# Physics 3204 Unit 1: Motion

**A cyclist follows a parabolic path through the air after riding off a ramp.**

**TOPICS:**

1. **Vectors in Two Dimensions**
2. **Review of Relative Motion**
3. **Relative Motion in Two Dimensions**
4. **Review of Kinematics**
5. **Projectile Motion**
6. **Review of Momentum and Collisions**
7. **Momentum and Collisions in Two Dimensions**

# 1.1: Vectors in Two Dimensions

At the end of this section, you should be able to:

- ☐ Draw diagrams to represent the sum of two vectors in two dimensions.

Last year, we added vectors using arrow diagrams, but only in one dimension. This year, we will be adding vectors in two dimensions.

Recall that in order to add two or more vectors using **arrow diagrams**, you must add the vectors “tip-to-tail.” That is, you draw the first arrow, then draw the second arrow starting from the arrow head of the first arrow, etc. The resulting “total” vector runs from the _____________ of the first vector to the _______ of the last vector.

*Sketch an arrow diagram for these vector additions. Then sketch the resulting total vector.*

*a) 35 m [E] + 26 m [E]*

*b) 20 km [N] + 16 km[N]*

*c) 75 km [S] + 53 km [N]*

*d) 63 m [E] + 24 m [W]*

When adding vectors in two perpendicular directions, a ______________________ is formed. We can use the properties of right triangles to calculate the length of the resulting vector (___________________________________) and the direction/angle of the vector (_________________________________).

*Add each set of vectors with a diagram, then calculate the length and direction of the overall vector.*

a) *42 m [E] + 28 m [N]*

b) *37 km [W] + 37 km [S]*

c) *64 km [E] + 52 km [S]*

d) *21 m [N] + 34 m [W]*

e) *13 m [E] + 21 m [S] + 4 m [W]*

f) *53 m [S] + 43 m [E] + 16 m [N]*

# 1.2: Review of Relative Motion

At the end of this section, you should be able to:

- ☐ Solve 1D relative motion problems with a single object moving within a moving medium.

We looked at two kinds of relative motion last year: 1) two objects moving relative to each other, and 2) an object moving within a moving medium. We will only deal with the second kind this year.

*A boat has a speed of 15 m/s relative to the water. The river's current flows at 5 m/s East.*

- *The boat travels on the river to the East. Sketch a diagram to show how the boat would appear to move relative to the river bank.*

- *How does going with the river's current affect the velocity of the boat relative to the ground?*________________________________________________________________

- *The boat now travels on the river to the West. Sketch a diagram to show how the boat would appear to move relative to the river bank.*

- *How does going against the river's current affect the velocity of the boat relative to the ground?* ________________________________________________________________

*An aircraft can fly at 100 km/h [S] in still air. The wind is blowing at 25 km/hr. Find the velocity of the plane relative to the ground if the wind is blowing to the:*

a) *South (tailwind)*

b) *North (headwind)*

*A train is moving at 15.0 m/s [E]. A passenger on the train can walk at 2.0 m/s. Calculate the velocity of the passenger relative to the ground if they walk towards:*

a) *the front of the train*

b) *the back of the train*

*A plane is flying at an airspeed of 50.0 m/s [E] while there is a headwind of 15.0 m/s.*

a) *What is the velocity of the plane relative to the ground?*

b) *How much time will it take the plane to fly 1200 m [E]?*

*Julia can swim at 2.30 m/s in still water. She is swimming in a river which has a current of 0.95 m/s [S].*

a) *What is her velocity relative to the ground if she swims upstream? What if she swims downstream?*

b) *How much time will it take her to travel 450 m upstream? What if she goes downstream?*

*A boat travels 450 m [N] up a river that flows at 3.0 m/s [S], and then returns. The boat has a speed of 7.0 m/s relative to the water. How much time will each leg of the trip take (up and back down the river)?*

*An aircraft has an airspeed (speed relative to the air) of 230 km/hr [N].*

a) *What is the plane's velocity relative to the ground if it:*
   i) *Flies with a tailwind of 50 km/h?*
   ii) *Flies into a headwind of 50 km/h?*
b) *What will be the displacement of the plane if it flies into the headwind for 2.5 h?*

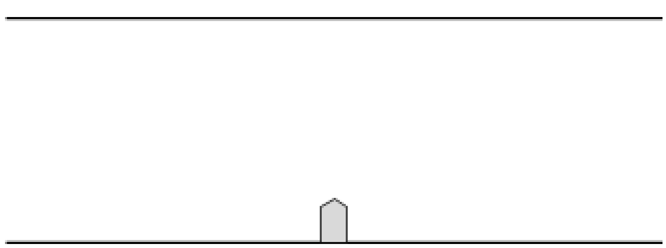
# 1.3: Relative Motion in Two Dimensions

At the end of this section, you should be able to:
- ☐ Given a situation, sketch the path of motion for a specified reference frame.
- ☐ Calculate relative velocity of an object using vector addition.
- ☐ Calculate required heading to reach an intended destination.
- ☐ Calculate time to travel a given distance.

