Online Appendix to: *The increase in the elasticity of substitution: A repeated cross-country investigation.*

Section A.

A.1 Derivation of Equation (6)

Equation (6) can be rephrased as:

$$\sigma = \frac{f(k) - kf'(k)}{-\frac{kf(k)f''(k)}{f'(k)}}$$

by multiplying both the numerator and the denumerator of the second term of the previous equation by $kf'(k)/f^2(k)$, we obtain:

$$\sigma = \frac{\frac{kf'(k)}{f(k)} - \frac{k^2(f'(k))^2}{f^2(k)}}{-\frac{k^2 f''(k)f(k)}{f^2(k)}} .$$

Considering the definition $\theta = kf'(k)/f(k)$, the numerator of the right handside of the previous equation collapses to:

$$\theta(1 - \theta)$$

Regarding the denumerator, by adding and subtracting the terms $kf'(k)/f(k)$ and $-k^2(f'(k))^2/f^2(k)$, we obtain:

$$\theta - \theta^2 - \frac{k[f'(k)f(k) + kf''(k)f(k) - k(f'(k))^2]}{f^2(k)}$$

where the third term is just $k\theta'$. Finally, we obtain:

$$\sigma = \frac{\theta(1-\theta)}{\theta(1-\theta) - k\theta'}$$

which is Equation (6).

Table A1: Rolling window estimates for p and related standard errors

		SW	US	NL	AU	CA	FIN	JAP	FR	IT
1950-1999	estimate	0.302	0.141	0.356	0.314	0.368	0.116	0.035	0.163	0.100
	s.e.	0.041	0.020	0.054	0.038	0.035	0.021	0.006	0.031	0.014
1951-2000	estimate	0.330	0.135	0.389	0.327	0.385	0.131	0.038	0.174	0.112
	s.e.	0.040	0.022	0.057	0.038	0.036	0.023	0.007	0.032	0.016
1952-2001	estimate	0.362	0.122	0.422	0.336	0.400	0.146	0.040	0.186	0.124
	s.e.	0.037	0.024	0.060	0.038	0.037	0.025	0.007	0.033	0.017
1953-2002	estimate	0.381	0.108	0.454	0.347	0.415	0.159	0.044	0.190	0.135
	s.e.	0.036	0.025	0.062	0.038	0.038	0.026	0.007	0.035	0.018
1954-2003	estimate	0.398	0.098	0.489	0.357	0.431	0.173	0.049	0.197	0.146
	s.e.	0.036	0.025	0.064	0.037	0.039	0.028	0.008	0.035	0.019
1955-2004	estimate	0.419	0.093	0.531	0.363	0.449	0.188	0.055	0.202	0.158
	s.e.	0.035	0.024	0.066	0.037	0.041	0.029	0.008	0.036	0.020
1956-2005	estimate	0.434	0.102	0.578	0.371	0.468	0.201	0.061	0.207	0.170
	s.e.	0.035	0.023	0.067	0.036	0.042	0.030	0.009	0.036	0.020
1957-2006	estimate	0.449	0.102	0.623	0.376	0.482	0.213	0.067	0.207	0.180
	s.e.	0.034	0.023	0.066	0.036	0.043	0.030	0.010	0.035	0.020
1958-2007	estimate	0.456	0.102	0.653	0.379	0.492	0.229	0.073	0.211	0.190
	s.e.	0.033	0.024	0.063	0.035	0.043	0.031	0.010	0.036	0.021
1959-2008	estimate	0.459	0.104	0.679	0.391	0.503	0.242	0.078	0.216	0.199
	s.e.	0.033	0.023	0.061	0.035	0.043	0.031	0.010	0.035	0.020
1960-2009	estimate	0.460	0.116	0.679	0.398	0.495	0.242	0.083	0.216	0.203
	s.e.	0.033	0.023	0.059	0.035	0.044	0.031	0.011	0.035	0.020
1961-2010	estimate	0.460	0.124	0.666	0.406	0.493	0.245	0.088	0.220	0.209
	s.e.	0.032	0.024	0.057	0.035	0.044	0.031	0.011	0.035	0.020
1962-2011	estimate	0.456	0.131	0.653	0.414	0.493	0.247	0.093	0.220	0.214
	s.e.	0.033	0.024	0.055	0.035	0.044	0.032	0.011	0.035	0.020
1963-2012	estimate	0.439	0.138	0.634	0.419	0.483	0.243	0.098	0.212	0.214
	s.e.	0.035	0.024	0.054	0.035	0.045	0.033	0.011	0.035	0.020
1964-2013	estimate	0.415	0.147	0.603	0.422	0.471	0.241	0.105	0.202	0.216
	s.e.	0.037	0.024	0.053	0.034	0.045	0.033	0.012	0.035	0.020
1965-2014	estimate	0.396	0.156	0.583	0.409	0.458	0.241	0.112	0.191	0.217
	s.e.	0.039	0.024	0.053	0.034	0.045	0.033	0.012	0.035	0.020
1966-2015	estimate	0.378	0.166	0.555	0.388	0.430	0.243	0.120	0.185	0.217
	s.e.	0.040	0.023	0.051	0.035	0.046	0.034	0.012	0.035	0.021
1967-2016	estimate	0.354	0.175	0.540	0.380	0.405	0.245	0.127	0.179	0.216
	s.e.	0.040	0.023	0.051	0.035	0.047	0.035	0.013	0.034	0.021
1968-2017	estimate	0.333	0.178	0.526	0.375	0.385	0.253	0.136	0.173	0.215
	s.e.	0.040	0.023	0.050	0.035	0.047	0.036	0.013	0.034	0.021

Table A2: Estimates of σ – No labour-augmenting technical change.

Country	1950-2017	1950-1979	1980-2017
Australia	1.279 (0.332)	0.510 (0.162)	1.339 (0.147)
Canada	1.004 (0.115)	0.827 (0.090)	0.961 (0.036)
Finland	1.770 (0.612)	0.321 (0.365)	1.858 (0.200)
France	1.626 (0.456)	0.482 (0.330)	1.559 (0.171)
Italy	1.284 (0.134)	0.788 (0.071)	1.225 (0.058)
Japan	1.303 (0.095)	0.656 (0.023)	1.321 (0.061)
Netherlands	1.760 (1.295)	0.400 (0.293)	1.751 (0.207)
Sweden	1.284 (0.296)	0.386 (0.126)	1.392 (0.158)
USA	2.568 (1.255)	0.332 (0.079)	3.119 (0.765)

Notes: The table reports the estimates of the elasticity of substitution for a production function without labour-augmenting technical change implemented with a Non-Linear Least Squares procedure. The period under examination is indicated in the column head. Bootstrapped standard errors are in parentheses.

