Anticipate

Consider how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that you would like the students to learn.

- Solve the problem yourself first. If possible, work with colleagues.
- Ask yourself the following questions:
  - What strategies have students used in the past?
  - What representations are students most likely to use?
  - What incorrect or unproductive strategies are students likely to try?
  - What things might get in the way of students being able to engage with the problem? How can you remove those barriers?
  - What questions will you ask those who struggle?

Monitor

Pay close attention to students' mathematical thinking and solution strategies as they work on the task.

- Create a list of strategies the students may produce.
- Circulate the room. Watch and listen to students as they work.
- If any students use strategies you anticipated, write their name or group number on your list.
- Ask questions that will help students make their thinking visible.
- Ask questions that will help students clarify their thinking.
- Press students to consider aspects of the task to which they need to attend.

Select

Select particular students to share their work with the rest of the class to get specific mathematics into the open for discussion. The selection of particular students and their solutions is guided by the previously anticipated strategies and your assessment of how each approach will contribute to that goal.

- Based on the previously anticipated strategies and the mathematical goal of the activity, decide which student strategies to highlight.
- Select students who will share their work with the class.

HANDOUT 2

The 5 Practices Overview

Sequence	Make purposeful choices about the order in which students' work is shared to maximize the chances of achieving the mathematical goals for the discussion.
	• Based on the mathematical goal, decide on the purpose for the sequence of work. For example: least efficient to most efficient, concrete to abstract, misconceptions to conceptions, or building representations. • Decide in which order students will present their work.
Connect	Help students draw connections between their solutions and other students' solutions as well as the key mathematical ideas in the lesson. Help students to make judgments about the consequences of different approaches for the range of problems that can be solved, one's likely accuracy and efficiency in solving them, and the kinds of mathematical patterns that can be most easily discerned. Know where you want the discussion to "land" and make choices that are likely to get you there. If necessary, you may have to demonstrate an approach that students didn't come up with themselves.
	• As students share, ask questions to elicit and clarify student thinking. • After each student shares, ask questions to connect it to previously shared work or ask a student to summarize what another student said in their own words. • Ask students to compare and contrast strategies or representations during the discussion. • If students did not come up with an approach that you need them to see in order for the discussion to "land," demonstrate this approach and connect it to the work that students did.

Return to Table of Contents

HANDOUT 3 Connecting to the 5 Practices

	Notes	Connections to Planning	Connections to the Lesson Experience
Anticipate	Planning, thinking about accommodations that would help provide access to barriers Planning/misconceptions Planning Gathering back ground knowledge Don't assume students have background knowledge or know/understand the computation involved in solving a problem.	Understanding the lesson from beginning to end Map out the pacing of the lesson Easier to anticipate misconceptions and strategies when you have taught the lesson in the past Guided questions to get them back on track Gives us an idea of what to amplify during the lesson	Students will know if you do NOT know the lesson Makes the lesson run more smoothly Students use prior knowledge about constant of proportionality and equation y=kx to help complete table. An awareness of how lesson will begin Be aware of the fact that students may not have the prior knowledge needed to solve the problem.
Monitor	Walking around/checking for data Checking for understanding Check for multiple strategies used to solve problem Strategies for students who	Map out which students ' work you want to see Create the list of strategies during anticipation (planning), but have the list available while circulating.	Monitor both student writing and verbally
Select	How do we select students? - There are problems to solve then..."Right way"		Selected two strategies that were different Be intentional about the students who you want to share and how you will use their work to highlight the learning goal

Sequence	Bring up strategies in a certain order.		Use an example that shows the concept, then use an example that shows the concept using vocabulary
Connect	Compare and contrast strategies Encourages active learning and is more engaging Introduce new learning	At the end of the lesson connect back to the learning goal. See which lessons this lesson builds on	Assign the cool down activity to check learning goal.

HANDOUT 4 Student Task Statement for Lesson Experience

Warm-up 10.1: Notice and Wonder: The Price of Sunglasses

From Grade 7 Unit 4 Lesson 10

You are on vacation and want to buy a pair of sunglasses for $10 or less. You find a pair with a price tag of $10. The cashier says the total cost will be $10.45.

What do you notice? What do you wonder?

10.2: Shopping in Two Different Cities

Different cities have different sales tax rates. Here are the sales tax charges on the same items in two different cities. Complete the tables.

