of a function at a point.* *• If a function is differentiable at a point, then it is continuous at that point. In particular, if a point is not in the domain of f, then it is not in the domain of f'.* • A continuous function may fail to be differentiable at a point in its domain. •Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form $f(x) = x^r$. •Sums, differences, and constant multiples of functions can be differentiated using derivative rules. • The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions. • Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions. • In some cases, recognizing an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit. *• Derivatives of products of differentiable functions can be found using the product rule.* • Derivatives of quotients of differentiable functions can be found using the quotient rule.

	• Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules.

Standards

NJSL Content Specific Student Learning Standards

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph.

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

***F.IF.C.9*:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables)

***F.LE.A.1.c*:** Recognize situations in which a quantity grows or decays proportionally.

***F.LE.A.2*:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.

***F.LE.A.3*:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function.

WIDA

ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity
- Analyzing data and owning problem-solving approaches
- Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles

ELD-MA 9-12 Explain Expressive:

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated assumptions
- Generalize logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments

Computer Science and Design Thinking

***8.1.12.DA.5*:** Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices.

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations.

Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.4: Identify issues associated with student loan debt, requirements for repayment, and consequences of failure to repay student loan debt. Analyze and calculate rates (financial rates of change, growth models).

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas(choosing appropriate differentiation strategies).

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: *Classwork and Homework assignments evaluating derivatives of basic functions*

Benchmark Assessment *Quizzes on basic derivatives, and limits of a difference quotients*

Summative Assessment *AP Style test covering Multiple Choice and Open Ended AP questions covering basic derivatives and the difference quotient*

Alternative Assessment: *Group presentations of classwork or homework. Students teach others about how to evaluate a derivative and what it represents.*

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

• *In Business:* Students will apply derivatives to cost and revenue functions. Derivatives show how much cost and profit varies while making or selling one more item.
• In physics: Students will relate derivatives to velocity and acceleration as measures of positional change.
• In Biology: Students will learn how to use derivatives to calculate how fast a population grows.

Accommodations and Modifications

Special Education	• Preferential seating • Give step-by-step examples of derivatives in the guided notes. • Provide assignments in chunks that concentrate on a single differentiation rule at a time.

	• Allow for extra time. • Accommodations consistent with IEP • Modified assignments • NHS Tutor (Mu Alpha Theta) • Provide a calculator
504	• *Assign less work while maintaining rigor.* • *Extended time.* • *Allow use of calculator* • NHS Tutor (Mu Alpha Theta)
At Risk/ I&RS	• *Offer structured practice problems transitioning from the basic power rule to the product and quotient rules* • *Frequent check-ins to assess progress and quickly clear up any confusion.* • *Pair students with peers who can help them.*
ML	• *Use graphs of tangent lines and slopes to help explain concepts.* • *Allow the use of translation tools or bilingual glossaries for key terms .* • *Provide vocabulary support for derivative notations* $\frac{dy}{dx}$, $f'(x)$, $and\ y'$. • *Working with partners or small group*
Gifted	• *Introduce advanced concepts like implicit differentiation at an earlier stage.* • *Give real-world problems to solve and let students try projects using derivatives such as using*

	derivatives to model speed, fuel use, and cost. •

Resources
• College Board AP Calculus Course and Exam Description (CED) • Graphing calculators (TI-84) • *WebAssign.* • *Khan Academy: AP Calculus AB Differentiation modules and Desmos.* • *Desmos for interactive graphing.* • *AP Classroom practice problem.*

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 3 Topic: Differentiation: Composite, Implicit, and Inverse Functions** **Duration AB: 3.5 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	In this unit, students learn how to differentiate composite functions using the chain rule and apply that understanding to determine derivatives of implicit and inverse functions. Students need to understand that for composite functions, y is a function of u while u is a function of x. Leibniz notation for the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, accounts for these relationships. Units analysis can strengthen the connection, as in $\frac{psi}{min} = \frac{psi}{m} \cdot \frac{m}{min}$. Saying, "times the derivative of what's inside," every time we apply the chain rule reminds students to avoid a common error. Mastering the chain rule is essential to success in all future units.
Essential Questions	• *How can implicit differentiation be used to find derivatives when a function is not explicitly solved for y?* • *How is the derivative of an inverse function related to the derivative of the original function?* • In what situations is it appropriate (or not appropriate) to apply L'Hôpital's Rule?
Misconceptions	*Students may think that…* • Forgetting the chain rule in composite functions. • Any limit can be evaluated with L'Hospital's rule • Students may stop half way leaving $\frac{dy}{dx}$ in multiple terms without solving for it completely.

	• Students may assume $(f^{-1})'(x) = \frac{1}{f'(x)}$ instead of the correct formula $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$
Transfer	*Students will be able to use what they learn to…* • *Differentiate with respect to a* *ny variable, which can help calculus transfer "rate of change" to many other subjects and contexts* • *Prepare for differentiating parametric curves in Unit 9.* • *Understand how to deal with functions that are not explicitly defined, as separable differential equations are in Unit 8.*
Important Facts and Key Concepts	*Students will know…* • *The chain rule provides a way to differentiate composite functions.* • The chain rule is the basis for implicit differentiation. • The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists. •The chain rule applied with the definition of an inverse function, or the formula for the derivative of an inverse function, can be used to find the derivatives of inverse trigonometric functions. • Differentiating f ' produces the second derivative f ", provided the derivative of f ' exists; repeating this process produces higher-order derivatives of f. • Higher-order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y ". Higher-order derivatives can be denoted $\frac{d^ny}{dx^n}$, $f^n(x)$.

Standards

NJSL Content Specific Student Learning Standards

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph.

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

***F.IF.C.9*:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables)

F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.A.2: Write arithmetic and geometric sequences both recursively and explicitly, use them to model situations, and translate between the two forms.

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) and for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.LE.A.1.c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

WIDA

ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity
- Analyzing data and owning problem-solving approaches

- Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles

ELD-MA 9-12 Explain Expressive:

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated assumptions
- Generalize logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments

Computer Science and Design Thinking

***8.1.12.DA.5*:** Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices.

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations.

8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a systemic plan of investigation, and propose an innovative sustainable solution.

Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.4: Identify issues associated with student loan debt, requirements for repayment, and consequences of failure to repay student loan debt.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and use problem solving processes.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.CT.3: Collaborate with individuals to analyze a variety of potential solutions.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: Daily warm-ups on chain rule practice and exit tickets for implicit differentiation and inverse functions.
Benchmark Assessment *: Quizzes on chain rule, implicit, and inverse differentiation.*
Summative Assessment : *Unit test including multiple-choice and free-response questions modeled according to AP Exam style.*
Alternative Assessment: Group presentations that show how the chain rule or implicit differentiation is used in the real world.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.
In physics: Students will apply differentiation techniques to solve related rates problems involving velocity and motion.
In technology: Students will use technology to verify and explore derivative relationships.