Section B. Elasticity of substitution as a function of capital intensity

In this section, since we showed the elasticity of substitution is not constant over time within each country, we try to understand if we can consider it a function of the capital intensity k, by following the standard functional form $\sigma = \sigma(k)$. We then consider two different cases, the more general one, in which $\sigma(k) = \alpha k^{\beta}$, and then the simpler linear case, in which $\sigma(k) = \alpha + \beta k$ is assumed. As we specified in the previous parts of this work, it is impossible to obtain an econometric estimate for the parameters α and β in both the assumed functional forms for the elasticity of substitution (non-linear and linear), so we need a different approach to understand if the two assumptions can be considered realistic. The methodology we use consists of finding values for α and β (in both the cases mentioned above) that minimise several distance measures between the time series of capital intensity and the capital share. Then, we use these optimal values for the two parameters to construct a simulated path for the elasticity of substitution, depending on the functional form assumed on k. The following steps summarize our procedure:

1. Choosing a functional form for the elasticity of substitution as a function of k (non-linear and linear[1] functional forms will be considered), i.e., $\sigma = f(\alpha, \beta, k)$.

2. Solving the non-linear functional form in Equation (11) and obtaining θ as a function of k, α and β, i.e., $\theta = \phi(k, \alpha, \beta)$

3. Finding the solution to the following expression:

$$(\alpha^{*}, \beta^{*}) = arg \min_{\alpha,\beta} \quad d(\theta; \phi(k, \alpha, \beta))$$

where $d(\cdot)$ is a generic time series distance function, θ is the observed series for the share of capital and k is the observed series for the capital intensity.

[1] In what follows, also the special case in which (k)=1+βk will be treated. Even if it is less generic (it does not allow a CES production function framework), this case is worth to study since it is the only one that can lead to the integration of a well-defined aggregate production function of the VES form.

4. Finally, choosing particular values of α and β depending on α^* and β^* obtained in the previous step, and computing $\sigma = f(\alpha, \beta, k)$.

We want to clarify that the described procedure is not a proper econometric procedure to estimate the parameters of interest, given the complicated non-linear forms we are involved in, but it is used to have a hint about the actual values that the parameters can take. We will use different functions for $d(\cdot)$ in the minimization problem: the Euclidean Distance ($d = \sqrt{\sum_{t=1}^{T} (\theta_t - \phi_t)^2}$), the Complexity-Invariant Distance, suggested by Batista et al. (2011), the Dynamic Time Warping Distance, developed by Kate (2016), and, finally, the Infinite Norm Distance ($d = max|\theta_t - \phi_t|$). Tables A2, A3 and A4 in the Online Appendix provide results on the parameter estimates.

By choosing $\sigma(k) = \alpha k^{\beta}$ and relying upon the derived functional form of the capital share w.r.t α, β and k, the minimisation of the distance will be of the following form:

$$min_{\alpha,\beta,c}\ d\left(\theta; \frac{1}{1+ck^{-1}e^{-(k^{-\beta}/\alpha\beta)}}\right) \tag{19}$$

where $d(\cdot)$ takes the aforementioned forms. With $\sigma(k) = \alpha + \beta k$, we get instead the following metric:

$$\min_{\alpha,\beta,c}\ d\left(\theta; \frac{1}{1+ck^{-1}\left[k(\alpha+\beta k)^{-1}\right]^{1/\alpha}}\right) \tag{20}$$

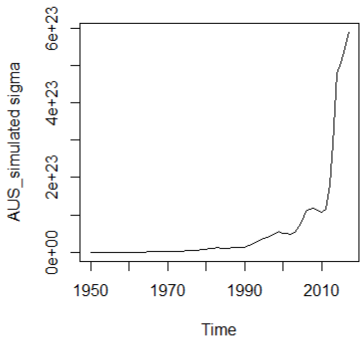
where $d(\cdot)$ takes the aforementioned forms.

Finally, if we consider that the function for the elasticity is $\sigma(k) = 1 + \beta k$, we have:

$$\min_{\beta,c}\ d\left(\theta; \frac{1}{1+\beta e^{c}k-e^{c}}\right) \tag{21}$$

The results obtained with this methodology, shown in Figures 3, 4 and 5, suggest that if the elasticity of substitution is considered a function of the capital intensity k (all the functional forms are presented), the corresponding values for the elasticity turn out to be exceedingly not realistic. Even if it seems that an increasing dynamic behaviour governs the elasticity of substitution in all cases, this strict functional relation between σ and k is economically meaningless. For example, an elasticity of substitution equal to 8×10^{14} would mean that a 1% change in the wage/rental price ratio w/r is associated with an 8×10^{14}% change in the capital/worker ratio K/L, which is unrealistic. It implies that the elasticity of substitution can be considered neither constant nor a function of the capital per worker. Thus, the implications of our findings would not support the plausibility of the class of VES production functions as a description of the production process.

Figure B1: Simulated values for the dynamic path of the elasticity of substitution (non-linear functional relation).



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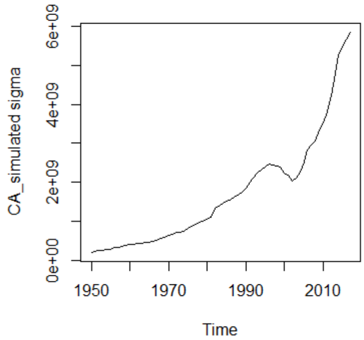

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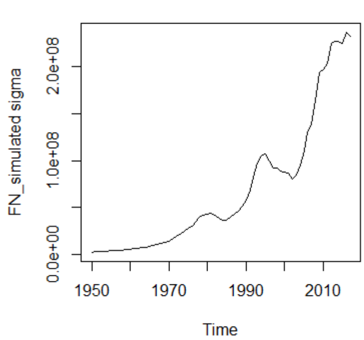
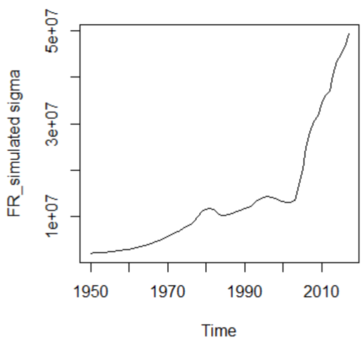


c) Finland



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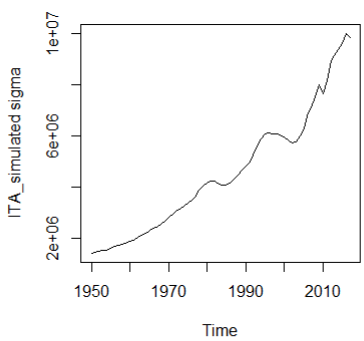

