Aleph-3 High School Problem-Solving Round

Do not open the test until instructed to do so.

Information:

- The Problem-Solving Round contains 10 questions for which you will be given 40 minutes to answer.
- There will be an 11th estimation question that will be used as a tiebreaker.
- There is no penalty for guessing, so make sure to give an answer to any of the questions, even if you are unsure.
- This test will be used along with the Speed Round to decide who is seeded in the buzzer round. Each question is worth 3 points counting towards this score.
- Do not expect to be able to solve every question. Likewise, do not be afraid to skip questions. The questions are roughly, but not strictly, in order of difficulty.
- Answers **do not** need to be integers. They can be numbers, words, coordinate points, or even dates.

Instructions:

- When starting the test, it is suggested to rip off the front page that contains the answer sheet.
- On the answer sheet, make sure to write your name, ID, and school.
- Only answers on the answer sheet will be graded, not answers in the test booklet.
- Answer with fractions reduced to simplest terms and with radicals having no squares in them.
- Units are not needed in the final answer, but make sure the number itself is in the correct units that is specified in the question.
- Good luck and have fun! If you have any questions, raise your hand.

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Aleph-3

1. Find $(3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9) \bmod 11$.

2. $\frac{\sqrt[3]{x^{10}}+\sqrt[3]{x}}{\sqrt[3]{x}} = 9$. Find x.

3. A triangle is formed by selecting three vertices of a regular pentagon. What is the smallest possible angle that can be formed between two sides of such a triangle, in degrees?

4. Alice is sending a positive integer N less than 26 to Bob over the internet. But since he wants to make sure that Eve, a person intercepting the message, does not see his number, he sends an encrypted integer E where E = 3N mod 26. Alice can decrypt the message back to its original form using N = 9E mod 26. Note that this encryption and decryption works for all possible values of N. If Bob instead chose an encryption method of E = 5N mod 26 and a decryption method N = cE mod 26, what is the smallest possible positive value of c that he could choose for his decryption method?

5. Alejan and Buni are playing a game with a fair spinner and a fair, standard, six-sided die. Alejan spins the spinner, and their score is equal to the angle made by the dial and the positive x-axis in radians (angle measured between 0 and 2π). Buni rolls the die, and their score is the number on the upward-facing side. What is the probability that Alejan has a higher score than Buni?

6. A right triangle ΔABC is constructed, where $\angle ABC = 90°$, and $\overline{AB} = 5$. A line $\overline{BD}$, where D is a point on $\overline{AC}$, is constructed, such that $\overline{BD}$ bisects $\angle ABC$. If ΔCBD is an isosceles triangle, what is the area of ΔABC?

7. Call a 2-digit number ab spectacular if it is equal to $a^2 + 2b + 1$. For instance, 43 is not spectacular because $4^2 + (2)(3) + 1 = 23 \neq 43$. There are two spectacular 2-digit numbers. What is their sum?

8. An ant stares up at an angle of 30° from the floor to look at an apple on a tree. When the ant travels 4 feet forward, it looks up at the apple again and now looks at an angle of 75° degrees from the floor. How far up is the apple in feet?

9. Oliua is drawing cards from a standard 52 card deck with 4 suits (spade, diamond, heart, club), each with 13 cards, until she draws a card with spades, after which she stops. On average, how many cards is Oliua going to draw (including the spades)?

10. What is $cos\frac{2\pi}{9} + cos\frac{4\pi}{9} + cos\frac{6\pi}{9} + cos\frac{8\pi}{9}$?

11. TIEBREAKER: For any 3-digit number <u>abc</u> that is between 000 and 999, inclusive, define a function f(<u>abc</u>) = $2^a + b^2 + 2c$. Xija chooses a random number <u>abc</u> between 000 and 999 as his starting number. He repeatedly applies the function f until he either gets a result greater than 999, or returns to a number he has already generated. What is the average amount of numbers he will generate? This is an estimation question that is likely not possible to be solved by hand. It will not be used to affect your score except for the event of a tiebreaker. It is recommended that at the very least, you provide a guess.