Electrical Systems Structure

Level 3 Physics Formula Sheet

- **D.C. Electricity**
  - **Current**

    Current is the rate of flow of charge

    $$I = \Delta Q/\Delta t$$

    I: Current (A)

    Q or q: Charge (C)

    T: time (s)

  - **Voltage**

    Voltage (Potential Difference) is the change in energy (work done) to each coulomb of charge between two points on a circuit, or two points across an electric field

    $$V = \Delta E/Q$$

    V: Voltage (V)

    ΔE or W: Change in energy/Work (J)

    Q or q: Charge (C)

  - **Resistance & Ohm's Law**

    Resists/limits the flow of current

    Each ohm (Ω) of resistance measures how many volts are required for each amp of current to pass through a resistor

Electrical Systems Structure

Level 3 Physics Formula Sheet

$$V = IR$$

V: Voltage (V)
I: Current (A)
R: Resistance (Ω)

- **Power**

  The rate of work done in a circuit

$$P = IV$$

$P = I^2R$ ←Ohm's Law substitutions→ $P = V^2/R$

P: Power (W)
V: Voltage (V)
I: Current (A)
R: Resistance (Ω)

- **Internal Resistance of a Battery**

  Batteries can be thought of as having an ideal voltage supply E.M.F. (Electromotive Force) in series with an internal resistance

$$V = \varepsilon - Ir$$

V: Voltage across battery (V)
ε: E.M.F. of battery (V)
I: Current running through battery(A)

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

r: Internal Resistance of battery (Ω)

- **Kirchhoff's Laws**
  - **Kirchhoff's Current Law**
    "*At any junction in a circuit, the total current entering the junction equals the total current leaving the junction*"
  - **Kirchhoff's Voltage Law**
    "*Around any closed path of a circuit, the total of all the potential differences,* V, *is zero*"

D.C. Circuit Construction Kit - PhET

D.C. Circuit Lab - PhET

- **Capacitors**

  Capacitors store energy in the form of an electric field

  Capacitance is the amount of Charge stored per Volt on capacitor plates

$$C = Q/V$$

C: Capacitance (F: Farad)

Electrical Systems Structure

Level 3 Physics Formula Sheet

NB: the Farad is a very large unit and is therefore usually given in:

$$\mu F = 10^{-6}\ F$$

$$nF = 10^{-9}\ F$$

$$pF = 10^{-12}\ F$$

Q or q: Charge stored on capacitor plates (C)
V: Voltage (Potential Difference) between capacitor plates (V)

Capacitance depends on:

- The area of the capacitor plates

$$C \propto A$$

- The distance between the plates

$$C \propto 1/d$$

- ∴ $C \propto A/d$

$$\therefore\ C = \varepsilon_o A/d$$

C: Capacitance (F: Farad)
A: Area of capacitor plates (m^2)
d: Distance between the plates (m)
ε_o: Permittivity of free space
$\varepsilon_o = 8.854 \times 10^{-12}\ Fm^{-1}$

Electrical Systems Structure

Level 3 Physics Formula Sheet

- **Dielectric**
  - An insulating material placed in between the capacitor plates to increase the Capacitance
  - Dielectric, ε_r, is a ratio that determines that factor that the Capacitance will increase by This is unique to each material used

$$C = \varepsilon_r \varepsilon_o A/d$$

PhET Application on Capacitors & Dielectrics

- **Networks of Capacitors**
  - **Capacitors in Series**

$$1/C_t = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$$

$1/C_t$: Inverse of Total Capacitance (C^{-1})

$1/C_1$ etc: Inverse of each individual capacitor (C^{-1})

  - **Capacitors in Parallel**

$$C_t = C_1 + C_2 + C_3 + \ldots$$

Electrical Systems Structure

Level 3 Physics Formula Sheet

C_t: Total Capacitance (F)

C_1 etc: each individual capacitor (F)

- **Energy Stored in a Capacitor**

$$E_p = \frac{1}{2} QV$$

$$E_p = \frac{1}{2} CV^2 \leftarrow \text{Substituting } Q = CV$$

$$E_p = Q^2/2C \leftarrow \text{Substituting } Q/C = V$$

E_p: Energy stored in a capacitor (J)

C: Capacitance (F: Farad)

Q or q: Charge stored on capacitor plates (C)

V: Voltage (Potential Difference) between capacitor plates (V)

- **Charging & Discharging a Capacitor (PhET App on Capacitors)**
  - **Time Constant**

The time it takes for a capacitor to charge up to 63% of the difference between the initial value and the final value.