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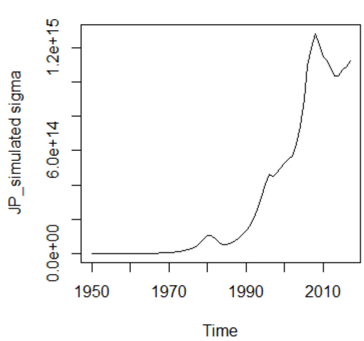

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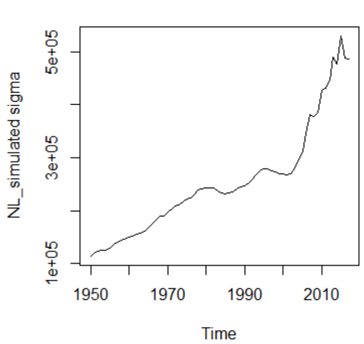
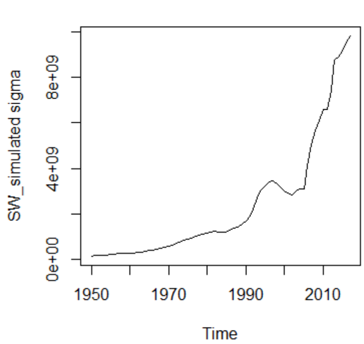
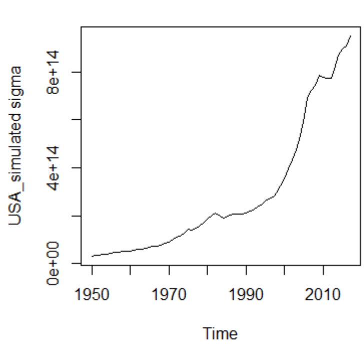


g) Netherlands

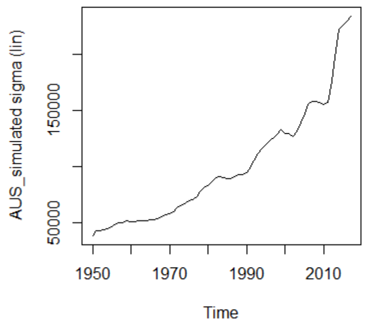


h) Sweden

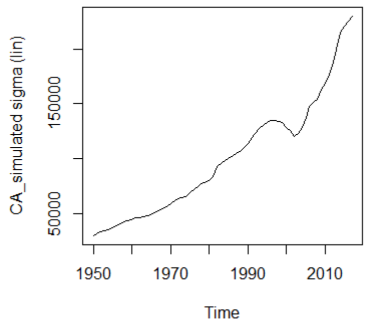


i) US

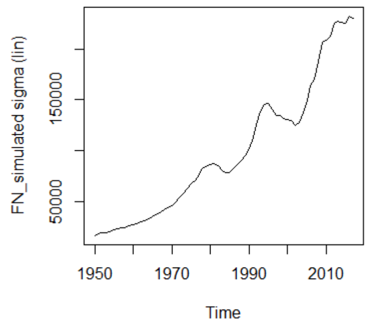
Figure B2: Simulated values for the dynamic path of the elasticity of substitution (linear functional relation).



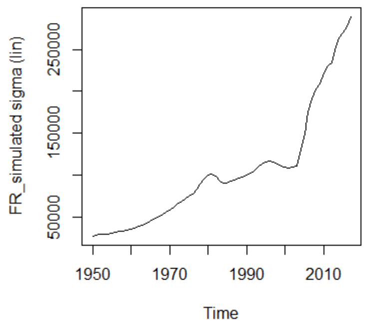
a) Australia



b) Canada



c) Finland



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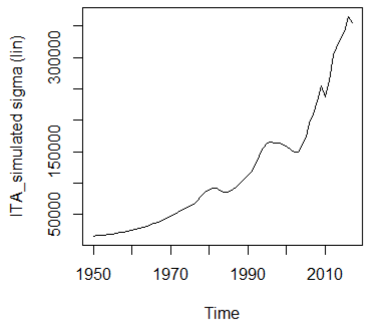

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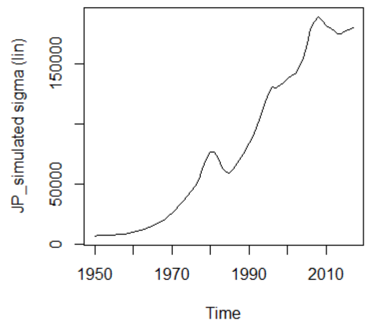

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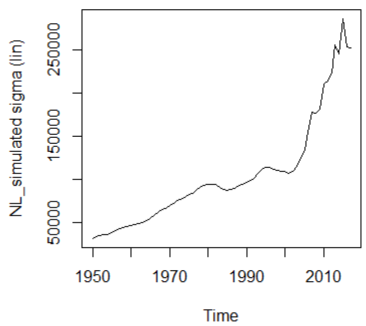

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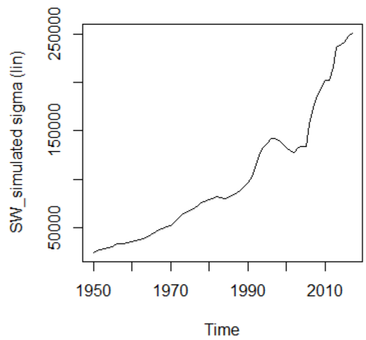

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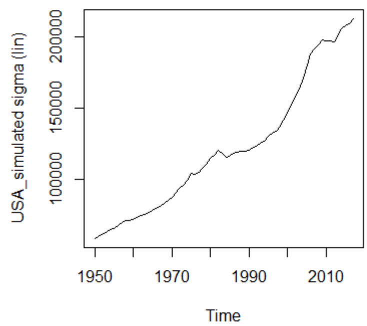
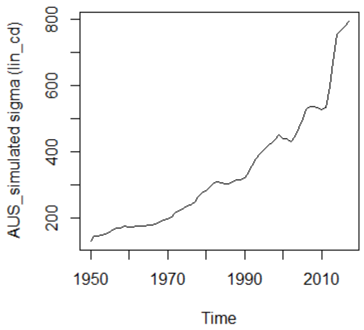


i) US

Figure B3: Simulated values for the dynamic path of the elasticity of substitution (linear functional relation, VES case).