Accommodations and Modifications

Special Education	• Preferential seating • Provide step-by-step guided notes for chain, product, and quotient rules. • Allow additional time for multi-step implicit differentiation problems. • Allow for extra time. • Accommodations consistent with IEP • Modified assignments • NHS Tutor (Mu Alpha Theta) • Provide a calculator

504	• *Assign less work while maintaining rigor.* • *Extended time.* • *Allow use of calculator* • NHS Tutor (Mu Alpha Theta)
At Risk/ I&RS	• Use small group teaching for challenging concepts like the chain rule. • More practice before moving to AP-level free response questions. • Frequent check-ins to correct misunderstandings at an early stage. • Use peer tutoring.
ML	• Allow peer discussion to enhance mathematics communications. • Allow the use of translation tools or bilingual glossaries for key terms . • Include notations for higher-order derivatives for example $\frac{d^2y}{dx^2}$, $f''(x)$, and y ''. • Provide vocabulary support for derivative notations $\frac{dy}{dx}$, $f'(x)$, $and\ y'$.
Gifted	• Assign students enrichment tasks with implicit differentiation and inverse functions from previous AP Calculus BC free-response questions. • Assign students more difficult and higher level applications such as parametric, polar, and vector functions earlier than the rest of class. • Have students explore proofs of the chain rule.
Resources	
• College Board AP Calculus Course and Exam Description (CED) • AP Classroom problem sets	

- WebAssign.
- Khan Academy
- Graphing calculators (TI-84)

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 4 Topic: Contextual Applications of Differentiation** **Duration AB: 3 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	Unit 4 begins by developing understanding of average and instantaneous rates of change in problems involving motion. The unit then identifies differentiation as a common underlying structure on which to build understanding of change in a variety of contexts. Students' understanding of units of measure often reinforces their understanding of contextual applications of differentiation. In problems involving related rates, identifying the independent variable common to related functions may help students to correctly apply the chain rule. When applying differentiation to determine limits of certain indeterminate forms using L'Hospital's rule, students must show that the rule applies.
Essential Questions	• How can derivatives be used to describe and predict motion? • How can derivatives help determine whether a quantity is increasing, decreasing, slowing down, or Speeding up? • In what ways do related rates connect one quantity's change to another's? How can we use derivatives to explain and justify the behavior of a function?
Misconceptions	Students may think that… • Students may think the derivative has no units. • Students may think that if $v(t) = 0$, the particle is not moving at all. In reality, this doesn't always mean

	the object stops moving, it often marks a moment where the object changes direction. • Students forget to add $\frac{dx}{dt}$ or $\frac{dy}{dt}$ In related rates, they must add it because every variable changes with time. We can have a quick reminder for them about it in advance.
Transfer	*Students will be able to use what they learn to…* • *This will appear again in Unit 6 (Integration & Accumulation): Understanding rates builds the foundation* *for calculating the accumulation of change.* • *This will come up again in Unit 7 (Integration Applications): Understanding derivatives helps students* *Apply calculus to area, volume, and motion.*
Important Facts and Key Concepts	*Students will know…* • *The derivative of a function can be interpreted as the instantaneous rate of change with respect to* *its independent variable.* • The derivative can be used to express information about rates of change in applied contexts. • The unit for $f'(x)$ is the unit for f divided by the unit for x. • The derivative can be used to solve rectilinear motion problems involving position, speed,velocity, and acceleration.

	• The derivative can be used to solve problems involving rates of change in applied contexts. • The chain rule is the basis for differentiating variables in a related rates problem with respect to the same independent variable. • Other differentiation rules, such as the product rule and the quotient rule, may also be necessary to differentiate all variables with respect to the same independent variable. • The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known. •The tangent line is the graph of a locally linear approximation of the function near the point of tangency. • For a tangent line approximation, the function's behavior near the point of tangency may determine whether a tangent line value is an underestimate or an overestimate of the corresponding function value. • When the ratio of two functions tends to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ in the limit, such forms are said to be indeterminate. • Limits of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.
Standards	
NJSL Content Specific Student Learning Standards **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph.	

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain properties of the
function.

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables)

F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.A.2: Write arithmetic and geometric sequences both recursively and explicitly, use them to model situations, and translate between the
two forms.

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) and for specific values of k (both positive
and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph
using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.LE.A.1.c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

WIDA
ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity (Velocity, acceleration, and rate in/ out)
- Analyzing data and owning problem-solving approaches

- Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles

ELD-MA 9-12 Explain Expressive:

Construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated assumptions
- Generalize logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments

Computer Science and Design Thinking

***8.1.12.DA.5*:** Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.IC.1: Evaluate the ways computing impacts personal, ethical, social, economic, and cultural practices.

8.2.12.ED.3: Evaluate several models of the same type of product and make recommendations.

8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a systemic plan of investigation, and propose an innovative sustainable solution.

Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.4: Identify issues associated with student loan debt, requirements for repayment, and consequences of failure to repay student loan debt.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and use problem solving processes.
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.CT.3: Collaborate with individuals to analyze a variety of potential solutions.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: Daily warm-ups on rate-in/rate-out and related rates setups, interpretation of derivatives in context.
Benchmark Assessment *: Quizzes on related rates, velocity/ acceleration, and interpreting derivative values.*
Summative Assessment : Unit test including AP-style multiple-choice and free-response questions on contextual applications (motion, rates in/out, related rates).
Alternative Assessment: Group presentations that show how to model rate of change scenario such as

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

Students will

- Connect calculus to several real world situations.
- Find the rate at which the area impacted by a wave radiating outwards is increasing if we know how quickly the radius is changing.
- Solve how quickly an ice cube vs ice frozen in a cylindrical pellet shape is melting and what volume it will be after a given time.
- Solve physics problems about rectilinear motion, for example, finding when an object is speeding up or slowing down, finding when an object is moving to the left or right.

Accommodations and Modifications	
Special Education	• Guided notes with step-by-step examples of rate problems • Allow use of calculators for derivatives computation. • Use diagrams and graphic organizers.
504	• Offer reduced problem sets without lowering rigor. • Allow oral explanation if needed.
At Risk/ I&RS	• Frequent check-ins to monitor progress. • Peer tutoring.
ML	• Include notations for higher-order derivatives for example $\frac{d^2y}{dx^2}$, $f''(x)$, and y '' with context Explanations (acceleration as rate of velocity). • Provide vocabulary lists for derivative notations $\frac{dy}{dx}$, $f'(x)$, $and\ y'$ with plain language definition.
Gifted	• Explore BC level materials such as parametric motion and polar coordinates. • Ask students to create their own related rates problems with full solutions.

Resources
• College Board AP Calculus Course and Exam Description (CED) • AP Classroom problem sets • WebAssign. • Khan Academy: Applied derivatives and related rates modules • Graphing calculators (TI-84)

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 5 Topic: Analytical Applications of Differentiation** **Duration AB: 4 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	*In this unit, the superficial details of contextual applications of differentiation are stripped away to focus on abstract structures and formal conclusions. Reasoning with definitions and theorems establishes that answers and conclusions are more than conjectures; they have been analytically determined. As when students showed supporting work for answers in previous units, students will learn to present justifications for their conclusions about the behavior of functions over certain intervals or the locations of extreme values or points of inflection. The unit concludes this study of differentiation by applying abstract reasoning skills to justify solutions for realistic optimization problems.*
Essential Questions	• In what ways do the first and second derivatives provide information about extrema, concavity, and inflection points? • Why are the Extreme Value Theorem and the Mean Value Theorem so important in the study of Calculus?

