

Lesson Plan: Graph Coloring and Applications

By Ayah Almousa and Avery St. Dizier

Abstract:

In this course, we will introduce students to graph theory and its applications. Our objective is to use guided problem solving techniques to aid students in exploring and discovering various properties of graphs. We will begin by introducing the concept of a graph, allowing students to explore properties of graphs through 3D models and guided worksheets. Along the way students will learn how to identify isomorphic graphs and planar graphs, and will eventually learn how to use graph theory to solve map coloring and other real-world problems.

This course does not require any background material, and it is appropriate for middle and high school students.

Many of the activities in this lesson plan were inspired by similar outreach activities by PhD student Melinda Lanius at University of Illinois at Urbana-Champaign. We credit her for allowing us to use some of her worksheets.

Lesson 1

Learning Objectives:

- To understand the concept of a graph
- To differentiate between the concept of a graph and a drawing of a graph
- To recognize planar graphs and produce examples of planar graphs

Plan:

- What is a graph? (10 minutes)
 - Examples:
 - Six degrees of separation (Social networks)
 - Subways
 - Airline routes
 - Phylogenetic trees / family trees
 - Circuit diagrams
 - Introduce the concept (5 minutes)
 - Discuss applications (5 minutes)
- Graph Isomorphisms: there is more than one way to draw the same graph (20 minutes)
 - Introduce the concept (5 minutes)
 - Activities (10 minutes)
 - Present 3D models of graphs made with wire and styrofoam: can they match them to graphs on their papers?
 - Identifying isomorphic graphs on their worksheets
 - Class Discussion (5 minutes)
- Planar Graphs (15 minutes)
 - Introduce the concept (5 minutes)
 - Activities (10 minutes)
 - Give students the chance to “untangle” the graphs on their papers
 - Perhaps have 3D examples with wire or string (still unsure)
 - Show examples of non-planar graphs

Lesson 2

Learning Objectives:

- To illustrate graph coloring
- To demonstrate the relationship between a map and a planar graph
- To apply the the four-color theorem to map coloring

Plan:

- Review worksheet: 5 minutes
- Map coloring (20 minutes)
 - Intro (5 minutes)
 - Activities (10 minutes)
 - Map of South America: Can you use only 3 colors?
 - When can you use only 3 colors?
 - Other maps: Do you ever need more than 4 colors?
 - Group discussion (5 minutes)
- Maps are planar graphs (20 minutes)
 - Intro (5 minutes)
 - Activities (10 minutes)
 - Examples of maps as planar graphs (including South America)
 - When is a graph 3-colorable?
 - Graph coloring, even for non-planar graphs
 - Discussion (5 minutes)

Lesson 3

Learning Objectives:

- To apply graph coloring to real-world problems
- To understand the 4-color theorem

Plan:

- Review worksheet (5 minutes)
- 4 color theorem (10 minutes)
 - Catch-up from last time
 - History
- Scheduling as a coloring problem
 - Explain (5 minutes)
 - Try it out (see worksheet) - best if students work in **pairs**
- Wrap up: Life as a mathematician (15 minutes)
 - about us, math research, careers, and answering questions

Bonus for students who finish scheduling problem quickly: Sudoku as a coloring problem (see worksheet)