Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Deduces that $A = \pm 50$ **or** $b = \frac{1}{4}$	B1	3.4
	Deduces that $A = \pm 50$ **and** $b = \frac{1}{4}$	B1	3.4
	Uses $t = 0, H = 1 \Rightarrow \alpha = ...$ E.g. $1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$	M1	3.4
	$H = \left\| \pm 50\sin\left(\frac{1}{4}t + 1.15\right)° \right\|$	A1	3.3
		(4)	
(b)	E.g. the minimum height above the ground of the passenger on the original model was 0 m or Adding "d" means the passenger does not touch the ground.	B1	3.5b
		(1)	
		(5 marks)	
Notes:			

(a) Note that B0B1 is not possible

B1: Uses the equation of the given model to deduce that $A = \pm 50$ **or** $b = \frac{1}{4}$ o.e.

May be seen embedded within their equation.

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May be seen embedded within their equation.

M1: Uses $t = 0$ and $H = 1$ in the equation of the model to find a value for α .

Follow through on their value for A. Allow for $\pm 1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$ where α is in degrees or radians.

Note that in radians $\sin^{-1}\left(\frac{1}{50}\right) \approx \frac{1}{50}$ (0.0200…) which may appear incorrect but is in fact ok.

Also in degrees a value of e.g. 1.14 (truncated) would indicate the method.

A1: Writes down the correct full equation of the model: $H = \left| "\pm" 50\sin\left(\frac{1}{4}t + 1.15\right)° \right|$ o.e.

Condone omission of degrees symbol and allow awrt 1.15 for α.

Allow if a correct equation is seen anywhere in their solution.

(b)

B1: Gives a suitable explanation with no contradictory statements.

Condone "so that pod/capsule/seat/passenger/ferris wheel/it etc. will not hit/touch the ground"

Responses that focus on the starting point of the model are likely to score B0

Q2.

Question	Scheme	Marks	AOs
(a)	Attempts to use $h^2 = at + b$ with either $t = 2, h = 2.6$ or $t = 10, h = 5.1$	M1	3.1b
	Correct equations $2a + b = 6.76$, $10a + b = 26.01$	A1	1.1b
	Solves simultaneously to find values for a and b	dM1	1.1b
	$h^2 = 2.41t + 1.95$ cao	A1	3.3
		(4)	
(b)	Substitutes $t = 20$ into their $h^2 = 2.41t + 1.95$ and finds h or h^2 Or substitutes $h = 7$ into their $h^2 = 2.41t + 1.95$ and finds t	M1	3.4
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h = 7.08\,(\text{m})$ and states that it is a good model as 7.08 (m) is close to 7 (m) E.g II Finds t =19.5 years and states that the model is accurate as 19.5 (years) $\approx$ 20 (years)	A1	3.5a
		(2)	
			(6 marks)
Notes:			

(a)

M1: For translating the problem into mathematics. Attempts to use the given equation o.e. with either of the pieces of information to form one correct equation.

Award for unsimplified equations as well, such as $2.6^2 = 2a + b$ or $2.6 = \sqrt{2a + b}$

A1: Two correct (and different) equations which may be unsimplified

dM1: Solves simultaneously to find values for a and b. It is dependent upon the previous M

Don't be too concerned with the process here as calculators may be used.

Score if values of a and b are reached from a pair of simultaneous equations

A1: Establishes **the full equation of the model** with values of a and b given to **exactly** 3sf. Award if seen in either (a) or (b). It is not scored for the values of a and b.

Allow either $h^2 = 2.41t + 1.95$ or $h = \sqrt{2.41t + 1.95}$

If they go on to square root each term from $h^2 = 2.41t + 1.95$ then it is A0. E.g. $h = 1.55t + 1.40$

Special case for candidates who mistakenly use $h = at + b$

For $2.6 = 2a + b,\ \ 5.1 = 10a + b \ \Rightarrow h = 0.3125t + 1.975$ or $h = 0.313t + 1.98$

can score M1 correct equations with attempt to solve and A1 for either correct answer shown above.
These are the only marks available to them for a maximum mark of 1100 00

(b)

M1: A full and valid attempt to

either substitute $t = 20$ into their $h^2 = 2.41t + 1.95$ o.e. and find a value for h or h^2

or substitute $h = 7$ into their $h^2 = 2.41t + 1.95$ o.e. and find a value for t

(to enable the candidate to compare real life data with that of the model.)

The equation of the model must be of the correct form, either $h^2 = at + b$ or $h = \sqrt{at + b}$

Do not be too concerned with the mechanics of the solution but the square or $\sqrt{}$ must have been used appropriately to enable the comparison to be made.

In cases with no working you will need to check the calculation

A1: Compares their h=7.08m to 7m o.e using h^2 or their t =19.5 years to 20 years and makes valid conclusion with reason.