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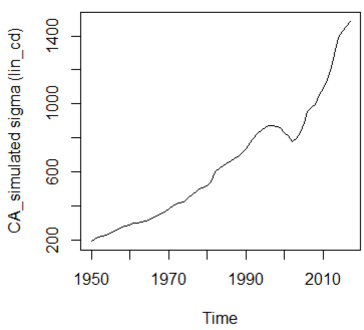
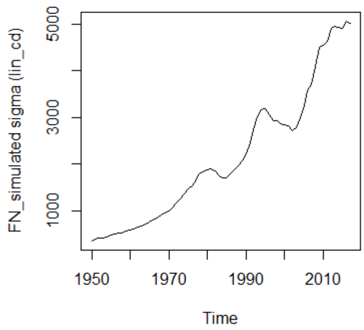
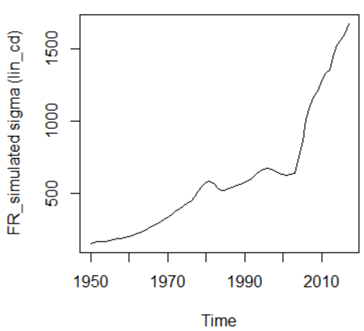


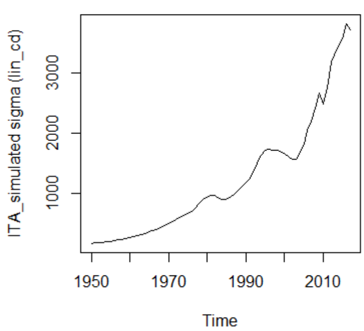
b) Canada



c) Finland



d) France



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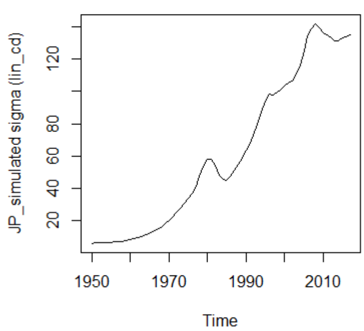

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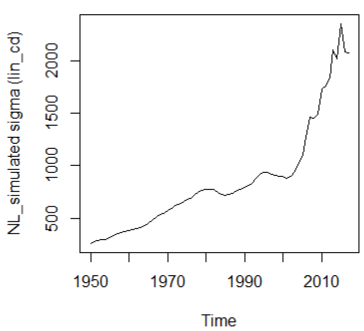

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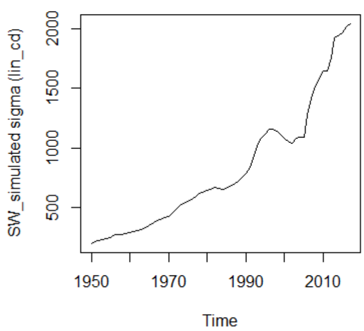

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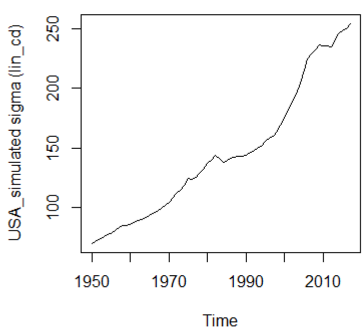


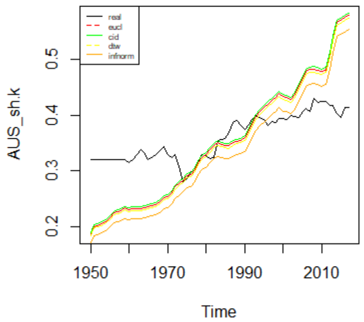
i) US

Table B1: Nonlinear functional relationship between elasticity of substitution and capital intensity.

Country	Distance	c	β	α
Australia	CID	315,496.700	9.669	5.578
	DTW	327,286.200	0.519	2.684
	EUC	320,829.900	0.500	106,612.400
	INF	356,830.700	0.524	2.435
Canada	CID	434,519.200	14.778	92.023
	DTW	405,360.400	0.507	2.562
	EUC	497,533.300	0.002	0.682
	INF	487,973.600	0.143	0.696
Finland	CID	NA	NA	NA
	DTW	128,481.200	0.001	2.994
	EUC	NA	NA	NA
	INF	310,143.300	0.089	0.449
France	CID	34,398.430	2.205	2.078
	DTW	356,164.000	0.500	114,236.500
	EUC	362,104.200	0.003	0.649
	INF	407,362.000	0.317	0.542
Italy	CID	NA	NA	NA
	DTW	59,133.270	6,845.809	2.502
	EUC	222,999.700	0.003	0.624
	INF	219,652.900	0.375	0.606
Japan	CID	4,293.958	0.845	2.783
	DTW	49,601.360	0.569	3.215
	EUC	461,197.600	0.009	0.468
	INF	NA	NA	NA
Netherlands	CID	374,747.000	159.304	4,337.066
	DTW	74,519.680	0.500	− 24,504.060
	EUC	369,681.300	0.003	0.699
	INF	NA	NA	NA
Sweden	CID	NA	NA	NA
	DTW	274,040.700	0.500	33,098.350
	EUC	249,021.700	0.003	0.661

	INF	221,901.200	0.419	1.853
United States	CID	349,999.800	6.093	5.024
	DTW	357,002.000	0.449	2.296
	EUC	358,388.800	0.003	0.704
	INF	342,664.600	0.508	2.506

Figure B4: Nonlinear functional relationship between elasticity of substitution and capital intensity.



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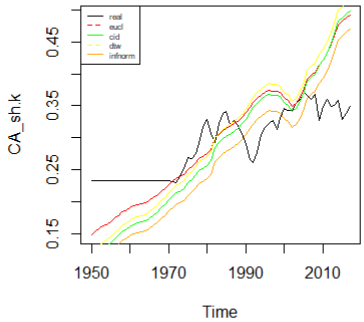

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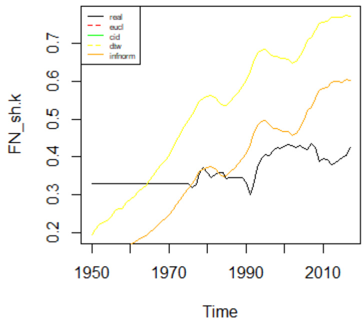

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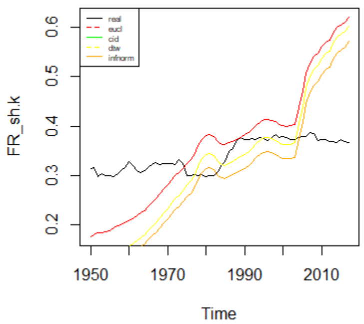