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

The time it takes for a capacitor to discharge down to 37% of its initial value.

$$\tau = RC$$

τ: Time Constant (s)
R: Resistance in circuit (Ω)
C: Capacitance (F)

Charging	Discharging
$V = V_b \times (1 - 0.37^n)$ **OR** $V = V_b \times (1 - e^{-t/\tau})$	$V = V_c \times 0.37^n$ **OR** $V = V_b \times e^{-t/\tau}$
Voltage increases from zero to battery voltage	Voltage decreases from initial voltage to zero
$V_b = V_C + V_R$	$V_C = V_R$
Current ∝ V_R	Current ∝ V_R
$I = I_i \times 0.37^n$ **OR** $I = I_i \times e^{-t/\tau}$	$I = I_i \times 0.37^n$ **OR** $I = I_i \times e^{-t/\tau}$
Current decreases from initial current to zero	

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

These formulas are not given on the formula sheet

V: Voltage across capacitor at a given time (V)
V_b: Voltage of charging battery (V)
V_c: Initial voltage across discharging capacitor (V)
V_R: Voltage across Resistor (V)
I: Current at a given time (A)
I_i: Initial Current (A)
n: Number of Time Constants that have passed
e: Euler's number is an irrational number
$e \approx 2.71828$
$e^{-1} \approx 0.367879$ close to 37%

$n = t/\tau$ ← (time in seconds ➗ Time Constant)

- **Inductors**
  Inductors store energy in the form of a magnetic field

Electrical Systems Structure

Level 3 Physics Formula Sheet

- **Magnetic Flux**

The product of the magnetic field, B, penetrating an area, A, normal to the field

$$\phi = BA$$

ϕ: Magnetic Flux (weber: Wb = Vs)
B: Magnetic Field (Tesla: T)
A: Area normal to the field (m^2)

When the magnetic field, B, and area, A, are not at right angles, then

$$\phi = BA\sin(\theta)$$

θ: Angle between magnetic field, B, and area, A

NB: Weber Unit

$$Wb = Tm^2 = Vs = J/A = (kgm^2)/(s^2A)$$

- **Lenz's Law**

Electrical Systems Structure

Level 3 Physics Formula Sheet

"*An induced current will always flow in the direction that will oppose the changing flux that caused it*"
This is an extension of the conservation of energy law

- **Faraday's Law**

The induced voltage, e.m.f., is directly proportional to the rate of change of flux

$$V = -\Delta\phi/\Delta t$$

OR

$$\varepsilon = -\Delta\phi/\Delta t$$

V *OR* ε: Induced e.m.f. (V)
$\Delta\phi$: Change in Magnetic Flux (Wb)
Δt: Change in time (s)
- : to remind you that the induced e.m.f. always opposes the change in flux - Lenz's Law

Faraday's Law PhET Application

- **Transformers**

The ratio of the secondary to primary voltage is equal to the ratio of the secondary to primary turns

Electrical Systems Structure

Level 3 Physics Formula Sheet

$$V_s/V_p = N_s/N_p$$

V_s: Secondary Voltage (V)
V_p: Primary Voltage (V)
N_s: Secondary Turns
N_p: Primary Turns

In an ideal transformer:

Secondary Power = Primary Power

In reality, energy is lost through heat from eddy currents generated in the soft iron core from the changing flux.

- **Inductance**

Mutual Inductance	Self Inductance
$V = -m\Delta I/\Delta t$	$V = -L\Delta I/\Delta t$
m: Mutual Inductance (H: Henrys)	L: Self Inductance (H: Henrys)
-: to remind you of Lenz's Law (inductor will react against	

Electrical Systems Structure

Level 3 Physics Formula Sheet

any change in flux)
V *or* e.m.f. : Induced Voltage *or* back e.m.f. (V)
ΔI: Change in Current (A)
Δt: Change in Time (s)

- **Charging & Discharging an Inductor** **(PhET App on Inductors)**
  - **Time Constant**

The time it takes for an inductor to discharge down to 37% of its initial value.

$$\tau = L/R$$

τ: Time Constant (s)
R: Resistance in circuit (Ω)
L: Inductance (H)

Powering Up	Powering Down
$V = V_L \times 0.37^n$ OR $V = V_L \times e^{-t/\tau}$	$V = V_L \times 0.37^n$ OR $V = V_L \times e^{-t/\tau}$

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

Voltage decreases from initial voltage ($V_L = - V_b$) to zero	Voltage decreases from initial voltage ($V_L = V_b$) to zero
$V_b = V_L + V_R$	$V_L = V_R$
Current $\propto V_R$	Current $\propto V_R$
$\mathbf{I = I_i (1 - 0.37^n)}$ **OR** $\mathbf{I = I_i (1 - e^{-t/\tau})}$	$\mathbf{I = I_i \times 0.37^n}$ **OR** $\mathbf{I = I_i \times e^{-t/\tau}}$
Current increases from zero current to a max	**Current decreases from initial current to zero**
These formulas are not given on the formula sheet	

V: Voltage across inductor at a given time (V)
V_b: Voltage of charging battery (V)
V_L: Initial voltage across inductor (V)
V_R: Voltage across Resistor (V)
I: Current at a given time (A)
I_i: Initial Current (A)
n: Number of Time Constants that have passed
e: Euler's number is an irrational number

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

$e \approx 2.71828$

$e^{-1} \approx 0.367879$ close to 37%

$n = t/\tau$ ← (time in seconds ➗ Time Constant)