For this mark you require

- a statement that it is a "good" or "accurate' ' model or similar wording
- a reason such as "the values are close", "the values are similar" or "the predicted values are within 5% of the true values."
- a model with equation $h^2 = a\,t + b$ o.e. where $a =$ awrt 2.4 and $b \in [1.9, 2.0]$
- correct calculations

Condone a statement like ' the model is pretty accurate as it predicted 7.08m and the actual value is 7m'

Do not allow incorrect statements such as the model is incorrect as it does not give 7 metres.

Do not allow just "the model gives an underestimate of the true value."

Do not allow 'bad' or 'poor' model

Q3.

Question	Scheme	Marks	AOs
(a) Way 1	$H = Ax(40-x)$ $\{$or $H = Ax(x-40)\}$	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \dfrac{3}{100}$	dM1	3.1b
	$H = \dfrac{3}{100}x(40-x)$ or $H = -\dfrac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) Way 2	$H = 12 - \lambda(x-20)^2$ $\{$or $H = 12 + \lambda(x-20)^2\}$	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \dfrac{3}{100}$	dM1	3.1b
	$H = 12 - \dfrac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) Way 3	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0, H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40, H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20, H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\dfrac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}$ $3 = \dfrac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ or $\{H = 3 \Rightarrow\}$ $3 = 12 - \dfrac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \dfrac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\left\{\text{chooses } 20 + \sqrt{300} \Rightarrow\right\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12-\lambda(x-20)^2$ or $H = 12+\lambda(x-20)^2$
	Way 3: Writes $H = ax^2+bx+c$, **and** uses $(0, 0)$ to deduce $c=0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c=0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking H to x E.g. $H = \frac{3}{100}x(40-x)$, $H = 12-\frac{3}{100}(x-20)^2$ or $H = -0.03x^2+1.2x$
Note:	Condone writing y in place of H for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2+1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12-(x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2+1.2x$ or $a=-0.03, b=1.2, c=0$ in an implied ax^2+bx or ax^2+bx+c (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H=3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x\pm\alpha)^2 = \beta;\ \alpha,\ \beta \neq 0$
Note:	E.g. $1.2x-0.03x^2=3$ or $40x-x^2=100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for (their A)$x^2 = 3 \Rightarrow x = ...$ or their (their A)x^2 + (their k) $= 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give *at least one solution*
A1:	Interprets their solution in the original context by selecting the larger correct value *and states correct units for their value*. E.g. Accept awrt 37.3 m or $\left(20+\sqrt{300}\right)$ m or $\left(20+10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • H (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Q4.

Question	Scheme	Marks	AOs
(a)	$(A=)\,55$	B1	3.4
		(1)	
(b)	$\left\{\frac{dH}{dt}=\right\}-ABe^{-Bt}$ or $\left\{\frac{dH}{dt}=\right\}-"55"Be^{-Bt}$	M1	3.1b
	$-B\times"55"=-7.5\Rightarrow B=...\ \left(\frac{3}{22}=\text{awrt }0.136\right)$	M1	1.1b
	$H=55e^{-0.136t}+30$	A1cso	3.3
		(3)	
			(4 marks)

Notes

(a)

B1: 55 only. Just look for this value e.g. "$A=$" is not required. Ignore any "units" if given e.g. 55 °C

(b)

M1: Differentiates to obtain an expression of the form $\pm ABe^{-Bt}$ which may have their A already substituted in so allow for $\pm ABe^{-Bt}$ or $\pm"55"Be^{-Bt}$

M1: Substitutes $t=0$ and their A into their $\frac{dH}{dt}$, sets $=\pm7.5$ and proceeds to find a value for B which may be implied by $\frac{3}{22}$ or awrt 0.136

Their $\frac{dH}{dt}$ must not be H. i.e. it must be a "changed" function.

A1cso: Correct **equation** which follows **fully correct work** $H=55e^{-0.136t}+30$ but condone $H=55e^{-\frac{3}{22}t}+30$

The final equation must be correct but you can ignore spurious notation within their solution such as integral signs and "+ c" which do not affect their solution.

<u>**Marking guidance is as follows for particular cases in (b)**</u>

Case 1: $\left\{\frac{dH}{dt}=\right\}-"55"Be^{-Bt},\ -"55"Be^{-Bt}=7.5\Rightarrow\ B=-0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: it should be – 7.5

Case 2: $\left\{\frac{dH}{dt}=\right\}"55"Be^{-Bt},\ "55"Be^{-Bt}=-7.5\Rightarrow\ B=-0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 3: $\left\{\frac{dH}{dt}=\right\}"55"Be^{-Bt},\ "55"Be^{-Bt}=7.5\Rightarrow\ B=0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 4: $\left\{\frac{dH}{dt}=\right\}-"55"Be^{-Bt},\ "55"B=7.5\Rightarrow\ B=0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A1**

No errors

Q5.