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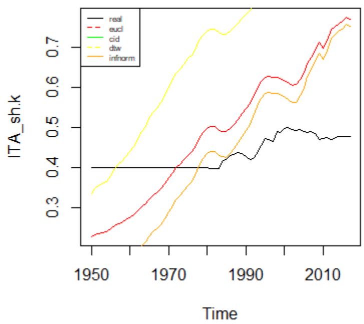
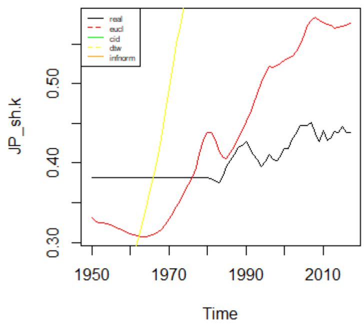
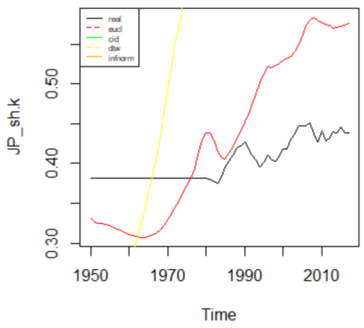


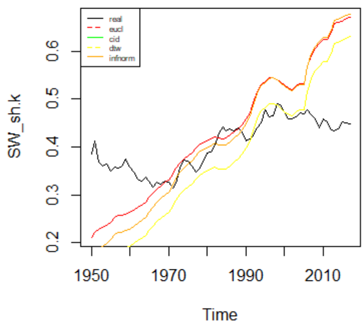
e) Italy



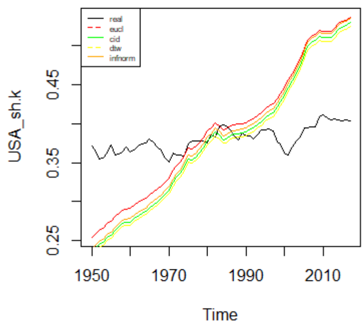
f) Japan



g) Netherlands



h) Sweden



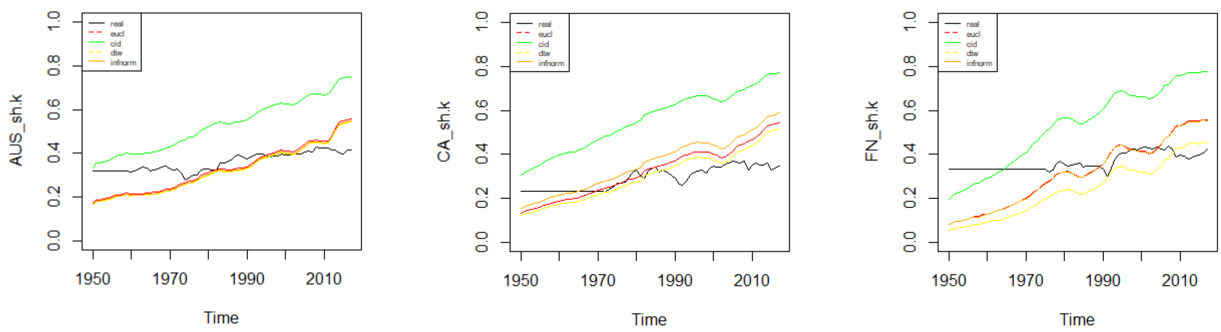
i) US

Table B2: Nonlinear functional relationship between elasticity of substitution and capital intensity.

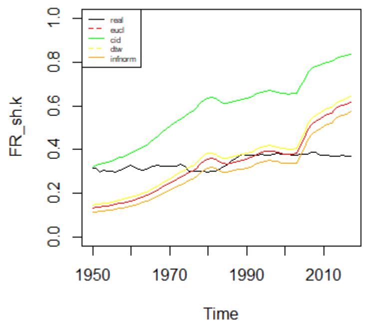
Country	Distance	*c*	β	α
Australia	CID	500.059	0.100	0.565
	DTW	500.012	0.100	0.515
	EUC	500.006	0.100	0.518
	INF	500.012	0.100	0.516
Canada	CID	500.078	0.100	0.574
	DTW	500.008	0.100	0.512
	EUC	500.009	0.100	0.517
	INF	499.996	0.100	0.527
Finland	CID	500.071	0.100	0.574
	DTW	499.999	0.100	0.497
	EUC	500.009	0.100	0.518
	INF	500.008	0.101	0.517
France	CID	500.072	0.100	0.584
	DTW	500.003	0.100	0.526
	EUC	500.018	0.100	0.521
	INF	500.007	0.100	0.512
Italy	CID	500.066	0.100	0.566
	DTW	500.019	0.100	0.505

	EUC	500.012	0.101	0.521
	INF	500.005	0.101	0.521
Japan	CID	500.028	0.099	0.540
	DTW	500.015	0.100	0.545
	EUC	500.021	0.101	0.543
	INF	500.001	0.101	0.526
Netherlands	CID	500.055	0.100	0.597
	DTW	499.994	0.100	0.524
	EUC	500.018	0.100	0.521
	INF	NA	NA	NA
Sweden	CID	500.032	0.101	0.587
	DTW	500.020	0.101	0.522
	EUC	500.011	0.101	0.522
	INF	500.010	0.101	0.518
United States	CID	500.030	0.100	0.593
	DTW	500.014	0.100	0.525
	EUC	500.011	0.100	0.522
	INF	500.005	0.100	0.521

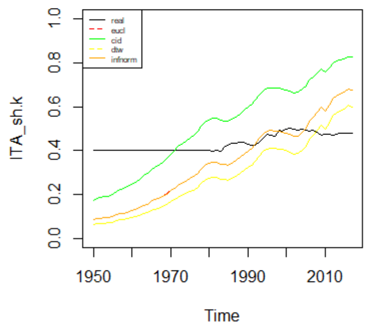
Figure B5: Linear functional relationship between elasticity of substitution and capital intensity.