City 1			
item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.48	8.48
lamp	25.00	1.50	
pack of gum	1.00		
laundry soap	1.20		
x			

City 2			
item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.64	8.64
lamp	25.00	2.00	
pack of gum	1.00		
laundry soap	12.00		
x			

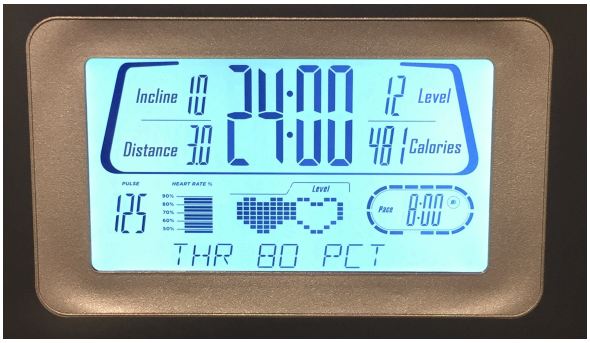
HANDOUT 5 Student Task Statement for Case Study

Warm-up 10.1: Treadmills

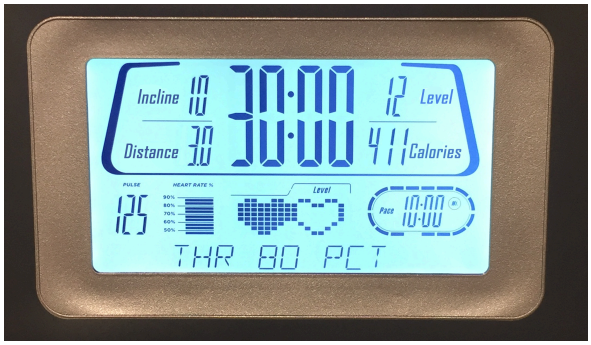
From Grade 6 Unit 2 Lesson 10

Mai and Jada each ran on a treadmill. The treadmill display shows the distance, in miles, each person ran and the amount of time it took them, in minutes and seconds.

Here is Mai's treadmill display:



Here is Jada's treadmill display:



1. What is the same about their workouts? What is different about their workouts?

2. If each person ran at a constant speed the entire time, who was running faster? Explain your reasoning.

Activity 10.2: Concert Tickets

Diego paid $47 for 3 tickets to a concert. Andre paid $141 for 9 tickets to a concert. Did they pay at the **same rate**? Explain your reasoning.

Activity 10.3: Sparkling Orange Juice

Lin and Noah each have their own recipe for making sparkling orange juice.

- Lin mixes 3 liters of orange juice with 4 liters of soda water.
- Noah mixes 4 liters of orange juice with 5 liters of soda water.

How do the two mixtures compare in taste? Explain your reasoning.

Cool-down 10.4: Comparing Runs

Andre ran 2 kilometers in 15 minutes, and Jada ran 3 kilometers in 20 minutes. Both ran at a constant speed.

Did they run at the same speed? Explain your reasoning.

HANDOUT 6 Case Study: Ms. Kim

In Ms. Kim's first block of Grade 6 Math students are in the midst of their second unit of the year, Introducing ratios. To this point, students have learned that a ratio is an association between two quantities, e.g., " 1 teaspoon of drink mix to 2 cups of water." Students have analyzed contexts that are often expressed in terms of ratios, such as recipes, mixtures of different paint colors, and constant speed (an association of time measurements with distance measurements). Students have used discrete diagrams and double number line diagrams to support thinking about equivalent ratios before they begin work with tables of equivalent ratios in upcoming lessons.

In a previous lesson, students learned that if two situations involve equivalent ratios, we can say that the situations are described by the **same rate**. In this lesson, students compare ratios to see if two situations in familiar contexts involve the same rate. The learning goals are as follows:

- compare two situations and explain whether they happen at the same rate.g(G) Choose and create diagrams to help
- (G) Justify that two situations do not happen at the same rate by finding a ratio to describe each situation where the two ratios share one value but not the other, i.e., *a:b* and *a:c*, or *x:z* and *y:z*.
- (G) Recognize that a question asking whether two situations happen "at the same rate" is asking whether the ratios are equivalent.

In each case, the numbers are purposely chosen so that reasoning directly with equivalent ratios is a more appealing method than calculating how-many-per-one and then scaling. The reason for this is to reinforce the concept that equivalent ratios describe the same rate, before formally introducing the notion of unit rate and methods for calculating it. However, students can use any method. Regardless of their chosen approach, students need to be able to explain their reasoning in the context of the problem.