Question	Scheme	Marks	AOs
(a)	$\log_{10} V = 3 \Rightarrow V = 10^3$	M1	1.1b
	$(V =)$ £1000	A1	3.4
		(2)	
(b)	e.g. $(\log_{10} b =) \dfrac{2.79-3}{10-0} = -0.021$ **or** $\log_{10} V = 3 - 0.021t$ **or** $10^{2.79} = \text{"}1000\text{"} b^{10}$	M1	1.1b
	e.g. $b = 10^{-0.021}$ $(= 0.952796....)$ **or** $V = 10^3 \times 10^{-0.021t}$ **or** $b = \sqrt[10]{\text{"}0.61659...\text{"}}$	M1	3.1b
	$V = 1000 \times 0.953^t$	A1ft	3.3
		(3)	
(c)	e.g. $V = 1000 \times \text{"}0.953\text{"}^{24}$ $(= £315)$ or e.g. $\log_{10} V = 3 - \text{"}0.021\text{"} \times 24 \Rightarrow V = ...(= £313)$	M1	3.4
	which is close (to £320) so it is a suitable model	A1	3.2b
		(2)	
			(7 marks)

Notes
(a) M1: Sets $\log_{10} V = 3$ and attempts to find a value for a or an expression for V when $t = 0$. Score for sight of 10^3 or implied by the correct answer. There may be more complicated routes to finding the initial value. e.g. finding a complete equation such as $\left(\log_{10} V = 3 + \frac{2.79-3}{10}t \Rightarrow \log_{10} V = 3 \Rightarrow V = \right) 10^3$ This mark can also be scored for the equation $V = 10^3 \times 10^{"-0.021"t}$ or $V = 1000 \times (....)^t$ but not $V = 10^{3-"0.021"t}$ (the 10^3 has not been split up from $10^{"-0.021"t}$) A1: £1000 cao (including units) do not accept £10^3
(b) Mark (b) and (c) together. Note work seen in (a) must be used in (b) to score M1: Either • finds the gradient between the two points. Score for the expression $\frac{2.79-3}{10-0}$ o.e. e.g. -0.021 Do not condone sign slips for this mark. May be implied by later work such as sight of $10^{-0.021}$. • finds the equation for $\log_{10} V$ in terms of t e.g. $\log_{10} V = 3 - "0.021"t$ which may be unsimplified. • forms the equation $10^{2.79} (= 616.5...) = "1000" b^{10}$ o.e. such as $2.79 = 3 + 10\log b$ M1: Attempts to find the value or an expression for b using their gradient or their equation Score for either: • the expression $10^{"-0.021"}$ o.e such as $10^{\frac{2.79-3}{10-0}}$ or may be implied by a correct value using their gradient. You may need to check this on your calculator. • correctly proceeding from $\log_{10} V = 3 - "0.021"t$ to $V = 10^{3-"0.021"t}$ and splitting this into $V = 10^3 \times 10^{-"0.021"t}$ • attempting to equate coefficients: $\log_{10} V = \log_{10} a + (\log_{10} b)t \Leftrightarrow \log_{10} V = 3 - "0.021"t \Rightarrow \log_{10} b = "-0.021" \Rightarrow b = 10^{"-0.021"}$ • using their equation $10^{2.79} = "1000" b^{10}$ or $2.79 = "3" + 10\log b$ and proceeding to e.g. $b = \sqrt[10]{"0.61659..."}$ or $b = 10^{"-0.021"}$ A1ft: Complete correct equation, follow through on their "1000" so score for $V = "1000" \times (\text{awrt } 0.953)^t$ or accept $V = "10^3" \times (\text{awrt } 0.953)^t$. Just stating the values of a and b is A0ft, but if the equation is written in (c) before substituting in $t = 24$ then this mark can be awarded.

(c) Mark (b) and (c) together

M1: A full and valid attempt to:

- either substitute $t = 24$ into their model of the form $V = ab^t$ where a is positive and finds a value for V
- or substitutes $t = 24$ into their model of the form $\log_{10} V = p + qt$ where p is positive and finds a value for V (if they only proceed as far as $\log_{10} V$ they would also have to find the value of $\log_{10} 320$)
- or substitutes $V = 320$ into their $V = 1000 \times 0.953^t$ o.e. and finds a value for t

(to enable the candidate to compare real life data with that of the model.)
Do not be too concerned with the mechanics of the solution but they must be attempting to find two values which can be compared (e.g. usually 320 and a value for V, but they could proceed to find $\log_{10} 320$ and compare with $\log_{10} V = 2.496$ when $t = 24$, or a value for t to compare with $t = 24$)
In cases with no working you will need to check the calculation.

A1: Compares their awrt £313-£315 with £320 or their awrt $t = 23.5 - 23.7$ with $t = 24$ or $\log_{10} 320 = 2.505...$ with 2.496 and makes a valid conclusion with a reason.
For this mark you require:

- correct calculations (if using percentage error allow this to be rounded to compare awrt £313-£315 with £320 then it will be in the range (1.4, 2.4). For £314.94 this is $=$ awrt 1.6%)
- a reason such as "the values are close", "the values are similar", "the values are approximately equal". Allow use of "$\approx$". Allow the calculation of the % error as reason.
- a statement that it is a "good" or "accurate" model or similar wording.