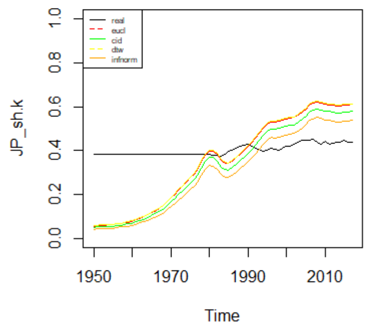
a) Australia　b) Canada　c) Finland



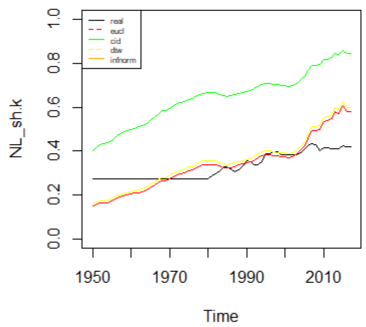
d) France



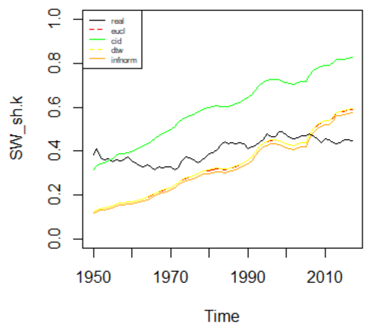
e) Italy



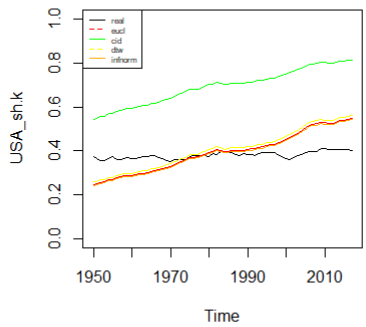
f) Japan



g) Netherlands



h) Sweden



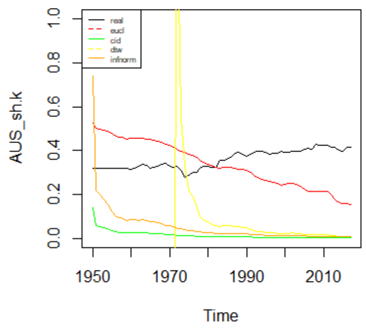
i) US

Table B3: Nonlinear functional relationship between elasticity of substitution and capital intensity.

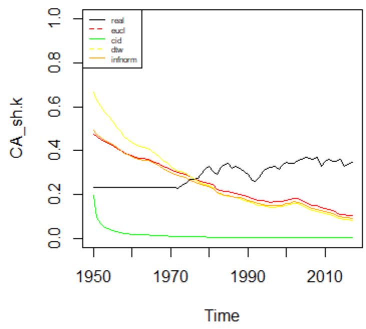
Country	Distance	c	β
Australia	CID	4.508	0.00001
	DTW	3.677	0.00001
	EUC	− 6.359	0.007
	INF	3.453	0.00001
Canada	CID	4.532	0.00002
	DTW	0.046	0.00003
	EUC	− 6.533	0.014

	INF	-1.311	0.0001
Finland	CID	4.517	0.00003
	DTW	3.405	0.00003
	EUC	-7.927	0.046
	INF	-2.681	0.0003
France	CID	4.540	0.00002
	DTW	-4.437	0.001
	EUC	-6.584	0.011
	INF	-1.597	0.0001
Italy	CID	4.526	0.00004
	DTW	3.677	0.00001
	EUC	-7.667	0.022
	INF	-3.582	0.0004
Japan	CID	4.518	0.0001
	DTW	-4.319	0.001
	EUC	3.501	0.0001
	INF	-2.912	0.001
Netherlands	CID	4.508	0.00002
	DTW	3.852	0.00000
	EUC	-6.756	0.013
	INF	NA	NA
Sweden	CID	4.524	0.00002
	DTW	3.574	0.00002
	EUC	-7.315	0.017
	INF	-2.746	0.0002
United States	CID	4.509	0.00001
	DTW	-3.389	0.0002
	EUC	-5.641	0.002
	INF	3.458	0.00001

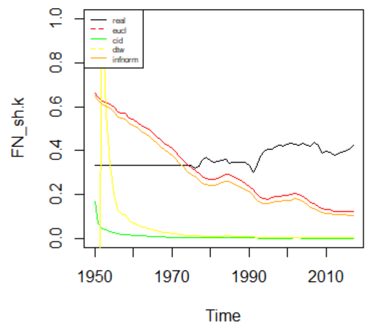
Figure B6: Linear functional relationship between elasticity of substitution and capital intensity. (VES case)



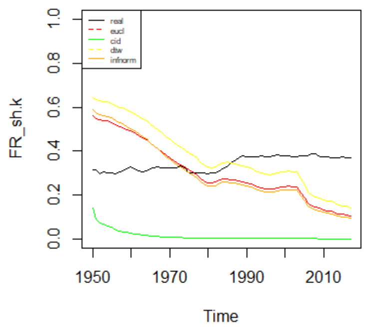
a) Australia



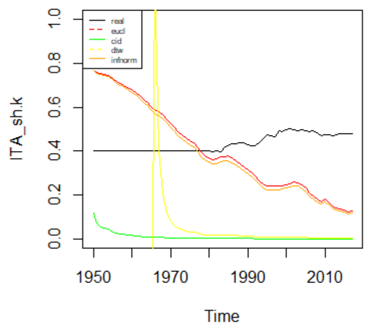
b) Canada



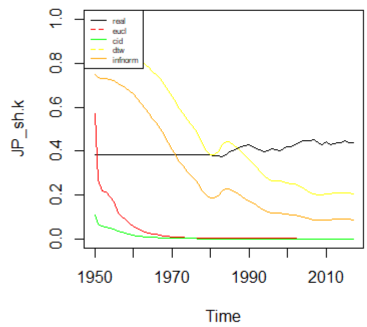
c) Finland



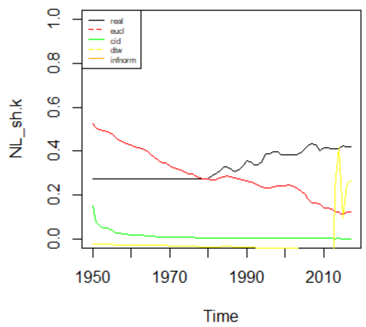
d) France



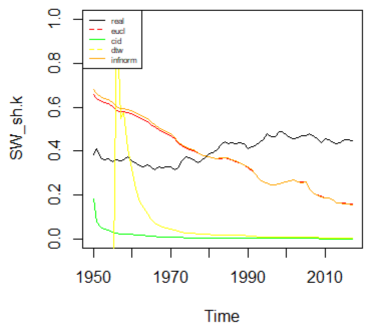
e) Italy



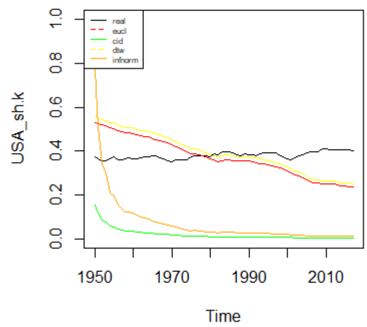
f) Japan



g) Netherlands



h) Sweden



i) US

Section C

Table C1: Estimates of σ (Kmenta approximation).