Ms. Kim: Good morning, good morning! I look forward to learning alongside you today as we continue to make sense of ratios. Before we begin the warm-up, I'd like to share a video of a friend of mine walking on a treadmill. A treadmill is an exercise machine for walking or running. I can see some of you know what I'm talking about by the nods you're giving me. Thank you. Let's take a look at my friend on a treadmill. (*Class watches video)* What are some things you noticed? *(Elicits and acknowledges student responses)* Thank

you. You may have noticed the blue screen at the center of the console. That screen gives a lot of information. Can I share Mai and Jada's screens with you, from one of their recent exercise sessions? Take a moment to study the information on their screens. What's the same about their workouts? What's different? You can write your thinking in your workbooks. Now I'd like to ask you to respond to question 2. Let's take a couple of minutes of quiet work time.

Activity 10.1 (Warm-up): Treadmills

Mai and Jada each ran on a treadmill. The treadmill display shows the distance, in miles, each person ran and the amount of time it took them, in minutes and seconds.

Here is Mai's treadmill display:

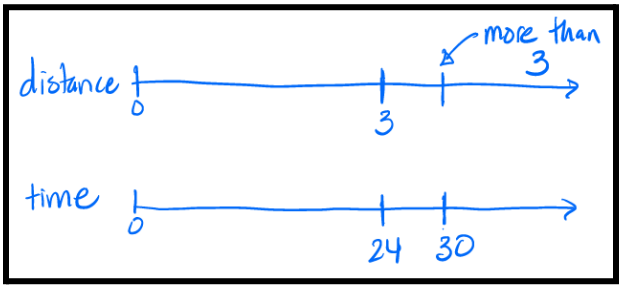
Here is Jada's treadmill display:

1. What is the same about their workouts? What is different about their workouts?
2. If each person ran at a constant speed the entire time, who was running faster? Explain your reasoning.

Ms. Kim: Now I'll invite you to discuss your thinking with a partner.

As students discuss, Ms. Kim notices most students agree Mai ran faster. She also notices Elena used double number lines to illustrate the speed of the runners. She brings the whole class together to discuss Elena's work. One number line illustrates distance and the second illustrates elapsed time. Elena shares that both runners ran 3 miles, yet Mai did it in 24 minutes while Jada did it in 30 minutes.

Elena: So, Jada was slower because it took her longer. If Mai had run for 30 minutes like Jada, she would have gone farther than 3 miles. Mai ran 3 miles in 24 minutes. It took Jada more time to run the same amount. (*Directs attention to the second number line*)

50	
51 52 53 54 55 56	Ms. Kim: Thank you Elena. Can y'all take a moment to share with a partner how Elena used the double number line to show us that Mai was faster? (*Students talk with their partners for 1 minute.)* Thank you for the great conversations. Please take a moment to write a "note to self" about something you heard or saw that might be helpful to you as we continue to study ratios in different contexts.
57	It was time for students to move into Activity 10.2.

Activity 10.2: Concert Tickets

Diego paid $47 for 3 tickets to a concert. Andre paid $141 for 9 tickets to a concert. Did they pay at the **same rate**? Explain your reasoning.

During her planning block the day before, Ms. Kim had worked through the activity, thinking about strategies students might use to determine whether Diego and Andre paid at the same rate. She knew that some students would use double number lines to illustrate the relationship between amount paid and number of tickets purchased. For this approach, she anticipated that students may begin with Diego's numbers and scale up to see if Andre's figures were the same. She knew students would look for a multiplicative relationship between Diego's values and Andre's values without the use of double number lines. She anticipated that some students might work to find the unit rate for each and then compare the unit rates. Knowing that unit rate is a focus of the next unit, she made a note to highlight this strategy based on the prevalence in the class.

Ms. Kim launched the activity as suggested in her teacher materials, arranging students in groups of 2 and asking them to work quietly for a few minutes before discussing their thinking with a partner. Students had access to the number line applet while they worked. She reminded them to make their thinking visible on paper using words, numbers, and any other tools that might be helpful in giving others a window into their thinking.

As she watched students work, she made a note of the number of students taking on the various approaches she had anticipated (M)

Ms. Kim: Thank you for respecting our quiet work time before sharing your ideas with a classmate. Now take some time to share your responses and reasons with your partner and also listen to the responses and reasons of your partner. (C) If you haven't yet come up with a strategy for answering this question, consider whether you might be able to use a strategy from your partner.

Ms. Kim listened in as students began to share their thoughts with each other. She noticed that Lin was using double number lines. Priya appeared to be using repeated addition, as she had written 3, 6, 9 and 47, 84, 141 on her paper. (M)Ms. Kim stopped and asked Priya to tell her more.(Seq)

Priya: I thought about it in groups: Diego bought 3 tickets, so that's one group. So, if he would've bought 2 groups of 3 tickets, that'd be 6 tickets for $94. Kind of like batches.

