Vito Volterra Meeting – 24-28 June 2024

Monday 24

9.00 Welcome!

9.30 **Figalli (cancelled)**

11.30 Coffee break

12.00 **Tione**

13.00 Lunch

14.30 **Skorobogatova**

15.30 **Olbermann**

Tuesday 25

9.00 **Mora**

11.00 Coffee break

11.30 **Slepcev**

Wednesday 26

10.30 Coffee break

11.00 **Slepcev**

13.00 Lunch

14.30 **Santambrogio**

16.30 **Solombrino**

Thursday 27

9.30 **Zeppieri**

10.30 Coffee break

11.00 **Mora**

13.00 Lunch

Friday 28

9.30 **Iacobelli**

10.30 Coffee break

11.00 **Santambrogio**

Book of Abstracts

Alessio Figalli

Title: Rigidity and stability in variational problems

Abstract: In the realms of analysis and geometry, geometric and functional inequalities are of paramount significance, influencing a variety of problems. Traditionally, the focus has been on determining precise constants and identifying minimizers. More recently, there has been a growing interest in investigating the stability of these inequalities. The central question we aim to explore is:
"If a function nearly achieves equality in a known functional inequality, can we demonstrate, in a quantitative way, its proximity to a minimizer?"
In this mini-course I will overview this beautiful topic and discuss some recent results.

Maria Giovanna Mora

Title: Shape optimization problems for nonlocal anisotropic energies

Abstract:
Nonlocal shape optimization problems involving interaction energies with competing repulsive and attractive terms are of interest in a variety of applications and have been extensively studied in the last decades in the mathematical community. In this short course I will focus on nonlocal energies defined on sets with prescribed mass, where the repulsive interaction is of Riesz type and the attractive interaction is a power law. I will first review some results in the isotropic case, that is, when both the repulsive and the attraction kernels are radially symmetric. In this setting existence of minimizers typically fails for small mass, whereas the unique minimizers are balls for large mass. I will then discuss some recent advancements for a class of anisotropic kernels and show how the occurrence of a critical mass for existence is related to the shape of minimizers when considering the energy on the larger class of measures with prescribed mass. If time permits, I will discuss some open questions and future perspectives.

Filippo Santambrogio

Title: OT methods for parabolic diffusion equations: the JKO scheme

Abstract: This mini-course is concerned with those PDEs which have a gradient flow structure in the Wasserstein space W_2 and can thus be attacked via the so-called Jordan-Kinerlehrer-Otto scheme, a sequence of iterated minimization problems in the space of measures which provide a time-discretization of the solution.
The first lecture will explain the convergence of this scheme to a solution of the PDE and present the main techniques to obtain it, including in the case where the distance W_2 is replaced with W_p, which lets non-linear PDEs of p-Laplace type appear.
The second lecture will introduce some tools to prove iterable estimates on the solution of the JKO scheme. We will then show how one can easily recover well-known results for linear diffusion but also new estimates which would be more difficult to guess using "continuous" methods rather than this time-discrete counterpart.

Dejan Slepčev

Title: Gradient flows for variational inference and their deterministic,

interacting-particle approximations

Abstract: Over the recent years deterministic interacting-particle approximations of gradient flows in Wasserstein and other geometries have gained popularity in applications to machine learning and other areas due to their simplicity and flexibility. In these lectures we will consider various gradient flows that evolve an initial measure towards the desired target measure, which is the basic problem of variational inference. In particular we will consider the blob regularization of the Fokker-Planck equation, Stein Variational Gradient Descent, gradient flow of Maximum Mean Discrepancy, birth-death dynamics and the new Radon-Wasserstein gradient flows. In investigating the flows we will discuss which flows and which metrics are suitable in high dimensions. This is turn motivates discussing the Sliced Wasserstein metric and some related metrics.

Mikaela Iacobelli

Title: Stability and singular limits in plasma physics

Abstract: In this talk, we will present two kinetic models that are used to describe the evolution of charged particles in plasmas: the Vlasov-Poisson system and the Vlasov-Poisson system with massless electrons. These systems model respectively the evolution of electrons, and ions in a plasma. We will discuss the well-posedness of these systems, the stability of solutions, and their behavior under singular limits. Finally, we will introduce a new class of Wasserstein-type distances specifically designed to tackle stability questions for kinetic equations.