- **A.C. Electricity**
  - **A.C. Power & Root Mean Squared (r.m.s.) Current and Voltage**

The average power in an a.c. circuits is half the maximum power

$$\text{Power}_{av} = \tfrac{1}{2}\,\text{Power}_{max}$$

$I_{r.m.s.}$ to I_{max}	$V_{r.m.s.}$ to V_{max}
$I_{av} \times V_{av} = (I_{max} \times V_{max})/2$ $I^2_{av} \times R = (I^2_{max} \times R)/2$ $\therefore I^2_{av} = (I^2_{max})/2$ $I_{av} = I_{max}/\sqrt{2}$ $\sqrt{2} \times I_{av} = I_{max}$ **$\therefore \sqrt{2} \times I_{r.m.s.} = I_{max}$**	$I_{av} \times V_{av} = (I_{max} \times V_{max})/2$ $V^2_{av}/R = (V^2_{max}/R)/2$ $\therefore V^2_{av} = (V^2_{max})/2$ $V_{av} = V_{max}/\sqrt{2}$ $\sqrt{2} \times V_{av} = V_{max}$ **$\therefore \sqrt{2} \times V_{r.m.s.} = V_{max}$**

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

∴ The average Current is equal to the maximum Current x $1/\sqrt{2}$

∴ The average Voltage is equal to the maximum Voltage x $1/\sqrt{2}$

$1/\sqrt{2} = 0.707$ (3s.f.) So $I_{r.m.s.}$ & $V_{r.m.s.}$ Are roughly 71% of their maximum values

- **A.C. Resistors**

$$V_{max} = I_{max} \times R$$

$$V_{r.m.s.} = I_{r.m.s.} \times R$$

Work in r.m.s. Or max but **not** both together

The voltage drop across a resistor is **always in phase** with the current running through the resistor

$$V \propto I$$

- **A.C. Capacitors**

$$V_c = I \times X_c$$

V_c: Voltage drop across the capacitor (V)

I: Current (A)

Electrical Systems Structure

Level 3 Physics Formula Sheet

X_c: Reactance of the capacitor (Ω)
NB: Again work in either r.m.s. Or max

$$X_c = 1/(\omega C) \quad \text{OR} \quad X_c = 1/(2\pi fC)$$

X_c: Reactance of the capacitor (Ω)
ω: Angular velocity of a.c. current ($rads^{-1}$)
$\omega = 2\pi f$
f: frequency of a.c. current (Hz)
C: Capacitance (F)

V_c is always 90° behind the current
(V_c lags I by π rad) - This is because $V_c \propto Q$ and it takes time for the charge to run on and off the plates

- **A.C. Inductors**

$$V_L = I \times X_L$$

V_L: Voltage drop across the inductor (V)
I: Current (A)
X_L: Reactance of the inductor (Ω)
NB: Again work in either r.m.s. Or max

Electrical Systems Structure

Level 3 Physics Formula Sheet

$$X_L = \omega L \quad \text{OR} \quad X_L = 2\pi f L$$

X_L: Reactance of the inductor (Ω)
ω: Angular velocity of a.c. current ($rads^{-1}$)
$\omega = 2\pi f$
f: frequency of a.c. current (Hz)
L: Inductance (H)

V_L is always 90° ahead of the current
(V_L leads I by π rad) - This is because $V_L \propto -\Delta I/\Delta t$, Faraday's Law, and the induced voltage reacts against any change in current. The rate of change of current is maximum when the current is zero (see a.c. current graph)

- **LCR Circuits and Resonance**

$$V_s = I \times Z$$

V_s: Voltage of the a.c. supply (V)
I: Current (A)
Z: Impedance of the whole circuit (Ω)

Electrical Systems Structure

[Level 3 Physics Formula Sheet]

$$V_s = \sqrt{((V_L - V_C)^2 + V_R^2))}$$

$$Z = \sqrt{((X_L - X_C)^2 + R^2))}$$

$V_s = IZ$

$V_L = IX_L$

$V_C = IX_C$

$V_R = IR$

A phasor diagram is a way to relate the phase relationships between the supply, inductor, capacitor and resistor in voltage drops or impedances, reactances and resistances

NB: Again work in either r.m.s. Or max

Angle of the Impedance, Z, or Source Voltage, Vs, to the current

$$\tan(\theta) = (X_L - X_C)/R = (V_L - V_C)/V_R$$

$$\theta = \tan^{-1}((X_L - X_C)/R) = \tan\text{-}1((V_L - V_C)/V_R)$$

[MIT Applet on RCL Phase Relationships of XL, XC & R]

Resonance in the LCR Circuit

Electrical Systems Structure

Level 3 Physics Formula Sheet

Because X_L and X_C are 180° apart, they directly oppose one another. Resonance will occur when $X_L = X_C$ and resonant frequency, f_o, can be found

$$X_L = X_C$$

$$2\pi fL = 1/(2\pi fC)$$

$$f_o = 1(2\pi\sqrt{(LC)})$$

NB: that Z and V_s will be in phase with the current