	• *How are a function, its first derivative, and its second derivative connected to one another?* • *How do derivatives help us analyze and interpret the behavior of functions?*
Misconceptions	*Students may think that…* •*"The derivative is increasing" means the same thing as "the function is increasing".* • *Local extrema are always global extrema* • *If a graph of the derivative is given, we do not have any information about the second derivative.* • *Looking for whether there is a "smile" or "frown" on any graph indicates that the original function is* *concave up or concave down.* • *The graph of a derivative having a positive slope means the function is increasing.* • *Seeing a local extrema on the graph of the derivative can help find the extrema for the original function.* • *Assuming the graph of f , f', f'' are unrelated* • *Confusing points of inflection with local minima/maxima* *Thinking and believing that the Mean Value Theorem always means there's a max or min*
Transfer	• *Relationships between a function and its derivative are examined in this unit but are further explored* *once the Fundamental Theorem of Calculus and the antiderivative is introduced in Unit 6. Then,*

	additional questions about a function, its antiderivative, and its derivative can be investigated. The skills used in justifying these connections through equations and written explanations are essential to mathematics. *• Knowledge of the first and second derivative can assist with solving optimization problems involving concrete economic and business applications by using the extrema of functions.* *• Understanding the relationship between a function and its first and second derivatives transitions nicely to using these concepts in kinetics in physics.*
Important Facts and Key Concepts	*Students will know…* *•If a function f is continuous over the interval [a, b] and differentiable over the interval (a, b), then the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.* *•If a function f is continuous over the interval (a, b), then the Extreme Value Theorem guarantees that f has at least one minimum value and at least one maximum value on [a, b].* *• A point on a function where the first derivative equals zero or fails to exist is a critical point of the function.*

	• All local (relative) extrema occur at critical points of a function, though not all critical points are local extrema. • *The first derivative of a function can provide information about the function and its graph, including intervals where the function is increasing or decreasing.* • The first derivative of a function can determine the location of relative (local) extrema of the function. •*Absolute (global) extrema of a function on a closed interval can only occur at critical points or at Endpoints.* • The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval. •*The second derivative of a function provides information about the function and its graph, including intervals of upward or downward concavity.* • The second derivative of a function may be used to locate points of inflection for the graph of the original function. •*The second derivative of a function may determine whether a critical point is the location of a relative*

	(local) maximum or minimum. *•When a continuous function has only one critical point on an interval on its domain and the critical point* *corresponds to a relative (local) extremum of the function on the interval, then that critical point also* *corresponds to the absolute (global) extremum of the function on the interval.* • Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations. *•Graphical, numerical, and analytical information from f' and f'' can be used to predict and explain the* *behavior of f.* • Key features of the graphs of f, f', and f'' are related to one another. *•The derivative can be used to solve optimization problems; that is, finding a minimum or maximum* *value of a function on a given interval.* *•Minimum and maximum values of a function take on specific meanings in applied contexts.* *•A point on an implicit relation where the first derivative equals zero or does not exist is a critical point of* *the function.* *•Applications of derivatives can be extended to implicitly defined functions.* *•Second derivatives involving implicit differentiation may be relations x, y, of and $\frac{dy}{dx}$.*

Standards

NJSL Content Specific Student Learning Standards

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities,
and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where
the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using
technology for more complicated cases. *

F.LE.A.1:

A.APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function
defined by the polynomial.

- **MP.2:** *Reason abstractly and quantitatively.*
- **MP.4:** *Model with mathematics.*
- **MP.6:** *Attend to precision.*

WIDA

ELD-MA.9-12 Explain Interpretive

Interpret mathematical explanations by

- Identifying concept or entity (Velocity, acceleration, and rate in/ out)

- Analyzing data and owning problem-solving approaches
- Evaluating rationales, models, and/or interpretations by comparing different justifications for classifying extrema (first vs

  Second derivative test) and critique reasoning that confuses inflection points with maxima/minima

ELD-MA.9-12.Argue.Expressive

Construct mathematics arguments that

- Create precise conjecture, using definitions, previously established results, and stated assumptions Generalize logical relationships across cases
- Justify (and refute) conclusions with evidence and mathematical principles
- Evaluate and extend others' arguments by proposing the relationships between first and second derivatives and the

  The shape of graph and prove whether a critical point is a local extrema.

Computer Science and Design Thinking

***8.1.12.AP.5*:** Decompose problems into smaller components through systematic analysis, using constructs such as procedures,
modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and use problem solving processes to solve authentics
problems.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment: Students will receive teacher feedback in groups or individually as problems are assigned during class as practice.
In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.
Benchmark Assessment *:* Students will complete questions in WebAssign and show work. Students are awarded points on the Free
Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach
their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of
written work for problems to better understand what is expected of them on exams and the AP Exam.

Summative Assessment : Students will complete a unit exam that has both multiple choice and free response question types. There will be
a calculator portion and a non-calculator portion to the Unit 6 exam. This follows the format of the AP Calculus exam. Sometimes the exam
questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different
modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented
with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 6 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical progression of steps that lead to an answer with

justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

Economics: Knowledge of the first and second derivative can assist with solving optimization problems involving concrete economic and business applications by using the extrema of functions. Maximization/ minimization (profit/cost)

Science: Understanding the relationship between a function and its first and second derivatives transitions nicely to using these concepts in kinetics in physics.

Art/ Design: Sketching curve and inflection in graphic design

Accommodations and Modifications

Special Education	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its second derivative, $f''(x)$. Labeling the overlapping graphs can help, but Desmos especially aids students as you can show or hide graphs so as not to overwhelm students with too much information at once. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
504	Dynamic graphs generated on Desmos can help students to visualize and understand the connection

	between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its second derivative, $f''(x)$. Labeling the overlapping graphs can help, but Desmos especially aids students as you can show or hide graphs so as not to overwhelm students with too much information at once. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
At Risk/ I&RS	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its second derivative, $f''(x)$. Labeling the overlapping graphs can provide vocabulary support for the type of justification wanted, but Desmos especially aids students as you can show or hide graphs so as not to overwhelm students with too much information at once. Encourage students to label graphs. If given a graph of the original function, draw tangent lines on the graph. Label with ++ for large positive slopes, + for positive slopes, 0 for horizontal slope, - for negative slopes, and - - for negative slopes with large magnitudes. Identify what is happening when the graph is labeled + (increasing) and - (decreasing). National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
ML	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between a function, $f(x)$ and its derivative, $f'(x)$ as well as between $f(x)$ and its second derivative, $f''(x)$. Labeling the overlapping graphs can provide vocabulary support for the type of justification wanted, but Desmos especially aids students as you can show or hide graphs so as not to overwhelm students with too much information at once. Encourage students to label graphs with symbols. If given a graph of the original function, draw tangent lines on the graph. Label with ++ for large positive slopes, + for positive slopes, 0 for horizontal slope, - for negative slopes, and - - for negative slopes with large magnitudes. Identify what is happening when the graph is labeled + (increasing) and - (decreasing). Giving many examples in complete sentences of the explanation and connections between derivatives that teachers are looking for are helpful for multilingual learners. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.
Gifted	Interested students may pursue these connections between all combinations of graphs. For example,

	on a first derivative graph, what would an extremum mean on the original graph? If the second derivative graph only has one x intercept on an interval, what does that prove about the original function on that same interval? If the first derivative is increasing, what does that say about the concavity of the original function? Students can also consider points of inflection where the second derivative is undefined but changes signs. How would that be represented on the graph of the first derivative? On the original function? Interested students can rigorously pursue additional optimization problems, which can be found in just about any subject when one wants to minimize or maximize different quantities given certain constraints. Interesting problems exist about optimizing the ratio of the height of a soda can to the radius of a soda can (or any cylinder) given construction materials, laying optimal roads and bridges or pipelines across a river, or maximizing profit.