Note: Condone as a minimum e.g. "£314.94 and £320 so good model" (we accept the two values being stated here as a comparison that they are similar)

Do not allow incorrect statements such as the model is incorrect as it does not give £320.
Do not allow just "the model gives an underestimate of the true value" (does not comment sufficiently on whether the model is reliable)
Do not allow comments suggesting that the model is not reliable.
Note using the full value for b leads to 313.3285724…

Q6.

Question	Scheme	Marks	AOs
	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	(1)	
(b) Way 1	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	(3)	
(b) Way 2	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \dfrac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	(3)	
(b) Way 3	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	

(c)	Allow any of {0.84, in £s} represents • the *cost* of {making} each extra bar {of soap} • the direct *cost* of {making} a bar {of soap} • the marginal *cost* of {making} a bar {of soap} • the *cost* of {making} a bar {of soap} (Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) Way 1	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) Way 2	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss $= 1.12$}	M1	3.4
	• Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit $= 0.04$} leading to an answer of 369 {bars}	A1	3.2a
		(2)	
			(7 marks)

Notes for Question	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. k, c, m etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points (800, their 1100) and (300, their 680)
A1:	Confirms $y = 0.84x + 428$ is true for both (800, 1100) and (300, 680) and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c,\ 80 = 300k + c \Rightarrow k = \dfrac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	<u>**Special Case:**</u> Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Q7.

Question	Scheme	Marks	AOs
(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t+180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes	
(a)	
B1:	$a=60$ (may be seen in their final equation of the model or implied by 60 substituted for a in the model)
M1:	Attempts to find b by substituting in $t=0$, $H=2$ and their a and proceeding to a value for b. May be seen as two simultaneous equations formed: $2=a-b(-20)^2$ and $60=a-b(20-20)^2$ proceeding to a value for b
A1:	$H=60-0.145(t-20)^2$ or equivalent such as $H=-\frac{29}{200}t^2+5.8t+2$ or $H=60-\frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $a=60,\ b=0.145$ then A0 A correct answer with no working seen scores full marks.
(b)	
B1:	2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)
(c)	
M1:	$(\alpha=)\ 180$ or $(\beta=)\ 31$ Condone $(\alpha=)\ \pi$
A1:	$H=29\cos(9t+180)°+31$ or equivalent e.g. $H=-29\cos(9t)+31$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $\alpha=180,\ \beta=31$ then A0. A correct equation with no working seen scores both marks. Does not require the degree symbol.
(d)	
B1:	Score for a reason which makes reference to any of • the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once") • the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H=2$") • the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen") • the original model after 2 minutes would not be back at the start Do not allow vague responses on their own e.g. "the original model is a parabola" If calculations are used then they must be correct using a correct model (allow rounded or truncated) Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390

Q8.

Question	Scheme	Marks	AOs
(a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t=10, V=-1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t=3 \Rightarrow V =$ awrt 10400 barrels	B1ft	3.4
		(5)	
			(7 marks)

Notes:

(a)(i)

B1: 10750 barrels

(a)(ii)

B1: See scheme

(b)(i)

M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.

dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$

With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.

M1: Uses a correct method to find all constants in the model.

A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example

$V = 16000e^{-0.144t}$ $\quad$ $V = 16000 \times (0.866)^t$ $\quad$ $V = 15800e^{-0.146t} + 200$

(b)(ii)

B1ft: Follow through on their exponential model

Q9.

Question	Scheme	Marks	AOs
(a)	Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times-0.002}$	dM1	1.1b
	$\text{Distance} = \text{awrt } 204(\text{m})\text{ only}$	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$	M1	1.1b
	$=-0.002\left((d-100)^2-10000\right)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
			(9 marks)

Notes:
(a)
M1: Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$
M1: Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900 \Rightarrow d=..$) or even allow answers coming from a graphical calculator
A1: Awrt 204 m only
(b)
B1: States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"
(c)
M1: Score for taking out a common factor of -0.002 from at least the d^2 and d terms
M1: For completing the square for their (d^2-200d) term
A1: $=21.8-0.002(d-100)^2$ or exact equivalent
(d)
B1ft: For their '21.8+0.3' =22.1m
B1ft: For their 100m

Q10.

Question	Scheme	Marks	AOs
(a)	$2<x<6$	B1	1.1b
		(1)	
(b)	States either $k>8$ or $k<0$	M1	3.1a
	States e.g. $\{k:k>8\}\cup\{k:k<0\}$	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y=ax(x-6)^2$ or $\mathrm{f}(x)=ax(x-6)^2$	M1	1.1b
	Substitutes $(2,8)$ into $y=ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y=\frac{1}{4}x(x-6)^2$ or $\mathrm{f}(x)=\frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
		(6 marks)	

Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence

(a)

B1: Deduces $2 < x < 6$ o.e. such as $x > 2, x < 6$ $x > 2$ and $x < 6$ $\{x : x > 2\} \cap \{x : x < 6\}$ $x \in (2,6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g. $\{x > 2\} \cap \{x < 6\}$ $\{x > 2, x < 6\}$. Allow just the open interval $(2,6)$