Country	1950-2017 (σ)	1950-1979 (σ_1)	1980-2017 (σ_2)
Australia	0.360	0.120	− 0.252
	[0.314: 0.414]	[0.076: 0.167]	[− 0.719: − 0.070]
Canada	0.398	0.158	− 0.478
	[0.366: 0.433]	[0.139: 0.179]	[− 45587.89: 22542.73]
Finland	0.755	0.545	− 1.893
	[0.633: 0.912]	[0.453: 0.659]	[− 492174.8: 115251.6]
France	0.248	0.201	0.286
	[0.232: 0.266]	[0.193: 0.210]	[0.100: 0.518]
Italy	0.364	0.433	0.183
	[0.353: 0.375]	[0.386: 0.484]	[0.037: 0.309]
Japan	0.585	0.372	0.241
	[0.550: 0.623]	[0.348: 0.397]	[0.031: 0.436]
Netherlands	0.224	0.921	− 0.327
	[0.199: 0.250]	[− 2898.665: 3009.09]	[− 1.615: − 0.055]
Sweden	0.747	0.311	− 0.919
	[0.633: 0.896]	[0.253: 0.380]	[− 210017: 50951.67]
USA	0.277	0.110	− 0.130
	[0.215: 0.363]	[0.087: 0.134]	[− 0.429: − 0.016]

Notes: For the linear Kmenta approximation, we present the range in which the estimated elasticity of substitution may vary according to the standard errors of the estimated coefficients of the linear regression model representing the linear approximation. See Tables C2-C10.

It is straightforward to notice that the estimates obtained with the Kmenta linear approximation have to be refused. First of all, comparing the estimates obtained for the elasticity of substitution for the overall period (σ) with the ones obtained with the non-linear least square methodology, we can observe that the first are always lower than the unity, while the last are always higher than one. However, the range in which the Kmenta estimates for σ are allowed to vary according to the standard errors of the regression coefficients is sometimes large, compared to the standard errors of the non-linear estimates for the same parameter. Second, the most important thing to be noticed is the fact that the Kmenta estimates for the elasticity of substitution in the two different periods (σ_1 and σ_2) are not precise and often not comparable with theoretical assumptions (especially regarding the Kmenta estimates for σ_2, very often lower than zero, which is meaningless form a theoretical perspective). There can be several reasons for this particular behaviour. One can be the simple fact that there are no sufficient observations to perform a well-behaved regression (while the non-linear estimation procedure seems to be well behaved, and it is confirmed by the bootstrap analysis on the standard errors) so that estimates tend to be inconsistent.

Another possible cause of these results can be that, running a simple linear regression, we are not able to pose implicit constraints to the coefficients we want to estimate (the constraints that have to be assumed are non-trivial, given that we have an assumption about the behaviour of the parameters $\tilde{\pi}$ and p, but we estimate the vector of coefficients β; box-constrained least squares methods are not so developed in literature and present lots of problematic issues). The last explanation for this result is that the linear approximation around the unitary elasticity of substitution is not representative of the real data: the Kmenta approximation is a good linear representation of the CES production function if we assume that the elasticity of substitution is very close to the unity, which implicitly implies that it works well when we assume a production function not too different from the Cobb-Douglas one. In other words, the resulting estimates for the elasticity of substitution obtained with this linear approximation have to be very close to one to be credible, and we have seen that this is not the case.

Table C2: Kmenta estimation for Australia (two subperiods)

	Dependent variable:		
	$logy$ (1)	$logy_{1950-1979}$ (2)	$logy_{1980-2017}$ (3)
$logk$	0.731*** (0.015)		
$(logk)^2$	−0.175*** (0.032)		
$logk_{1950-1979}$		0.300*** (0.112)	
$(logk_{1950-1979})^2$		−0.764*** (0.162)	
$logk_{1980-2017}$			1.197*** (0.113)
$(logk_{1980-2017})^2$			−0.586*** (0.102)
Constant	0.042*** (0.011)	0.002 (0.017)	-0.058** (0.028)
Observations	68	39	29
R^2	0.973	0.938	0.940
Adjusted R^2	0.972	0.934	0.935
Residual Std. Error	0.060 (df = 65)	0.057 (df = 36)	0.040 (df = 26)
F Statistic	1,154.939*** (df=2;65)	270.151*** (df = 2; 36)	202.562 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C3: Kmenta estimation for Canada (two subperiods)

	Dependent variable:		
	$logy$ (1)	$logy_{1950-1979}$ (2)	$logy_{1980-2017}$ (3)
$logk$	0.569^{***}		
	(0.013)		
$(logk)^2$	-0.185^{***}		
	(0.025)		
$logk_{1950-1979}$		0.271^{***}	
		(0.033)	
$(logk_{1950-1979})^2$		-0.523^{***}	
		(0.038)	
$logk_{1980-2017}$			1.364^{***}
			(0.362)
$(logk_{1980-2017})^2$			-0.766^{***}
			(0.286)
Constant	0.059^{***}	0.038^{***}	-0.171
	(0.011)	(0.007)	(0.102)
Observations	68	39	29
R^2	0.970	0.972	0.982
Adjusted R^2	0.969	0.970	0.980
Residual Std. Error	0.061 (df = 65)	0.036 (df = 36)	0.021 (df = 26)
F Statistic	1,035.167*** (df = 2; 65)	1,365.531*** (df = 2; 36)	32.604 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C4: Kmenta estimation for Finland (two subperiods)

	Dependent variable:		
	$logy$ (1)	$logy_{1950-1979}$ (2)	$logy_{1980-2017}$ (3)
$logk$	0.769***		
	(0.015)		
$(logk)^2$	−0.029		
	(0.020)		
$logk_{1950-1979}$		0.643***	
		(0.043)	
$(logk_{1950-1979})^2$		−0.096***	
		(0.034)	
$logk_{1980-2017}$			1.547***
			(0.485)
$(logk_{1980-2017})^2$			−0.647*
			(0.318)
Constant	0.018	-0.029**	-0.153
	(0.016)	(0.011)	(0.169)
Observations	68	39	29
R^2	0.979	0.988	0.703
Adjusted R^2	0.978	0.987	0.681
Residual Std. Error	0.091 (df = 65)	0.049 (df = 36)	0.107 (df = 26)
F Statistic	1,506.124*** (df = 2; 65)	1,435.276*** (df = 2; 36)	30.837 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C5: Kmenta estimation for France (two subperiods)