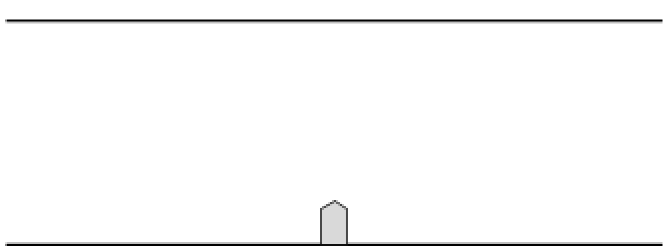
The main type of **relative motion** scenario we will look at this year is where an object is moving through some medium that is moving perpendicular to the object's motion. The two easiest and most common examples are:

1. ______________________________________________
2. ______________________________________________

*A boat is trying to cross a river, which is 50 m wide. The boat can move at 10 m/s relative to the water. Sketch a vector diagram to show how the boat will move across the river if the water is still.*



*Now the river has a current of 5 m/s East. Sketch a vector diagram to show how the boat will move as it crosses the river.*



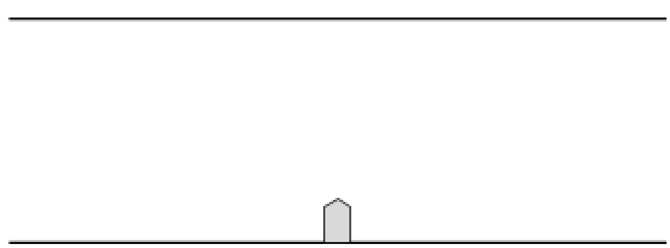
*What happened to the boat as it crossed the river, due to the river's current?*

______________________________________________

*How could the boat prevent this from happening?*

______________________________________________________________________

*Sketch another diagram to represent what the boat needs to do in order to actually go straight across the river.*



*How are these two diagrams different?*

______________________________________________________________________

______________________________________________________________________

*Is the angle at which the boat was pushed off course by the current (diagram 2) the same as the angle at which the boat needed to turn to counteract the current (diagram 3)?* ________________________________________________________________

*In each scenario, calculate the velocity of the boat relative to the ground.*

*An airplane can fly at 118 km/h [S] in still air. The wind blows at 28 km/h [E].*

a) *Find the velocity of the plane relative to the ground (including direction!). Draw a diagram to help keep things straight.*

b) *What must be the plane's heading in order to land due South of its original position? Draw a diagram to help solve this.*

*A small plane flies with a speed of 225 km/h relative to the air. The plane experiences a crosswind blowing at 55 km/h [E].*

a) *What heading must the pilot use to fly due North?*

b) *What is the resulting speed of the plane relative to the ground?*

c) *How much time would it take for the plane to reach an airport 575 km [N] of its starting point?*

*A boat can move at 8.0 m/s in still water. It is travelling in a river where the current flows at 3.0 m/s [S].*

a) *What will be the velocity of the boat relative to the ground if it is pointed due West?*

b) *How much time will it take to cross the river if the river is 650 m wide?*

c) *How far downstream will the boat land?*

d) *What heading must the boat use to sail due West?*

e) *What is the speed of the boat relative to the ground?*

*Bobby can swim in still water at 2.4 m/s. If he is swimming across a river with a current of 1.1 m/s [S].*

a) *What will be his overall velocity if he points himself straight across the river?*

b) *What direction will he have to point himself in order to actually go straight across the river?*

# 1.4: Review of Kinematics

At the end of this section, you should be able to:

- ☐ Use the proper kinematics formulae to calculate an unknown displacement, velocity, acceleration, or time for a given scenario.