Resources

College Board AP Calculus Course and Exam Description (CED)

- AP Classroom problem sets
- WebAssign.
- Khan Academy
- Graphing calculators (TI-84)

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 6 Topic: Integration and Accumulation** **Duration AB: 4 weeks Duration BC: 2 weeks BC**
Unit Summary (Enduring Understandings)	The College board delineates that this unit establishes the relationship between differentiation and integration using the Fundamental Theorem of Calculus. Students begin by exploring the contextual meaning of areas of certain regions bounded by rate functions. Integration determines accumulation of change over an interval, just as differentiation determines instantaneous rate of change at a point. Students should understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes). Future units will apply the idea of accumulation of change to a variety of realistic and geometric applications.
Essential Questions	*What strategies can I use to break down or simplify an integral?* *How are derivatives and integrals related?*
Misconceptions	*Students may think that...* ● Antiderivatives and integrals are exactly the same ● Definite and indefinite integrals are the same ● The chain rule does not impact antiderivatives ● The power rule can be used to find every antiderivative function ● Every integral can be solved
Transfer	*Students will be able to use what they learn to…* ● Students should be able to clearly explain how integrals represent net change and total accumulation, justifying their solutions both graphically and algebraically. This supports later units in this course (especially Unit 7: Applications of Integration) and other STEM and STEM adjacent fields (engineering, economics) where communication of quantitative reasoning is essential. ● Recognizing the integral as the area under a curve, students can analyze cost functions, profit over time, or energy usage—transferring the skill into economics, business, and sustainability studies. ● In more advanced STEM courses (including AP Physics), students will need to choose from numerical,

	graphical, or analytical methods based on given information. This will also be useful in later units, including units 7 and 8 of AP Calculus.
Important Facts and Key Concepts	*Students will know…* ● *The area of the region between the graph of a rate of change function and the x axis gives the accumulation of change.* ● *In some cases, accumulation of change can be evaluated by using geometry.* ● *If a rate of change is positive (negative) over an interval, then the accumulated change is positive (negative).* ● *The unit for the area of a region defined by rate of change is the unit for the rate of change multiplied by the unit for the independent variable.* ● *Definite integrals can be approximated for functions that are represented graphically, numerically, analytically, and verbally.* ● *Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.* ● *Definite integrals can be approximated using numerical methods, with or without technology.* ● *Depending on the behavior of a function, it may be possible to determine whether an approximation for a definite integral is an underestimate or overestimate for the value of the definite integral.* ● *The limit of an approximating Riemann sum can be interpreted as a definite integral.* ● *A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.* ● *The definite integral of a continuous function f over the interval [a, b], denoted by* $\int_a^b f(x)dx$*, is the limit of Riemann sums as the widths of the subintervals approach 0. That is,* $\int_a^b f(x)dx = \lim_{max\Delta xi \to 0} \sum_{i=1}^{n} f(x_i{}^*)\Delta x_i i$ *, where n is the number of subintervals, x∆ i is the width of the ith subinterval, and xi * is a value in the ith subinterval.* ● *A definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.* ● *The definite integral can be used to define new functions.* ● *If f is a continuous function on an interval containing a, then* $\frac{d}{dx}\int_a^x f(t)dt = f(x)$*, where x is in the interval.* ● *Graphical, numerical, analytical, and verbal representations of a function f provide information about the function*

	g defined as $g(x) = \int_a^x f(t)dt$ • *In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.* • *Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.* • *The definition of the definite integral may be extended to functions with removable or jump discontinuities.* • *An antiderivative of a function f is a function g whose derivative is f.* • *If a function f is continuous on an interval containing a, the function defined by* $F(x) = \int_a^x f(t)dt$ *is an antiderivative of f for x in the interval.* • *If f is continuous on the interval [a, b] and F is an antiderivative of f, then* $\int_a^x f(t)dt = F(a) - F(b)$. • $\int f(x)$ *is an indefinite integral of the function f and can be expressed as* $\int f(x) = F(x) + C$ *, where F'(x)=f(x) and C is any constant.* • *Differentiation rules provide the foundation for finding antiderivatives.* • *Many functions do not have closed-form antiderivatives.* • *Substitution of variables is a technique for finding antiderivatives.* • *For a definite integral, substitution of variables requires corresponding changes to the limits of integration.*

Standards

NJSL Content Specific Student Learning Standards

G.MG.A.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

F.IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.C.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

A.REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.SSE.A.1

Interpret expressions that represent a quantity in terms of its context.

A.SSE.A.1.a

Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.A.1.b

Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.

N.Q.A.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

WIDA

ELD-MA 9-12 Explain Interpretive: Interpret mathematical explanations by analyzing data and owning problem-solving approaches

ELD-MA 9-12 Explain Expressive: Construct mathematical explanations that introduce a mathematical concept or entity, share solutions with others, and describe data and/or approach used to solve a problem

Computer Science and Design Thinking

- *8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.*

Career Readiness, Life Literacies, and Key Skills

- *9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas*

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to better understand what is expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to solve or statements to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new material covered since the last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a calculator portion and a non-calculator portion to the Unit 6 exam. This follows the format of the AP Calculus exam. Sometimes the exam questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 6 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical progression of steps that lead to an answer with justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

By recognizing the integral as the area under a curve, students can analyze cost functions, profit over time, or energy usage—transferring the skill into economics, business, and sustainability studies. This can connect calculus to many different content areas, including physics, biology, climate studies, economics, and business.

Accommodations and Modifications

Special Education	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between definite integrals and the area under a curve.

	Students can be supported with guided questioning to help students use strategies presented in class as to evaluating integrals either analytically or graphically. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
504	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between definite integrals and the area under a curve. Students can be supported with guided questioning to help students use strategies presented in class as to evaluating integrals either analytically or graphically. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
At Risk / I&RS	Video lectures on AP Classroom are available for students who miss course content face to face. Being able to rewind and pause videos to support note taking can benefit students at risk of falling behind in the curriculum. Complex tasks can be chunked into manageable steps. Acknowledge effort and improvement. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
ML	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between definite integrals and the area under a curve. Encourage the use of bilingual dictionaries as needed.

	Vocabulary building activities can be incorporated. Complex tasks can be chunked into manageable steps.
Gifted	Interested students can extend this unit by studying additional types of antiderivatives not covered in AP Calculus: trig substitution, partial fraction decomposition, improper integration, and/or integration by parts. Additionally, students can find data where there is not a known function for the area under a curve but collected data can be analyzed with Riemann sums.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.org/courses/ap-course-audit and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multiple choice questions from previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via physical copy (available for students to bring home) or electronic access through WebAssign.