Heiner Olbermann

Title: Phase separation on varying surfaces and convergence of diffuse interface approximations

Abstract: In this talk we will consider phase separations on generalized hypersurfaces in Euclidean space. For a diffuse surface area (line tension) energy of Modica-Mortola type, we prove a compactness and lower bound estimate in the sharp interface limit. We use the concept of generalized BV functions over currents as introduced by Anzellotti et al. [Annali di Matematica Pura ed Applicata, 170, 1996] to give a suitable formulation in the limit and achieve the necessary compactness property. We also consider an application to phase separated biomembranes where a Willmore energy for the membranes is combined with a generalized line tension energy.
For a diffuse description of such energies we give a lower bound estimate in the sharp interface limit.
Joint work with Matthias Röger, TU Dortmund.

Anna Skorobogatova

Title: Structure of flat singularities for mod(p) area-minimizing surfaces

Abstract: One possible framework in which to study the Plateau problem is by using currents with mod(p) coefficients, for a fixed integer p. This setting allows for minimizing surfaces to exhibit codimension 1 singularities like triple junctions (p=3), and the known regularity theory for general stable minimal surfaces is so far consistent with that for mod(p) minimizers, unlike for area-minimizing integral currents, which exhibit better

regularity properties. For mod(p) minimizing hypersurfaces, a reasonably complete characterization of the structure of the interior singular set has recently been established.
In this talk, I will discuss joint work in progress with Camillo De Lellis and Paul Minter towards establishing a structural result on the interior singular set when the surface has higher codimension, which is an analogue of that for hypersurfaces. I will emphasize the difficulties that arise here in contrast to the codimension 1 setting.

Francesco Solombrino

Title: Non-local approaches to effective energies for materials' singularities

Abstract: We discuss in two relevant case-studies the rigorous derivation via Gamma-convergence of asymptotic energies accounting for singularities in elastic materials from non-local models (convolution-type integral functionals).In the first part, we are concerned with free-discontinuity functionals à la Griffith, coupling a linearly elastic behavior in the uncracked part of a reference configuration with an energy concentrated on lower-dimensional submanifolds accounting for crack formation. The models we propose feature a nonlocal linearly elastic energy coupled with a truncating potential accounting for the breaking of elastic bonds. Their asymptotic behavior in suitable spaces of weakly differentiable functions with surface discontinuities is analyzed and further research directions are proposed.
In the second part, a strain-gradient theory for plasticity is derived as a limit of discrete dislocation fractional energies, avoiding the excision of a so-called core-radius from the reference configuration. Away from dislocations, the stored elastic energy is given in terms of a fractional gradient of order 1-alpha of the strain, extended to the case of incompatible strain fields via the usage of Riesz-type singular convolution operators. As alpha goes to 0, we show that a suitable rescaling of the energies converge to a macroscopic strain-gradient model featuring a positively 1-homogeneous plastic energy on the dislocation density, which coincides with the one recovered by Garroni, Leoni, and Ponsiglione (2010) through a core-radius approach.
From joint works with Roberta Marziani (L'Aquila), Stefano Almi (Napoli), Maicol Caponi (Napoli) and Manuel Friedrich (Erlangen).

Riccardo Tione

Title: Unique continuation for differential inclusions

Abstract: The Sobolev regularity of solutions to the Monge-Ampère equation in the plane can be rephrased in terms of a unique continuation property of differential inclusions. After an overview of the known results concerning the Monge-Ampère equation, I will explain this connection to differential inclusions and sketch a proof of the unique continuation property. Based on joint work with G. De Philippis and A. Guerra.

Caterina Ida Zeppieri

Title: Homogenisation of nonlinear Dirichlet problems in randomly perforated domains

Abstract: In this talk we present some recent results on the convergence of nonlinear Dirichlet problems for systems of variational elliptic PDEs defined on randomly perforated domains of R^n.
Under the assumption that the perforations are small balls whose centres and radii are

generated by a stationary short-range marked point process, we obtain in the critical-scaling limit an averaged nonlinear analogue of the extra term obtained in the classical work of Cioranescu and Murat. In analogy to the random setting recently introduced by Giunti, Hoefer, and Velazquez to study the Poisson equation, we only require that the random radii have finite $(n-p)$-moment, where $1<p<n$ is the growth-exponent of the associated energy functionals. This assumption on the one hand ensures that the expectation of the nonlinear p-capacity of the spherical holes is finite, and hence that the limit problem is well defined. On the other hand, it does not exclude the presence of balls with large radii that can cluster up. We show however that the critical rescaling of the perforations is sufficient to ensure that no percolating-like structures appear in the limit.