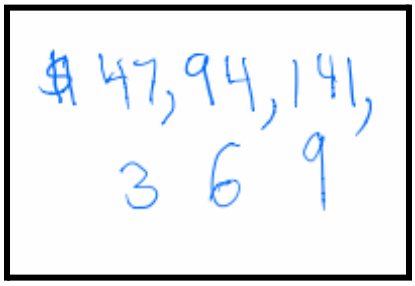
Ms. Kim: (Sel) I see. Thank you for sharing your thinking with me. When the time comes, would you share your thinking with the class? *(Priya agrees)* Thank you.

As Ms. Kim continued to circulate, she noticed Noah had taken a scaling approach similar to Lin's, without double number lines. (M)

After about five minutes of partner sharing time, the talking began to subside, and so she got the attention of the class for a whole-group discussion.

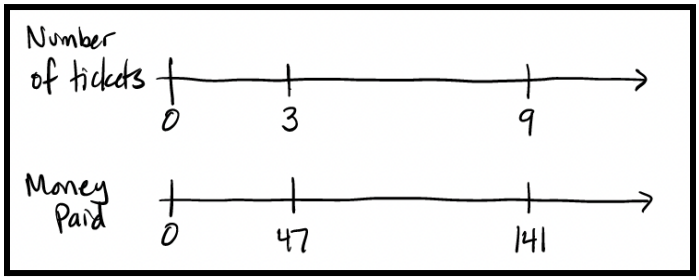
Ms. Kim: Let's spend some time learning from each other. Priya, will you share with us how you thought about this question?

Priya: Yes. I thought about it in groups, Diego bought 3 tickets, that's one group. So, if he would've bought 2 groups of 3 tickets, that'd be 6 tickets for $94. Then 3 groups would've been 9 tickets for a total of $141. So that means Diego and Andre paid at the same rate.



Ms.Kim: Thank you Priya. I noticed how you lined up your numbers to keep track of tickets and amount paid. Lin, I think you had a different way of organizing the numbers you focused on. Would you share your thinking with us?

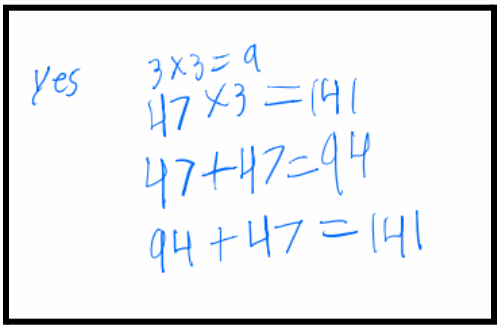
Lin: I drew a number line for the number of tickets and a number line for the amount of money paid. Then, kinda like Elena did on the warm-up, I put Diego on the number lines, here. *(Points to 3 and 47 respectively)*. Then I knew 3 times 3 is 9, so I checked to see if 47 times 3 is 141. It is. So, this is Andre here. *(Points at 9 and 141 respectively).*



Ms. Kim: Thank you Lin. Take a moment to discuss with your partner a couple of things. First, how are Priya's and Lin's strategies the same? Second, how did they share with us that Diego and Andre paid at the same rate?

After about 2 minutes, Ms. Kim calls the class together to discuss a third strategy. She invites Noah to share his thinking with the class. (Seq)

Noah: I was looking for the numbers to be the same.The 3 with the 9 and the 47 with the 141. So, the 3 was easy. Like Lin said, 3 times 3 is 9. But I added to find 47 times 3, that's why you see this here. *(Points to* 47 + 47 = 94 *and* 94 + 47 = 141*)*



Ms. Kim: (Connect)Thank you Noah. Why was it important to you for, as you said, "the numbers to be the same"? *(Turns to class)* Can y'all think about that too? Why do you think it was important, as Noah said "for the numbers to be the same"? Take a couple of minutes to discuss with your partner. Noah, you can think about it here if you'd like or you can discuss with your partner while everyone thinks about this.

After some partner time, Ms. Kim calls the class back together. She invites Clare to share her thoughts on the question posed.

Clare: Noah's way looks like Lin's without the number lines, they both used multiplication to see if Andre's price was right. But, my partner Kiran said it's kind of like what Priya said about batches and groups. Right, Kiran?

Kiran: Yeah, I think Noah was trying to see if 3 groups of Diego's tickets would cost the same as Andre's 9 tickets. And it did. So, they paid the same.

Ms. Kim: Noah, what are your thoughts about what Clare and Kiran just added to the conversation?