Do not allow for incorrect inequalities such as e.g. $x > 2$ or $x < 6$, $\{x : x > 2\} \cup \{x : x < 6\}$ $x \in [2,6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either $k > 8$ (condone $k \geqslant 8$) or $k < 0$ (condone $k \leqslant 0$)

Condone for this mark $y \leftrightarrow k$ or $f(x) \leftrightarrow k$ and $8 < k < 0$

A1: Fully correct solution in the form $\{k : k > 8\} \cup \{k : k < 0\}$ or $\{k | k > 8\} \cup \{k | k < 0\}$ either way around

but condone $\{k < 0\} \cup \{k > 8\}$, $\{k : k < 0 \cup k > 8\}$, $\{k < 0 \cup k > 8\}$. It is not necessary to mention $\mathbb{R}$, e.g. $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$ Look for $\{\ \}$ and $\cup$

Do not allow solutions not in set notation such as $k < 0$ or $k > 8$.

(c)

M1: Realises that the equation of C is of the form $y = ax(x-6)^2$. Condone with $a = 1$ for this mark.

So award for sight of $ax(x-6)^2$ even with $a = 1$

dM1: Substitutes $(2,8)$ into the form $y = ax(x-6)^2$ and attempts to find the value for a.

It is dependent upon having an equation, which the ($y =$..) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for C $y = \frac{1}{4}x(x-6)^2$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x(x-6)^2$ but not $C = \frac{1}{4}x(x-6)^2$.

Allow this to be written down for all 3 marks

Examples of alternative methods

Alternative I part (c):
Using the form $y = ax^3 + bx^2 + cx$ and setting up then solving simultaneous equations.
There are various versions of this but can be marked similarly

M1: Realises that the equation of C is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in a, b and c. Condone with a =1 for this mark.

Note that the form $y = ax^3 + bx^2 + cx + d$ is M0 until d is set equal to 0.

There are four equations that could be formed, only two are necessary for this mark.

Condone slips

Using $(6, 0)$ $\Rightarrow 216a+36b+6c=0$

Using $(2, 8)$ $\Rightarrow 8a+4b+2c=8$

Using $\frac{dy}{dx}=0$ at $x = 2$ $\Rightarrow 12a+4b+c=0$

Using $\frac{dy}{dx}=0$ at $x = 6 \Rightarrow 108a+12b+c=0$

dM1: Forms and solves three different equations, one of which must be using $(2, 8)$ to find values for a, b and c. A calculator can be used to solve the equations

A1: Uses all of the information to form a correct equation for C $y=\frac{1}{4}x^3-3x^2+9x$ o.e.

ISW after a correct answer. Condone $\mathrm{f}(x)=\frac{1}{4}x^3-3x^2+9x$

Alternative II part (c)
Using the gradient and integrating

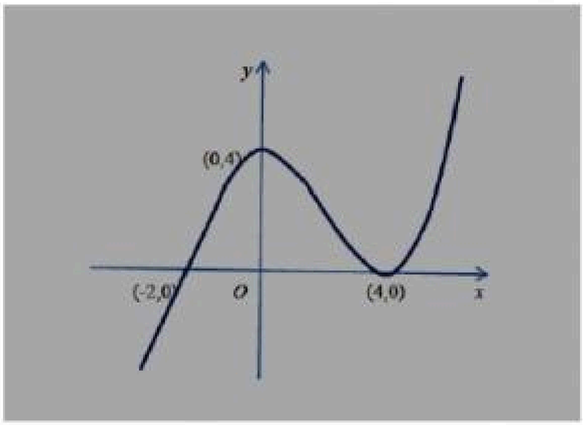
M1: Realises that the gradient of C is zero at 2 and 6 so sets $\mathrm{f}'(x) = k(x-2)(x-6)$ oe **and** attempts to integrate. Condone with $k = 1$

dM1: Substitutes $x = 2, y = 8$ into $\mathrm{f}(x) = k\left(...x^3+...x+...\right)$ and finds a value for k

A1: Uses all of the information to form a correct equation for C $y=\frac{3}{4}\left(\frac{1}{3}x^3-4x^2+12x\right)$ o.e.

ISW after a correct answer. Condone $\mathrm{f}(x)=\frac{3}{4}\left(\frac{1}{3}x^3-4x^2+12x\right)$

Q11.