Last year we derived five kinematics formulae that describe non-uniform motion - any motion that has a constant acceleration. Here they are all in one place:

| | | | | |
|---|---|---|---|---|
| $v_2 = v_1 + a\,t$ | $d = \frac{1}{2}(v_1 + v_2)\,t$ | $d = v_1\,t + \frac{1}{2}a\,t^2$ | $d = v_2\,t - \frac{1}{2}a\,t^2$ | $v_2^{\,2} = v_1^{\,2} + 2\,a\,d$ |

*Solve each of these problems using the kinematics formulae.*

*Perry the platypus reaches a final velocity of 128 m/s after accelerating for 13.5 s and traveling 425 m. What was his acceleration?*

*Quinn's drag race car slows down at a rate of 11.8 m/s$^2$ and it takes 5.32 s to come to a complete stop. What was the initial velocity of the car?*

*Rob travels 48 m as he comes to a stop in 6.3 s. What was his initial velocity?*

*At what minimum velocity must Steph throw a tennis ball straight up in order for it to rise to a height of at least 2.8 m?*

*Starting from rest, Tony accelerates at 11.7 m/s$^2$ for 8.4 s. How far does he travel during this time?*

*Victoria is driving at 35 m/s when she sees a herd of moose crossing the road 95 m ahead of her car. If her brakes can slow the car down at 6.8 m/s$^2$, what must her reaction time be before she applies the brakes in order to avoid hitting the moose?*

*Caleb is accelerating from rest at 2.54 m/s$^2$ for 21.5 s. After he reaches this maximum speed, he continues at that speed for 34.7 s. How far does he travel in total?*

# 1.5: Projectile Motion

At the end of this section, you should be able to:

- ☐ Recognize that projectiles follow a parabolic path where the horizontal component of motion is uniform and the vertical component is free fall.
- ☐ Resolve velocity vectors into components and solve problems where a projectile is launched.
- ☐ Solve for range, time of flight, velocity at any point (including initial and final velocity), and maximum height.

A **projectile** is ______________________________________________________________

______________________________________________________________________________

*Imagine a projectile is launched at an angle over a flat surface. Given the above definition, draw vectors to represent the horizontal and vertical components of the projectile's velocity, as well as the acceleration.*

When dealing with projectile motion, we have several key assumptions:

1. ______________________________________________________________________
2. ______________________________________________________________________
3. ______________________________________________________________________
4. ______________________________________________________________________

*A projectile rolls horizontally off a 12.0 m high ledge with an initial velocity of $v_x$ = 2.5 m/s.*

a) *Sketch a diagram including the path of the projectile.*

b) *Calculate the range, or horizontal displacement, of this projectile. List your givens, keeping x and y separate!*

c) *Find the overall velocity at t = 0.83 s (magnitude and direction).*

*A ball rolls horizontally off a ledge, 9.0 m high, with a velocity of 2.0 m/s.*

a) *For how much time is the projectile in the air?*

b) *What is the horizontal range of the projectile?*

c) *What is the overall velocity of the ball at 0.56 s after it leaves the ledge?*

*A football is kicked with an initial velocity of 21.0 m/s at an angle of 53° above the horizontal. It lands at the same height from which it was kicked.*

*a) Find the range of the football.*

*b) Find the maximum height reached by the football.*

*A rock is thrown from the top of a 23.0 m tall cliff with an initial velocity of 18.0 m/s at 63° above the horizontal.*

a) *If the ball hits the ground at the bottom of the cliff, how much time will it spend in the air?*

b) *What will be the range of the projectile?*

c) *What will be its maximum height?*

*A soccer ball is kicked at 16.6 m/s at 57° above horizontal. Will the ball go over the fence, which is 3.0 m tall and 20.0 m away?*

*A cannon on a 125 m high cliff fires a projectile with an initial velocity of 42 m/s at an angle of 27° below the horizontal.*

a) *What is the range of this projectile?*

b) *What is the final velocity of the projectile as it hits the ground (it's not 0!)?*

*Ryan fires his paintball gun at 92 m/s at 17° below the horizontal from the top of the school, which is 6.4 m tall. Will he hit Caleb, who is standing 17.4 m away from the school and is 1.75 m tall?*

# 1.6: Review of Momentum and Collisions

At the end of this section, you should be able to:

- ☐ Write an equation using the law of conservation of momentum describing an interaction between two objects
- ☐ Use the law of conservation of momentum to solve for some missing quantity

Remember from last year, the basic formula for **momentum**:

We also used the **Law of Conservation of Momentum**, which states that:

________________________________________________________________

________________________________________________________________

Or in math terms:

*A train car with a mass of 1500 kg and an initial velocity of 6.0 m/s [East] collides with a train car which is at rest and has a mass of 1200 kg. What is the final velocity of the cars after the collision if they stick together?*

*A 1.0 kg ball moving with a velocity of 3.0 m/s [right] collides with a stationary 2.0 kg ball. After the collision the 2.0 kg ball moves at 2.0 m/s [right]. What is the velocity of the 1.0 kg ball after the collision?*

*A 50.0 kg figure skater is standing at rest on the ice. A 95.0 kg hockey player, also at rest, pushes the skater. What is the velocity of the figure skater if the hockey player moves backwards at 2.00 m/s after the push?*

*With what speed must a 12.0 kg Super-Monkey be running in order to completely stop a 12000.0 kg freight train moving at 140.0 km/hr?*

*A grenade of mass 290 g at rest blows up into two fragments. If one fragment has a mass of 120 g and is moving at 220 m/s, what is the mass and velocity of the other fragment?*

# 1.7: Momentum and Collisions in Two Dimensions

At the end of this section, you should be able to:

- ☐ Solve problems involving 2D collisions and explosions where momentum is conserved.

Even though last year we only dealt with interactions in one dimension, the **Law of Conservation of Momentum** still applies in two or more dimensions. This year we will see collisions and explosions in two dimensions. The main difference this year is that we will look at the two components of momentum (x and y) separately.

This makes the problems much more complicated, but the basic ideas are the same - you just need to keep the x-components and y-components of velocity and momentum separate when doing most of the calculations.

*A 2.0 x $10^3$ kg car travelling at 20.0 m/s [N] is struck at an intersection by a 2500 kg pickup truck travelling at 14.0 m/s [W]. If the vehicles stick together upon impact, what will be the velocity of the car-truck combination immediately after the collision?*

*A 0.400 kg grenade is stationary when it explodes into three fragments:*

- *One fragment (A), of mass 0.237 kg, goes off at 19.7 m/s [35° S of E].*
- *A second fragment (B), of mass 0.066 kg, travels due North.*
- *A third fragment (C), of mass 0.097 kg travels due West.*

*Determine the velocity of the second and third fragments after the explosion.*

*A 0.170 kg cue ball is moving to the east at 1.30 m/s when it encounters a stationary 8-ball which has a mass of 0.160 kg. After the collision the 8-ball moves off at 1.14 m/s [15°S of E]. What is the final velocity of the cue ball?*

*An 86.4 kg defenseman skating at 2.46 m/s [S] collides with an 84 kg centreman skating directly West. The players get stuck together and slide at 1.57 m/s [52° S of W]. What was the initial velocity of the centreman?*

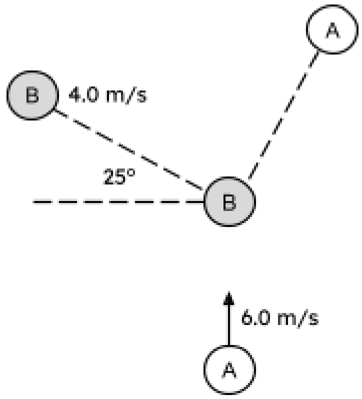
*A 3.2-kg hawk soaring at 23 m/s [N] collides with a 0.50-kg sparrow flying at 5.0 m/s [W]. If both the hawk and sparrow are on the same horizontal plane, find their velocity if the hawk hangs on to the sparrow after collision.*

*Radioactivity is the result of atoms that decay or break apart spontaneously. A stationary parent nucleus of mass 1.2 x $10^{-24}$ kg decays into three particles.*

- *One particle of mass 3.0 x $10^{-25}$ kg moves away with a velocity of 2.0 x $10^{7}$ m/s [E].*
- *Another particle of mass 2.3 x $10^{-25}$ kg moves at a speed of 4.2 x $10^{7}$ m/s [N].*

*Calculate the mass and velocity of the third particle.*

*From the diagram below, determine the final velocity of the ball A after the collision if both A and B have the exact same mass.*



*A stationary 1.00 kg grenade explodes into four pieces, all moving parallel to the ground:*

- *The first piece (A) of mass 0.20 kg moves East at 24 m/s.*
- *The second piece (B) of mass 0.30 kg flies North at 18 m/s.*
- *A third piece (C) of mass 0.25 kg is directed West at 32 m/s.*

*What is the velocity of the fourth piece (D) which has a mass of 0.25 kg?*