WebAssign.com (assignments from textbook)

*Note that this is Unit 5 for BC, but Unit 6 for AB

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 7 Topic: Applications of Integration** **Duration AB: 4 weeks Duration BC: 2 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students will learn how to find the average value of a function, model particle motion and net change, and determine areas, volumes, and lengths bc only defined by the graphs of functions. Emphasis will be placed on developing an understanding of integration that can be transferred across these and many other applications. Understanding that the area, volume, and length problems studied in this unit are limiting cases of Riemann sums of rectangle areas, prism volumes, or segment lengths saves students from memorizing a long list of seemingly unrelated formulas and develops meaningful understanding of integration.
Essential Questions	*How can I relate the position, distance, and displacement to the velocity of an object?* *How can I use integrals to represent accumulation in many different situations and with different units?* *How can I calculate the volume of a three dimensional object using two dimensional representations?*
Misconceptions	*Students may think that...* ● Only one function is needed to calculate the volume of any solid of revolution ● Volumes of solids of revolution and volumes of perpendicular cross sections are calculated using the same formula ● The axis of revolution does not factor into the integral setup ● Only one function is needed to calculate the volume of any solid of revolution ● The base of a solid of perpendicular cross sections and radius of a solid of revolution are the same since each quantity is squared in solving for volume ● They do not need to visualize the three dimensional shapes but that they can memorize a formula to use for every volume problem ● they can just memorize integrand expressions rather than understand the different area formulas and how the functions relate to the volume ● The axis of revolution does not factor into the integral setup for volumes of solids of revolution
Transfer	*Students will be able to use what they learn to…*

	● Break down three dimensional objects into two dimensional shapes that are stacked, as we do when 3D printing objects. ● Calculate averages in complex situations, like the average temperature of the earth over a year. ● Calculate the position, displacement, and distance traveled over a time interval when given the velocity of an object, as in physics and in unit 9 of AP Calculus BC.
Important Facts and Key Concepts	*Students will know…* ● *The average value of a continuous function f over an interval [a, b] is* $\frac{1}{b-a}\int_a^b f(x)dx.$ ● *For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.* ● *A function defined as an integral represents an accumulation of a rate of change.* ● *The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.* ● *The definite integral can be used to express information about accumulation and net change in many applied contexts.* ● *Areas of regions in the plane can be calculated with definite integrals.* ● *Areas of regions in the plane can be calculated using functions of either x or y.* ● *Areas of certain regions in the plane may be calculated using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions.* ● *Volumes of solids with square and rectangular cross sections can be found using definite integrals and the area formulas for these shapes.* ● *Volumes of solids with triangular cross sections can be found using definite integrals and the area formulas for these shapes.* ● *Volumes of solids with semicircular and other geometrically defined cross sections can be found using definite integrals and the area formulas for these shapes.* ● *Volumes of solids of revolution around the x- or y-axis may be found by using definite integrals with the disc method.* ● *Volumes of solids of revolution around any horizontal or vertical line in the plane may be found by using definite integrals with the disc method.* ● *Volumes of solids of revolution around the x- or y-axis whose cross sections are ring shaped may be found using definite integrals with the washer method.*

	• *Volumes of solids of revolution around any horizontal or vertical line whose cross sections are ring shaped may be found using definite integrals with the washer method.*

Standards

NJSL Content Specific Student Learning Standards

G.SRT.C.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.MG.A.1
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
G.MG.A.2
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
G.GMD.B.4
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
F.IF.B.6
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
A.CED.A.3
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
A.CED.A.4
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V=IR to highlight resistance R.
A.SSE.A.1.b
Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
N.Q.A.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in

formulas; choose and interpret the scale and the origin in graphs and data displays.

WIDA
ELD-MA 9-12 Argue Interpretive: Evaluating relationships among evidence and mathematical principles to create generalizations

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to better understand what is expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to solve or statements to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new material covered since the

last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a calculator portion and a non-calculator portion to the Unit 7 exam. This follows the format of the AP Calculus exam. Sometimes the exam questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 7 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical progression of steps that lead to an answer with justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

Students will

Break down three dimensional objects into two dimensional shapes that are stacked, as we do when 3D printing objects.

Calculate averages in complex situations, like the average temperature of the earth over a year for climate change.

Students will solve simple physics problems, such as calculating the position, displacement, and distance traveled over a time interval when given the velocity of an object.

Students will generate integral expressions based on a given rate of change to solve for the net change from one or more functions, such as a supplier receiving new inventory and shipping it out at different rates in business.

Accommodations and Modifications	
Special Education	Provide students with examples of free response question answers about volumes of solids of revolution and perpendicular cross sections. Ask them to identify which type of question goes with the format of each answer. Dynamic graphs generated on Desmos can help students to visualize and understand the connection between the curve being rotated and the solid generated. Physical representations of volumes of solids of revolution and volumes with perpendicular cross sections can be presented so that students can better visualize described shapes by seeing the models in real life. Complex tasks can be chunked into manageable steps. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
504	Dynamic graphs generated on Desmos can help students to visualize and understand the connection between the curve being rotated and the solid generated. Physical representations of volumes of solids of revolution and volumes with perpendicular cross sections can be presented so that students can better visualize described shapes by seeing the models in real life. Complex tasks can be chunked into manageable steps. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring.

	AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
At Risk/ I&RS	There are various murals on the 4th floor of Morristown High School that can be visited that have applied calculus problems from this unit (spinning top, Remy cooking soup, Totoro eating acorns, bees creating honey, etc) to help engage students. Teachers can create context specific problems that may appeal to at risk students to help them engage with the content. Complex tasks can be chunked into manageable steps. Acknowledge effort and improvement. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
ML	Encourage the use of bilingual dictionaries as needed, especially to translate the context of the problems. Focus on vocabulary for motion, including position, velocity, acceleration, speeding up, slowing down, moving left, moving right, standing still, etc. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
Gifted	Interested students can extend this unit to calculating volumes of solids of revolution around a slant axis. Students can also create problems with an appropriate real world context of “rate in / rate out” applications.
Resources	

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.org/courses/ap-course-audit and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multiple choice questions from previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via physical copy (available for students to bring home) or electronic access through WebAssign.

WebAssign.com (assignments from textbook)

*Note that this is unit 7 for AB, but Unit 6 for BC

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus AB and BC** **Unit 8 Topic: Differential Equations** **Duration AB: 2 weeks Duration BC: 2 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay and logistic growth bc only, students build understanding of topics introduced in Algebra II and other courses.
Essential Questions	*When given a differential equation with two variables, how can we find an antiderivative?* *If we cannot solve an antiderivative, what methods can be used to approximate an answer graphically or analytically?*
Misconceptions	*Students may think that...* ● All differential equations are only dependent on one variable ● All differential equations can be solved analytically ● Graphing slope fields is the same as finding a solution curve ● Tangent approximations are the same as exact solutions
Transfer	*Students will be able to use what they learn to…* ● Recognize situations, such as population growth, radioactive decay, drug concentrations in the bloodstream, compound interest, or Newton's law of cooling as governed by differential equations. ● Use differential equations to model real life scenarios and find the equilibrium of a population or temperature ● Recognize that directed graphs can be used to predict the weather
Important Facts and	*Students will know…*

Key Concepts	

- *Differential equations relate a function of an independent variable and the function's derivatives.*
- *Derivatives can be used to verify that a function is a solution to a given differential equation.*
- *There may be infinitely many general solutions to a differential equation.*
- *A slope field is a graphical representation of a differential equation on a finite set of points in the plane.*
- *Slope fields provide information about the behavior of solutions to first-order differential equations.*
- *Solutions to differential equations are functions or families of functions.*
- *Some differential equations can be solved by separation of variables.*
- *Antidifferentiation can be used to find general solutions to differential equations.*
- *A general solution may describe infinitely many solutions to a differential equation. There is only one particular solution passing through a given point.*
- *The function F defined by* $F(x) = y_0 + \int_a^x f(t)dt$ *is a particular solution to the differential equation* $\frac{dy}{dx} = f(x)$*, satisfying F(a)=*y_0*.*
- *Solutions to differential equations may be subject to domain restrictions.*
- *Specific applications of finding general and particular solutions to differential equations include motion along a line and exponential growth and decay.*
- *The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is* $\frac{dy}{dt} = ky$*.*
- *The exponential growth and decay model,* $\frac{dy}{dt} = ky$*, with initial condition y =* y_0*. when t = 0, has solutions of the form y =* $y_0 . e^{kt}$*.*

For BC Calculus Only:

- *Euler's method provides a procedure for approximating a solution to a differential equation or a point on a*
- *solution curve.*
- *The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is* $\frac{dy}{dt} = ky$*(a-y) .*
- *The logistic differential equation and initial conditions can be interpreted without solving the differential equation.*
- *The limiting value (carrying capacity) of a logistic differential equation as the independent variable approaches infinity can be determined using the logistic growth model and initial conditions.*
- *The value of the dependent variable in a logistic differential equation at the point when it is changing fastest can*

	be determined using the logistic growth model and initial conditions.