Question Number	Scheme	Notes	Marks
(a)	$f(x) = (x+1)(x-2)^2$	M1: Either stating or writing down that $(x \pm 1)$ **or** $(x \pm 2)$ is a factor – may be implied by their f(*x*) A1: **Both** $(x + 1)$ **and** $(x - 2)$ are factors - may be implied by their f(*x*) B1: *y* or f(*x*)= $(x + 1)(x - 2)^2$	M1A1B1
	$= (x+1)(x^2 - 4x + 4) = x^3 - 3x^2 + 4$	M1: Multiplying out a quadratic to get 3 terms and then multiplying by the linear term to form a cubic. A1: $x^3 - 3x^2 + 4$ or $a = -3$, $b = 0$, $c = 4$	M1A1
			(5)
(b)		Same shape and position (ignore any coordinates) with the maximum on the y-axis	B1
		y intercept = 4 or their '*c*'	B1ft
		x coordinates at -2 and 4 or marked as coordinates. Allow (0, -2) and (0, 4) if they are marked in the correct position. The curve must cross or at least stop at *x* = -2	B1
			(3)
			[8]
(a) Way 2	$x = 0, y = 4 \Rightarrow c = 4$	Uses (0, 4) to obtain $c = 4$ (can be just stated)	B1
	$x = -1, y = 0 \Rightarrow -1 + a - b + c = 0$ $x = 2, y = 0 \Rightarrow 8 + 4a + 2b + c = 0$	Uses both (-1, 0) and (2, 0) in $y = x^3 + ax^2 + bx + c$ to form 2 simultaneous equations. Allow the equations to contain *c* here.	M1
	$a - b = -3$ $4a + 2b = -12$ $\Rightarrow a = .. \text{ or } b = ...$	Solves simultaneously with a value for *c* to obtain a value for *a* or a value for *b*	M1
	Either $a = -3$ **or** $b = 0$		A1
	Both $a = -3$ **and** $b = 0$		A1

(a) **Way 3**	$\frac{dy}{dx} = 3x^2 + 2ax + b$	M1: $x^n \to x^{n-1}$ at least once including c → 0	M1
	$x = 0 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow b = 0$	Correct value for b	A1
	$x = 0, y = 4 \Rightarrow c = 4$	Uses (0, 4) to obtain $c = 4$ (can be just stated)	B1
	$3(2)^2 + 2a(2) + b = 0$ or $(-1)^3 + a(-1)^2 + b(-1) + 4 = 0$	Obtains an equation in a	M1
	$a = -3$	Correct value for a	A1
			(5)
	Special case: A common incorrect approach is to assume the cubic is of the form e.g. $f(x) = x(x \pm 1)(x \pm 2) + 4$ This scores B1 only for $c = 4$		
			[8]

Q12.

Question Number	Scheme	Marks
(a)	$R = \sqrt{20}$	B1
	$\tan\alpha = \frac{4}{2} \Rightarrow \alpha = \text{awrt } 1.107$	M1A1
		(3)
(b)(i)	$'4 + 5R^2' = 104$	B1ft
(ii)	$3\theta - '1.107' = \frac{\pi}{2} \Rightarrow \theta = \text{awrt } 0.89$	M1A1
		(3)
(c)(i)	4	B1
(ii)	$3\theta - '1.107' = 2\pi \Rightarrow \theta = \text{awrt } 2.46$	M1A1
		(3)
		(9 marks)

(a)

B1 Accept $R = \sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47
Do not accept $R = \pm\sqrt{20}$
This could be scored in parts (b) or (c) as long as you are certain it is R

M1 for sight of $\tan\alpha = \pm\frac{4}{2}, \tan\alpha = \pm\frac{2}{4}$. Condone $\sin\alpha = 4, \cos\alpha = 2 \Rightarrow \tan\alpha = \frac{4}{2}$

If R is found first only accept $\sin\alpha = \pm\frac{4}{R}, \cos\alpha = \pm\frac{2}{R}$

A1 $\alpha =$ awrt 1.107. The degrees equivalent $63.4°$ is A0.
If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if R was incorrect allow for the numerical value of their $'4+5R^2'$.
Allow a tolerance of 1 dp on decimal R's.

(b)(ii)

M1 Using $3\theta \pm \text{their}'1.107' = \frac{\pi}{2} \Rightarrow \theta = ..$

Accept $3\theta \pm \text{their}'1.107' = (2n+1)\frac{\pi}{2} \Rightarrow \theta = ..$ where n is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107') = \frac{\pi}{2} \Rightarrow \theta = ..$

The degree equivalent is acceptable $3\theta - \text{their}'63.4°' = 90° \Rightarrow \theta =$
Do not allow mixed units in this question

A1 awrt 0.89 radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta \pm \text{their}'1.107' = 2\pi \Rightarrow \theta = ...$
Accept $3\theta \pm \text{their}'1.107' = n\pi \Rightarrow \theta = ..$ where n is an integer , including 0
Allow slips on the lhs with an extra bracket such as
$3(\theta \pm \text{their}'1.107') = 2\pi \Rightarrow \theta = ..$
The degree equivalent is acceptable $3\theta - \text{their}'63.4°' = 360° \Rightarrow \theta =$ but
Do not allow mixed units in this question

A1 $\theta =$ awrt 2.46 radians or $141.1°$ Do not allow multiple solutions for this mark.

Q13.