Noah: Yeah, it's like I said. They are the same, so they paid the same.

Ms. Kim: Thank you to all for sharing your thinking. We just looked at a couple of ways to show that Diego and Andre paid for their concert tickets at the same rate. I'd like to ask you to take a moment to write a "note to self" now.

Ms. Kim then launched the next instructional activity. As she watched students work, she paid particular attention to ways in which students were attending to the sets of ratios in the problem. She was curious to see how students might work with the first recipe to then evaluate the second recipe.

Ms. Kim spent the last few minutes of class synthesizing the lesson. Before giving students the cool-down, she asked students how they would rate themselves on the learning targets:

1. I can decide whether or not two situations are happening at the same rate.
2. I can explain what it means when two situations happen at the same rate.
3. I know some examples of situations where things can happen at the same rate.

When Ms. Kim reflected on the lesson after the students left, she remembered that Activity 10.2 was tagged as a 5 Practices activity—Anticipate, Monitor, Select, Sequence, Connect. During class, she had been paying attention to which students were using which strategies, and she even sequenced them purposefully during the whole-class discussion, but in the moment, she had forgotten about trying to connect the students' strategies to the learning goals in the lesson.

1. What do you see in Ms. Kim's classroom that is similar to and different from your own classroom?

2. Based on what you have read about Ms. Kim's class, what would you like to ask any of her students, and what would you hope to learn by asking?
3. What would you like to ask Ms. Kim about her facilitation of the class, and what would you hope to learn by asking?
4. What opportunities do you see in the work that students shared for Ms. Kim to explicitly help students make connections between the strategies and the key mathematical ideas in the lesson?

HANDOUT 7 5 Practices in the Concert Tickets Task

Select an Appropriate Task	Grade 6 Unit 2 Lesson 10 Activity 2: Concert Tickets
Understand the Learning Goal	Looking for per(same rate) and equivalent ratios.Note to self looking at skip counting for struggling students in order to grasp the concept of the double number line.
Anticipate	
Monitor	
Select	
Sequence	
Connect	

HANDOUT 8 Looking for the 5 Practices

Grade 6 | Grade 7 | Grade 8 | Algebra 1 | Grade 6 (2) | Grade 7 (2) | Grade 8 (2)

Grade 6

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

Grade 7

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

Grade 8

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

Algebra 1

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

Grade 6 Group 2

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

Grade 7 Group 2

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

Grade 8 Group 2

Select an Appropriate Task	
Understand the Learning Goal	
Anticipate	
Monitor	
Select	
Sequence	
Connect	

HANDOUT 9 Monitoring Sheet Template

Lesson/Task:	Learning Goals:	
Connections I want to make to the learning goals and/or between strategies:		
Student Names and Sequence	**Strategy**	**Questions to ask M→ while monitoring C→ while connecting**

Notes:

HANDOUT 10

Learning Goals

1. Paraphrase the key ideas of each of the 5 Practices named in the framework for productive discussion.
2. Explain the teacher's role in orchestrating discussion, both in planning and during the lesson.
3. Give examples of how the materials support teachers to incorporate the 5 Practices in their planning.
4. Articulate how the framework supports student understanding through discourse.

HANDOUT 11

My Reflections

5 Practices	
	Connect: How do these ideas **connect** to what you already know?
	Extend: What new ideas did you get that **extend** or push your thinking in new directions?
	Challenge: What is now a **challenge** for you to get your mind around? What questions do you now have?
	Next step: What **next step** will you take back to your classroom?

HANDOUT 11 Are You Ready for More?

Subscribe to the IM Blog

- How the 5 Practices Changed my Instruction
- Using the 5 Practices with Instructional Routines
- The Illustrative Mathematics Team Reflect on the 5 Practices
- Engaging All Students in Meaningful Mathematics
- Explicit Classroom Norms to Teach Kids How to Learn From Solving Problems

Once you have clicked on a blog post, subscribe to the blog by entering your email address on the right sidebar. Subscribing will ensure you receive email notifications of future blog posts.

Joining the IM Community

The IM Community Hub is the home for teachers of Illustrative Mathematics authored curricula and people who support them. Here you will find resources and connections with other IM educators, as we work together toward a world where learners know, use, and enjoy mathematics. Register at https://hub.illustrativemathematics.org/home.

You'll find resources such as cool-down guidance, unfinished learning adaptation packs, distance learning section guides, unit math story videos, and video lesson summary videos.

Follow us @IllustrateMath and share your learning publicly using the #LearnWithIM hashtag.

Follow us at Illustrative Mathematics.