Standards

NJSL Content Specific Student Learning Standards

F.IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.C.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *

A.CED.A.2

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

WIDA Standards for Multilingual Learners

- ELD-MA 9-12 Explain Expressive: Share solutions with others

Computer Science and Design Thinking

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to better understand what is expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to solve or statements to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new material covered since the last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a calculator portion and a non-calculator portion to the Unit 8 exam. This follows the format of the AP Calculus exam. Sometimes the exam questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 8 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical progression of steps that lead to an answer with justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

- Recognize situations in biology, ecology, chemistry, finance, engineering, and physics, such as population growth, radioactive decay, drug concentrations in the bloodstream, compound interest, or Newton's law of cooling as governed by differential equations.
- Use differential equations to model real life scenarios and find the equilibrium of a population or temperature
- Recognize that directed graphs can be used to predict the weather

Accommodations and Modifications

Special Education	Use graphic organizers to identify paths for solving differential equations step-by-step. Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*dx$, by filling in a table with n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the sides with x_n, y_n m, dx, dy and y_{n+1}.
504	Use structured peer discussions to explain solution steps in solving differential equations with collaborative learning. Use graphic organizers to identify paths for solving differential equations step-by-step. Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*dx$, by filling in a table with n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the sides with x_n, y_n m, dx, dy and y_{n+1}.
At Risk/ I&RS	Use graphic organizers to identify paths for solving differential equations step-by-step. Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*dx$, by filling in a table with n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the sides with x_n, y_n m, dx, dy and y_{n+1}. Implement chunked instruction: break up lessons into short, manageable tasks. Use real-world modeling contexts (e.g., population growth, cooling, interest) to make abstract concepts more tangible. Use structured peer discussions to explain solution steps in solving differential equations with collaborative learning to engage students with their peers.

ML	Use graphic organizers to identify paths for solving differential equations step-by-step. Present Euler's methohd in multiple modalities–by using a formula $y_{n+1}=y_n+(dy/dx)*dx$, by filling in a table with n, x_n, y_n, dy/dx, dy, dx and y_{n+1}, and by drawing connected triangles and labeling the sides with x_n, y_n m, dx, dy and y_{n+1}. Implement chunked instruction: break up lessons into short, manageable tasks. Use structured peer discussions to explain solution steps in solving differential equations with collaborative learning to engage students with their peers.
Gifted	For interested students, offer extension problems involving programming computers to perform several iterations of Euler's Method. Interested students can also learn more general population modeling with differential equations where a population may have multiple equilibria Interested students may study non-separable differential equations, such as homogeneous differential equations.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.org/courses/ap-course-audit and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multiple choice questions from previously released AP Calculus exams)

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WebAssign.com (assignments from textbook)

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus BC Only** **Unit 9 Topic: Parametric and Polar Calculus** **Duration: 3 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.
Essential Questions	*How do we calculate slope for curves not constructed in the Cartesian plane?* *How do we calculate the area under curves not graphed in the Cartesian plane?* *How can we notate the different rates of change suggested by multiple derivatives?*
Misconceptions	*Students may think that...* ● X and t can be used interchangeably for parametric equations ● Y must always be dependent on x to be a function ● Integrating polar area has area swept from left to right or top to bottom. ● The order of bounds does not matter for polar integration. ● The term “derivative” always means dy/dx instead of dy/dt, dx/dt, dr/dt, dy/dx, dy/dr, dx/dr, or any other rate of change.
Transfer	*Students will be able to use what they learn to…* ● Translate between multiple representations of curves. In Multivariable Calculus, students will need to graph in spherical and cylindrical coordinates. Graphing in polar helps to prepare students for graphing

	in three dimensional systems other than Cartesian. ● Learning to work with parameters will also help prepare students for parameterizing a curve in multivariable calculus. Parameters are also often used in computer programming and basic computer animation. ● Model circular or spiral motion in physics and multivariable calculus with parametric and polar calculus.
Important Facts and Key Concepts	*Students will know…* ● *Methods for calculating derivatives of real-valued functions can be extended to parametric functions.* ● *For a curve defined parametrically, the value of dy/dx at a point on the curve is the slope of the line tangent to the curve at that point. dy/dx, the slope of the line tangent to a curve defined using parametric equations, can be determined by dividing dy/dt by dx/dt , provided dx/dt does not equal zero.* ● $\frac{d^2y}{dx^2}$ *can be calculated by dividing* $\frac{d}{dt}$ $(\frac{dy}{dx})$*by dx/dt.* ● *The length of a parametrically defined curve can be calculated using a definite integral.* ● *Methods for calculating derivatives of realvalued functions can be extended to vectorvalued functions.* ● *Methods for calculating integrals of real-valued functions can be extended to parametric or vector-valued functions.* ● *Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions.* ● *For a particle in planar motion over an interval of time, the definite integral of the velocity vector represents the particle's displacement (net change in position) over the interval of time, from which we might determine its position. The definite integral of speed represents the particle's total distance traveled over the interval of time.* ● *Methods for calculating derivatives of realvalued functions can be extended to functions in polar coordinates.* ● *For a curve given by a polar equation r = f(θ), derivatives of r, x, and y with respect to θ, and first and second derivatives of y with respect to x can provide information about the curve.*

	● *The concept of calculating areas in rectangular coordinates can be extended to polar coordinates.* ● *Areas of regions bounded by polar curves can be calculated with definite integrals.*

Standards

NJSL Content Specific Student Learning Standards

G.C.B.5

Find arc lengths and areas of sectors of circles

N.VM.A.1

Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).

N.VM.A.2.

Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.A.3.

Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.B.5

Multiply a vector by a scalar.

WIDA Standards for Multilingual Learners

ELD-MA 9-12 Explain Expressive

- Construct mathematical explanations that share solutions with others
- Construct mathematical explanations that describe data and/or approach used to solve a problem

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to better understand what is expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to solve or statements to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new material covered since the last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. There will be a calculator portion and a non-calculator portion to the Unit 9 exam. This follows the format of the AP Calculus exam. Sometimes the exam questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 9 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical

progression of steps that lead to an answer with justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

- Translate between multiple representations of curves. In Multivariable Calculus, students will need to graph in sperical and cylindrical coordinates. Graphing in polar helps to prepare students for graphing in three dimensional systems other than Cartesian.
- Learning to work with parameters will also help prepare students for parameterizing a curve in multivariable calculus. Parameters are also often used in computer programming and basic computer animation.
- Model circular or spiral motion in physics and multivariable calculus with parametric and polar calculus.