Question	Scheme	Marks	AOs
(a)	$R = \sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5 + \sqrt{5}\sin\left(\frac{\pi t}{12} + 1.107 - 3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12} + 1.107 - 3 = \frac{\pi}{2} \Rightarrow t =$	M1	3.1b
	$t =$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R = \sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}, \sin\alpha = \pm\frac{2}{"R"}$ OR $\cos\alpha = \pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha =$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C Remember to isw

Condone a lack of units. Follow through on their value of R so allow $(5 + "R")$ °C

(c)

M1: An complete strategy to find t from $\frac{\pi t}{12} \pm 1.107 - 3 = \frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t \pm 63.4 - 171.9 = 90$

A1: awrt $t = 13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

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It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$$\frac{d\theta}{dt} = \frac{\pi}{12}\cos\left(\frac{\pi t}{12} - 3\right) - \frac{2\pi}{12}\sin\left(\frac{\pi t}{12} - 3\right) = 0 \Rightarrow \tan\left(\frac{\pi t}{12} - 3\right) = \frac{1}{2} \Rightarrow t = 13.23 = 13{:}14 \text{ scores M1 A1 A1}$$

$$\frac{d\theta}{dt} = \cos\left(\frac{\pi t}{12} - 3\right) - 2\sin\left(\frac{\pi t}{12} - 3\right) = 0 \Rightarrow \tan\left(\frac{\pi t}{12} - 3\right) = \frac{1}{2} \Rightarrow t = 13.23 = 13{:}14 \text{ they can score M1 A0 A1 (SC)}$$

A value of $t = 1.23$ implies the minimum value has been found and therefore incorrect method M0.

Q14.

Question Number	Scheme			Marks
(a)	$\dfrac{dN}{dt} = \dfrac{(kt-1)(5000-N)}{t}, \quad t>0,\ 0<N<5000$			
	$\displaystyle\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or } = \int\left(k-\frac{1}{t}\right)dt\right\}$		See notes	B1
	$-\ln(5000-N) = kt - \ln t;\ +c$		See notes	M1 A1; A1
	then eg either...	*or...*	*or...*	
	$-kt + c = \ln(5000-N) - \ln t$	$kt + c = \ln t - \ln(5000-N)$	$\ln(5000-N) = -kt + \ln t + c$	
	$-kt + c = \ln\left(\dfrac{5000-N}{t}\right)$	$kt + c = \ln\left(\dfrac{t}{5000-N}\right)$	$5000 - N = e^{-kt+\ln t+c}$	
	$e^{-kt+c} = \dfrac{5000-N}{t}$	$e^{kt+c} = \dfrac{t}{5000-N}$	$5000 - N = te^{-kt+c}$	
	leading to $N = 5000 - Ate^{-kt}$ **with no incorrect working/statements. See notes**			A1 * cso
				[5]
(b)	$\{t=1,\ N=1200 \Rightarrow\}\ 1200 = 5000 - Ae^{-k}$ $\{t=2,\ N=1800 \Rightarrow\}\ 1800 = 5000 - 2Ae^{-2k}$		At least one correct statement written down using the boundary conditions	B1
	So $Ae^{-k} = 3800$ and $2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$			
	Eg. $\dfrac{e^{-k}}{2e^{-2k}} = \dfrac{3800}{3200}$ or $\dfrac{2e^{-2k}}{e^{-k}} = \dfrac{3200}{3800}$ So $\dfrac{1}{2}e^{k} = \dfrac{3800}{3200}$ or $2e^{-k} = \dfrac{3200}{3800}$		An attempt to eliminate A by producing an equation in only k.	M1
	$k = \ln\left(\dfrac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\dfrac{19}{8}\right)\right\}$		At least one of $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent	A1
	$\left\{A = 3800(e^{k}) = 3800\left(\dfrac{19}{8}\right) \Rightarrow\right\}\ A = 9025$		Both $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent	A1
				[4]
	Alternative Method for the M1 mark in (b)			
	$e^{-k} = \dfrac{3800}{A}$ $2A\left(\dfrac{3800}{A}\right)^2 = 3200$		An attempt to eliminate k by producing an equation in only A	M1
(c)	$\left\{t=5,\ N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$			
	$N = 4402.828401... = 4400$ (fish) (nearest 100)		anything that rounds to 4400	B1
				[1]
				10

	Question Notes	
(a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda\neq 0$ is a constant.
	A1	For $-\ln(5000-N)=kt-\ln t$ or $\ln(5000-N)=-kt+\ln t$ or $-\frac{1}{k}\ln(5000-N)=t-\frac{1}{k}\ln t$ oe **which is dependent on the 1st M1 mark being awarded.**
	A1	For applying a constant of integration, eg. $+c$ or $+\ln e^c$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or "$\ln e^c$" "$\ln c$" or and applies a fully correct method to prove the result $N=5000-Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example • **either** $5000-N=e^{\ln t-kt+c}$ • **or** $5000-N=te^{-kt+c}$ • **or** $5000-N=te^{-kt}e^c$ or equivalent needs to be stated before achieving $N=5000-Ate^{-kt}$
(b)	**B1**	At least one of **either** $1200=5000-Ae^{-k}$ (or equivalent) **or** $1800=5000-2Ae^{-2k}$ (or equivalent)
	M1	• **Either** an attempt to eliminate A by producing an equation in only k. • or an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k=\ln\left(\frac{19}{8}\right)$ or $k=-\ln\left(\frac{8}{19}\right)$ or $k=\ln 7600-\ln 3200$ or $k=-\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k=0.8649...$ without a correct exact equivalent is A0.
(c)	**B1**	anything that rounds to 4400

Q15.

