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B.Sc. (Hons.) Math (Semester – 4th)
NUMERICAL METHODS
Subject Code: BMATS1423
Paper ID: [22131217]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1 Attempt the following:

a What is the rate of convergent and explain convergent condition in Newton Raphson method?

b Explain in brief: Convergence of iteration methods.

c Solve the system of equations using the Gauss-Seidel Method $45x_1 + 2x_2 + 3x_3 = 58$, $-3x_1 + 22x_2 + 2x_3 = 47$, $5x_1 + x_2 + 20x_3 = 67$

d Write a sufficient condition for Gauss Seidel method to converge?

e Give the multistep methods available for solving ordinary differential equation. What is the error in Milne's Predictor formula.

f A third degree polynomial passes through the points (0, -1), (1, 1), (2, 1) and (3, -2), using Newton's forward interpolation formula finds the polynomial. Hence, find the value at 1.5

g Show that $\Delta\left[\frac{1}{f(x)}\right] = \frac{-\Delta f(x)}{f(x)f(x+1)}$.

h What is inverse Interpolation? What is the assumption we make when Lagrange's formula is used?

i What are the errors in trapezoidal and Simpson's rules of numerical integration? State three point Gaussian Quadrature formula.

j Apply Euler's modified method to approximate the solution of the initial value problem and calculate y(1.3) by using h=0.1: $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \quad y(1) = 1.$

Section – B **(5 marks each)**

Q2 Using Newton-Raphson iterative formula establishes the iterative formula $x_{n+1} = \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]$ to calculate the root of N where N is a real number, hence find the cube-root of 12 to 4 decimal places.

Q3 Determine the largest eigen value and the corresponding eigen vector of the matrices using Rayleigh power method: $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$

Q4 Given the values

x:	5	7	11	13	17
f(x):	150	392	1452	2366	5202

Evaluate f (x), using **(a)** Lagrange's interpolation formula, and **(b)** Newton's divided difference formula.

Q5 Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using **(i)** Trapezoidal rule, **(ii)** Weddle's rule.

Q6 Apply Taylor series method to obtain approximate value of y at x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$, Compare the numerical solution obtained with the exact solution.

Section – C **(10 marks each)**

Q7 **(i)** A real-root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval (0,1). Perform four iterations of the Regula-Falsi method to obtain this root.

(ii) Solve the following equations by Gauss Elimination Method.2x+ 3y – z =5,3x + 2y +z =10 , x- 5y + 3z =0

Q8 **(i)** Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data: f (−0.75) = −0.07181250, f (-0.5) = -0.02475, f(-0.25) = 0.33493750, f (0) = 1.10100. Hence find f (- 1/3).

(ii) Given $\frac{dy}{dx} = x^2 - y,\ y(0) = 1$ and the starting values y(0.1) = 0.90516, y(0.2) = 0.82127, y(0.3) = 0.74918, evaluate y(0.4) using Adam-Bashforth method.

Q9 **(i)** Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0)=1 at x = 0.2, 0.4.

(ii) From the table below, for what values of x, y is minimum? Also find this value of y.

x:	3	4	5	6	7	8
f(x):	0.205	0.240	0.259	0.262	0.250	0.224