Accommodations and Modifications

Special Education	Provide color-coded visuals for parametric curves and vector components in Desmos. Show area sweeps in Wolfram Demonstrations to illustrate how area is accumulated in polar graphing. Allow access to Desmos for graphing when a graph is not provided. Provide extended time as directed by individual education plans. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
504	Provide color-coded visuals for parametric curves and vector components in Desmos. Show area sweeps in Wolfram Demonstrations to illustrate how area is accumulated in polar graphing. Allow access to Desmos for graphing when a graph is not provided. Provide extended time as directed by 504 plans. National Honor Society tutors are available to help with one-on-one tutoring.

	Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
At Risk/ I&RS	Use chunked instruction in first teaching parametric derivatives and then derivatives and integrals of vectors. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
ML	Provide examples of common wording of questions for Free Response Questions involving parametric calculus. Provide color-coded visuals for parametric curves and vector components in Desmos. Show area sweeps in Wolfram Demonstrations to illustrate how area is accumulated in polar graphing. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
Gifted	Interested students can program a simple animation based on parameters. Interested students can also further study limacons and cardioids in polar form.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.org/courses/ap-course-audit and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multiple choice questions from previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

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WebAssign.com (assignments from textbook)

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus BC Only** **Unit 10 Topic: Power Series for Approximation** **Duration: 3 weeks**
Unit Summary (Enduring Understandings)	In this unit, students will extend the idea of a tangent line approximating a curve to a polynomial approximating a given curve. Since the first derivatives of the tangent line and the curve match at a point of tangency and provide a decent approximation, power series extend the concept to matching second derivatives so that the concavity of our polynomial approximation matches, and so on to third, fourth, and nth derivatives. In fact, an infinite power series exactly matches a given function. Students will use sigma notation to describe the patterns of power series with the general term.
Essential Questions	*How can a polynomial match a given function?* *How closely does a polynomial approximate a given function?* *How can I notate the pattern for an infinite series?*
Misconceptions	*Students may think that...* ● The number of terms in a series is the same as the order of the series ● A memorized Maclaurin series can be substituted with (x-a) to build a Taylor series. ● Multiplying a series is the same as substituting a variable ● We cannot intuit any information about a derivative from a given Taylor series ● Error bound is always based on the next degree term, even if it is 0
Transfer	*Students will be able to use what they learn to…* ● Model oscillations or wave behavior, such as sound waves when recording music ● Understand that a signal can be broken down into a series (Fourier series) for signal processing in electrical engineering ● Understand Euler's identity $e^{i\pi} + 1 = 0$
Important Facts and	*Students will know…*

Key Concepts	● The nth partial sum is defined as the sum of the first n terms of a series ● An infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S. ● A geometric series is a series with a constant ratio between successive terms. ● If a is a real number and r is a real number such that |r| <1, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ ● The nth term test is a test for divergence of a series. ● In addition to geometric series, common series of numbers include the harmonic series, the alternating harmonic series, and p-series. ● If an alternating series converges by the alternating series test, then the alternating series error bound can be used to bound how far a partial sum is from the value of the infinite series. ● The coefficient of the nth degree term in a Taylor polynomial for a function f centered at x = a is $\frac{f^{(n)}(a)}{n!}$. ● In many cases, as the degree of a Taylor polynomial increases, the nth degree polynomial will approach the original function over some interval. ● Taylor polynomials for a function f centered at x = a can be used to approximate function values of f near x = a. ● The Lagrange error bound can be used to determine a maximum interval for the error of a Taylor polynomial approximation to a function. ● In some situations, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the value of a function. ● A power series is a series of the form $\sum_{n=0}^{\infty} a_n(x - r)$, where n is a non-negative integer, $\{a_n\}$ is a sequence of real numbers, and r is a real number. ● A Taylor polynomial for f(x) is a partial sum of the Taylor series for f(x). ● The Maclaurin series for $\frac{1}{1-x}$ is a geometric series. ● The Maclaurin series for sin x, cos x, and e^x provides the foundation for constructing the Maclaurin series for other functions. ● Using a known series, a power series for a given function can be derived using operations such as term-by-term differentiation or term-byterm integration, and by various methods (e.g., algebraic processes, substitutions, or using properties of geometric series).

Standards

NJSL Content Specific Student Learning Standards

F.BF.A.1

Write a function that describes a relationship between two quantities.

F.BF.A.1.c

(+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.A.2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situationns, and translate between the two forms. *

F.IF.A.3

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0)=f(1)=1, f(n+1)=f(n)+f(n-1) for n>1.

F.IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

A.REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.SSE.A.1.a

Interpret parts of an expression, such as terms, factors, and coefficients.

WIDA Standards for Multilingual Learners

- ELD-MA.9-12.Argue.Expressive: Construct mathematics arguments that Generalize logical relationships across cases
- ELD-MA.-12 Explain Interpretive: Interpret mathematical explanations by identifying concept or entity

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using constructs such as procedures,

modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment

Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment

Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to better understand what is expected of them on exams and the AP Exam.

Students will complete three quizzes in this unit. Two of the quizzes will consist of open-ended problems for students to solve or statements to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new material covered since the last quiz or exam. The other quiz will be a memorization quiz on the four series (sinx, cosx, e^x, 1/(1-x)) mentioned in the course and exam description.

Summative Assessment

Students will complete a unit exam that has both multiple choice and free response question types. The unit 10 exam will be non-calculator. This follows the format of the AP Calculus exam questions on sequences, series, and Taylor Series. Sometimes the exam questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 10 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical progression of steps that lead to an answer with justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Describe how one content area may connect to the core content area in this unit.

- Model oscillations or wave behavior, such as sound waves when recording music in music or audio engineering
- Understand that a signal can be broken down into a series (Fourier series) for signal processing in electrical engineering

Accommodations and Modifications

Special Education	Students can develop a graphic organizer for the unit to understand what series must be memorized and what can be done to manipulate known series into specific functions. Dynamic representations of Taylor polynomials of up to 10 degrees modeling particular series (sine, cosine, exponential, rational) on Desmos can help to visually illustrate the closer and closer approximations as well as the error bound. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.

504	Dynamic representations of Taylor polynomials of up to 10 degrees modeling particular series (sine, cosine, exponential, rational) on Desmos can help to visually illustrate the closer and closer approximations as well as the error bound. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
At Risk/ I&RS	Priming with sigma notation, geometric series, and arithmetic series from PreCalculus can help students prepare to succeed in this unit. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another and an AP teacher.
ML	Dynamic representations of Taylor polynomials of up to 10 degrees modeling particular series (sine, cosine, exponential, rational) on Desmos can help to visually illustrate the closer and closer approximations as well as the error. National Honor Society tutors are available to help with one-on-one tutoring. Mu Alpha Theta tutors are available to help with one-on-one tutoring. AP Collaboration Hub is available after school for students to collaborate with one another in a small group setting and under the guidance of an AP teacher.
Gifted	Interested students can explore the concept of a Fourier transform for modeling waves. They can also design end of year projects creating models from Taylor polynomials.
Resources	
College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.org/courses/ap-course-audit and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multiple choice questions from previously released AP Calculus exams)	

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via physical copy (available for students to bring home) or electronic access through WebAssign.