Question	Scheme	Marks	AOs
(a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20\,000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(a) Option 1

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20000$ into their exponential model

Candidates may start by simply writing $V = 20000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares V = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20000e^{-0.223t} \Rightarrow 2000 = 20000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Uses $t = 0 \Rightarrow A = 20000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16000 = 20000r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20000$

You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of V when $t = 10$. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares (£) 2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of r in the range $(0.8, 1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8, 1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20000 \Rightarrow A = 18000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16000 = 2\,000 + 18000e^{k} \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0.251t} + 2000$

(b)

M1: Uses their model to find the value of V when $t = 10$.
Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251 \times 10} + 2000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of *k* or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$
Condone 'make the value of *k* or the -0.251 greater (or less negative)'

....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

Q16.

Question Number	Scheme	Marks
(a)	$D = 10, t = 5, \quad x = 10e^{-\frac{1}{8}\times 5}$ $= 5.353$ awrt	M1 A1 (2)
(b)	$D = 10 + 10e^{-\frac{5}{8}}, t = 1, \quad x = 15.3526\ldots \times e^{-\frac{1}{8}}$ $x = 13.549$ (✱)	M1 A1 cso (2)
Alt.(b)	$x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ M1 $\quad x = 13.549$ (✱) A1 cso	
(c)	$15.3526\ldots e^{-\frac{1}{8}T} = 3$ $e^{-\frac{1}{8}T} = \dfrac{3}{15.3526\ldots} = 0.1954\ldots$ $-\dfrac{1}{8}T = \ln 0.1954\ldots$ $T = 13.06\ldots$ or 13.1 or 13	M1 M1 A1 (3) **(7 marks)**

Notes: (b) (main scheme) M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or $\{10 + \text{their(a)}\}\, e^{-\frac{1}{8}}$

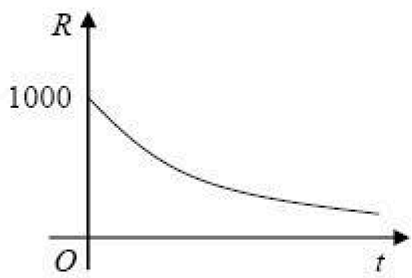
N.B. The answer is given. There are many correct answers seen which deserve M0A0 or M1A0

(c) 1st M is for $(10 + 10e^{-\frac{5}{8}})\, e^{-\frac{T}{8}} = 3$ o.e.

2nd M is for converting $e^{-\frac{T}{8}} = k$ $(k > 0)$ to $-\dfrac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme

Q17.

Question Number	Scheme		Marks	
	(a) 1000		B1	(1)
	(b) $1000\,e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	(2) **[9]**

Q18.

Question	Scheme	Marks	AOs
(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \dfrac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes
(a) M1: Takes logs of both sides and shows the addition law. Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$ A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer. Also allow t rather than T and A rather than a. **Allow working backwards e.g.** $\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$ $\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b$ * M1: Uses the given answer and uses the power law and addition law correctly A1: Reaches the given equation with no errors as above (b) B1: Deduces the correct value for b (Allow awrt 0.495 or $\frac{45}{91}$) M1: Correct strategy to find the value of a. E.g. substitutes one of the given points and their value for b into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of a. Allow slips in rearranging their equation but must be correct log work to find a. Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of a. E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$ A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$ (Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$) **Must see the <u>equation</u> not just correct values as it is a requirement of the question.** (c) B1: Correct interpretation

Q19.

Question	Scheme	Marks	AOs
(a) Way 1	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ or $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\quad 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = \ldots$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = \ldots$		
	$\{n = \text{awrt } 2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	(3)	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)

ADVICE: Ignore labelling (a), (b), (c) when marking this question

Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part

Notes for Question	
Note:	In their solution to (a) and/or (b) condone writing log in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model (**correct way round**)
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their d to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of d
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their d to find a total stopping distance and gives a correct ft conclusion or finds their d and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of d in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Q20.

Question	Scheme	Marks	AOs
(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
			(9 marks)

Notes:

(a)

M1: Takes logs of both sides and shows the addition law

M1: Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$

(b)

M1: Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** $b =$ gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$

M1: Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** $b =$ gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$

M1: Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$

A1: Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work.

There is an alternative to this using a graphical approach.

M1: Finds the value of $\log_{10} T$ from T =3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$

M1: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48"

Accept $\log_{10} N \approx 2.9$

M1: Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$

A1: Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work

(c)

M1 For using $N = 1000000$ and stating that $\log_{10} N = 6$

A1: Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate"

There is an alternative approach that uses the formula.

M1: Use $N = 1000000$ in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$.

A1: The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds

(d)

B1: Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at T =1