	Dependent variable:		
	$logy$ (1)	$logy_{1950-1979}$ (2)	$logy_{1980-2017}$ (3)
$logk$	0.778*** (0.011)		
$(logk)^2$	-0.261*** (0.015)		
$logk_{1950-1979}$		0.506*** (0.025)	
$(logk_{1950-1979})^2$		−0.495*** (0.026)	
$logk_{1980-2017}$			0.712*** (0.190)
$(logk_{1980-2017})^2$			−0.256* (0.140)
Constant	0.114*** (0.010)	0.070*** (0.005)	0.174*** (0.047)
Observations	68	39	29
R^2	0.988	0.997	0.858
Adjusted R^2	0.988	0.997	0.847
Residual Std. Error	0.060 (df = 65)	0.023 (df = 36)	0.061 (df = 26)
F Statistic	2,662.948*** (df = 2; 65)	7,007.800*** (df = 2; 36)	78.483 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C6: Kmenta estimation for Italy (two subperiods)

	Dependent variable:		
	$logy$ (1)	$logy_{1950-1979}$ (2)	$logy_{1980-2017}$ (3)
$logk$	0.706*** (0.006)		
$(logk)^2$	-0.181*** (0.007)		
$logk_{1950-1979}$		0.790*** (0.020)	
$(logk_{1950-1979})^2$		-0.108^{***} (0.014)	
$logk_{1980-2017}$			0.844*** (0.127)
$(logk_{1980-2017})^2$			-0.293^{***} (0.067)
Constant	0.148*** (0.007)	0.137*** (0.006)	0.138** (0.054)
Observations	68	39	29
R^2	0.996	0.998	0.884
Adjusted R^2	0.996	0.998	0.875

Residual Std. Error	0.042 (df = 65)	0.024 (df = 36)	0.042 (df = 26)
F Statistic	8,898.138*** (df = 2; 65)	10,877.40*** (df = 2; 36)	99.087 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C7: Kmenta estimation for Japan (two subperiods)

	Dependent variable:		
	$logy$ (1)	$logy_{1950-1979}$ (2)	$logy_{1980-2017}$ (3)
$logk$	0.690***		
	(0.010)		
$(logk)^2$	-0.076***		
	(0.010)		
$logk_{1950-1979}$		0.446***	
		(0.032)	
$(logk_{1950-1979})^2$		−0.208***	
		(0.019)	
$logk_{1980-2017}$			0.840***
			(0.141)
$(logk_{1980-2017})^2$			−0.211***
			(0.075)
Constant	0.096***	0.068***	0.119*
	(0.016)	(0.013)	(0.062)

Observations	68	39	29
R^2	0.991	0.993	0.956
Adjusted R^2	0.991	0.993	0.953
Residual Std. Error	0.078 (df = 65)	0.059 (df = 36)	0.026 (df = 26)
F Statistic	3,786.260*** (df = 2; 65)	2,555.661*** (df = 2; 36)	284.208 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C8: Kmenta estimation for Netherlands (two subperiods)

	Dependent variable:		
	$logy$	$logy_{1950-1979}$	$logy_{1980-2017}$
	(1)	(2)	(3)
$logk$	0.814***		
	(0.015)		
$(logk)^2$	-0.262***		
	(0.022)		
$logk_{1950-1979}$		0.975***	
		(0.043)	
$(logk_{1950-1979})^2$		−0.001	
		(0.049)	
$logk_{1980-2017}$			1.279***
			(0.193)
$(logk_{1980-2017})^2$			−0.724***
			(0.162)

Constant	0.079***	0.053***	0.046
	(0.011)	(0.007)	(0.040)
Observations	68	39	29
R^2	0.978	0.994	0.855
Adjusted R^2	0.978	0.993	0.844
Residual Std. Error	0.068 (df = 65)	0.028 (df = 36)	0.073 (df = 26)
F Statistic	1,467.107*** (df = 2; 65)	2,862.435*** (df = 2; 36)	76.616 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C9: Kmenta estimation for Sweden (two subperiods)

	Dependent variable:		
	$logy$	$logy_{1950-1979}$	$logy_{1980-2017}$
	(1)	(2)	(3)
$logk$	0.706***		
	(0.015)		
$(logk)^2$	-0.035		
	(0.023)		
$logk_{1950-1979}$		0.406***	
		(0.067)	
$(logk_{1950-1979})^2$		−0.266***	
		(0.063)	
$logk_{1980-2017}$			1.416***
			(0.267)

$(logk_{1980-2017})^2$			-0.614^{***}
			(0.194)
Constant	0.015	-0.053^{***}	-0.136
	(0.014)	(0.013)	(0.081)
Observations	68	39	29
R^2	0.973	0.972	0.830
Adjusted R^2	0.972	0.971	0.817
Residual Std. Error	0.080 (df = 65)	0.048 (df = 36)	0.084 (df = 26)
F Statistic	1,160.255*** (df = 2; 65)	634.897*** (df = 2; 36)	63.453 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

Table C10: Kmenta estimation for USA (two subperiods)

	Dependent variable:		
	$logy$	$logy_{1950-1979}$	$logy_{1980-2017}$
	(1)	(2)	(3)
$logk$	0.868^{***}		
	(0.015)		
$(logk)^2$	-0.150^{***}		
	(0.040)		
$logk_{1950-1979}$		0.362^{***}	
		(0.077)	
$(logk_{1950-1979})^2$		-0.927^{***}	
		(0.136)	

$logk_{1980-2017}$			1.128^{***}
			(0.104)
$(logk_{1980-2017})^2$			-0.624^{***}
			(0.149)
Constant	0.021^{***}	-0.026^{***}	0.023^{*}
	(0.008)	(0.009)	(0.014)
Observations	68	39	29
R^2	0.981	0.972	0.982
Adjusted R^2	0.981	0.970	0.980
Residual Std. Error	0.046 (df = 65)	0.036 (df = 36)	0.021 (df = 26)
F Statistic	1,715.141*** (df=2;65)	617.520*** (df = 2; 36)	704.759 *** (df = 2; 26)

Note: *p<0.1; **p<0.05; ***p<0.01

References

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Kate, R. J. (2016). Using dynamic time warping distances as features for improved time series classification. *Data Mining and Knowledge Discovery*, *30*(2), 283–312.