WebAssign.com (assignments from textbook)

Unit Plan Explanatory Document

Grade Level: High School	**AP Calculus BC Only** **Unit 11 Topic: Series Convergence** **Duration: 2.5 weeks**
Unit Summary (Enduring Understandings)	The College board delineates that in this unit, students need to understand that a sum of infinitely many terms may converge to a finite value. They can develop intuition by exploring graphs, tables, and symbolic expressions for series that converge and diverge and for Taylor polynomials that converge to a function. Students should build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration. Students who rely solely on memorizing a long list of tests and procedures typically find little success achieving a lasting conceptual understanding of series.
Essential Questions	*What can cause an infinite series to sum to a finite value?* *How can we know if a series is getting small enough quickly enough to converge?* *How can we find the values of x that will ensure that a power series converges?*
Misconceptions	*Students may think that...* ● All series with fractions converge. ● P series is any series with a polynomial in the denominator of the term. ● The integral test tells us the value that the corresponding series converges to rather than whether it converges or not. ● Comparison tests can be used with any series rather than positive, decreasing series. ● Getting an infinite limit (or zero) from the ratio test implies an infinite radius of convergence (or none). ● They do not need to check endpoints in the interval of convergence
Transfer	*Students will be able to use what they learn to…* ● Test endpoints of an interval of convergence for convergence with the other convergence tests learned in the unit. ● Model probability distributions with series in statistics and probability courses ● Model in physics and engineering with waveforms and electrical circuits

	• Select appropriate methods for testing abstract concepts in analysis, quantitative finance, statistical analysis, etc as students learn to apply convergence tests to infinite series
Important Facts and Key Concepts	*Students will know…* • The nth term test is a test for divergence of a series. • The integral test is a method to determine whether a series converges or diverges. • In addition to geometric series, common series of numbers include the harmonic series, the alternating harmonic series, and p-series. • The comparison test is a method to determine whether a series converges or diverges. • The limit comparison test is a method to determine whether a series converges or diverges. • The alternating series test is a method to determine whether an alternating series converges. • The ratio test is a method to determine whether a series of numbers converges or diverges. • A series may be absolutely convergent, conditionally convergent, or divergent. • If a series converges absolutely, then it converges. • If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value. • If a power series converges, it either converges at a single point or has an interval of convergence. • The ratio test can be used to determine the radius of convergence of a power series. • The radius of convergence of a power series can be used to identify an open interval on which the series converges, but it is necessary to test both endpoints of the interval to determine the interval of convergence. • If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval. • The radius of convergence of a power series obtained by term-by-term differentiation or termby-term integration is the same as the radius of convergence of the original power series.

Standards

NJSL Content Specific Student Learning Standards

A.SSE.A.1.a

Interpret parts of an expression, such as terms, factors, and coefficients.

A.REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
F.IF.A.3
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0)=f(1)=1, f(n+1)=f(n)+f(n-1) for n>1.

WIDA Standards for Multilingual Learners

- ELD-MA 9-12 Explain Expressive: Construct mathematical explanations that state reasoning used to generate own or alternate solutions
- ELD-MA.9-12.Argue.Expressive: Construct mathematics arguments that create precise conjecture, using definitions, previously established results, and stated assumptions

Computer Science and Design Thinking

- 8.1.12.AP.5: Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Career Readiness, Life Literacies, and Key Skills

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas

Assessment Evidence

Teachers may choose from the following list of assessments

Formative Assessment
Students will receive teacher feedback in groups or individually as problems are assigned during class as practice. In this way, the teacher can assess class-wide competency as students develop and demonstrate their understanding and skills.

Benchmark Assessment
Students will complete questions in WebAssign and show work. Students are awarded points on the Free Response Questions of the AP Calculus Exam for correct mathematical thinking. This requires showing all steps that they take to reach their solution. Answers without supporting work will usually not receive credit. Thus, students can receive feedback on their submission of written work for problems to better understand what is

expected of them on exams and the AP Exam.

Students will complete two quizzes in this unit. These two quizzes will consist of open-ended problems for students to solve or statements to justify in a timed setting and without reference materials. The quizzes are not typically cumulative, but focus on new material covered since the last quiz or exam.

Summative Assessment
Students will complete a unit exam that has both multiple choice and free response question types. The unit 11 exam will be non-calculator. This follows the format of the AP Calculus exam questions on convergence of series. Sometimes the exam questions will combine content in surprising ways requiring students to show synthesis of ideas and concepts presented in different modalities. Students may not have seen problems identical in format during class to those on the exam, but will have been presented with all the concepts, skills, and tools necessary to solve the problems presented on the Unit 11 Exam.

Alternative Assessment: Students are expected to present previously assigned homework problems at the board. Groups and problems will be randomly assigned at the beginning of class to provide a representative sample of student understanding of previously taught concepts. Misunderstandings can be corrected immediately, before covering additional material depending on prior content. Being able to verbalize understanding of previously assigned problems allows students to process the mathematics at play in each question and appreciate the logical progression of steps that lead to an answer with justification. This also gives students who are not presenting the opportunity to ask clarifying questions.

Interdisciplinary Connections

Students will be able to use what they learn to…

- Model probability distributions with series in statistics and probability courses
- Model in physics and engineering with waveforms and electrical circuits
- Select appropriate methods for testing abstract concepts in analysis, quantitative finance, statistical analysis, etc as students learn to apply convergence tests to infinite series

Accommodations and Modifications

Special Education	Physical representations of a p series converging (tearing a piece of paper in half over and over again infinitely many times) can help students to understand that for an infinite series to converge to a finite number, each term must be decreasing. Students can create a concept map or flowchart as a graphic organizer to help decide which convergence test is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections and differences between different series convergence tests.
504	Physical representations of a p series converging (tearing a piece of paper in half over and over again infinitely many times) can help students to understand that for an infinite series to converge to a finite number, each term must be decreasing. Students can create a concept map or flowchart as a graphic organizer to help decide which convergence test is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections and differences between different series convergence tests.
At Risk/ I&RS	Physical representations of a p series converging (tearing a piece of paper in half over and over again infinitely many times) can help students to understand that for an infinite series to converge to a finite number, each term must be decreasing. Students can create a concept map or flowchart as a graphic organizer to help decide which convergence test is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections and differences between different series convergence tests.
ML	Physical representations of a p series converging (tearing a piece of paper in half over and over again infinitely many times) can help students to understand that for an infinite series to converge to a finite number, each term must be decreasing. The physical representation can help anchor vocabulary for “series” and “term”. Students can create a concept map or flowchart as a graphic organizer to help decide which convergence test is appropriate to use in which situation and in what order to use the different tests. Chunking material appropriately can help students to recognize the connections and differences between different series convergence tests.

Gifted	Interested students can explore the Riemann rearrangement theorem for conditionally convergent series. Interested students could also explore different sizes of infinity with cardinality.

Resources

College Board (2012-2025). AP Calculus Exam. College Board. Retrieved from https://apcentral.collegeboard.org/courses/ap-course-audit and https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions and https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions (Free response and multiple choice questions from previously released AP Calculus exams)

Desmos.com (graphing calculator capabilities to visualize relations and evaluate expressions)

Stewart, J. (2016). Single variable calculus: Early transcendentals. Cengage Learning. Accessed either via physical copy (available for students to bring home) or electronic access through WebAssign.

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