

B. Sc (Hons. Math) (Semester – 1st)
CALCULUS-I
Subject Code: BMATS1121
Paper ID: 22131201

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a) Find the value of $\frac{5x^3+8x^2}{3x^4-16x^2}$.
- b) If $\sqrt{3-2y^2} \leq f(y) \leq \sqrt{3-y^2}$ for $-1 \leq y \leq 1$, find $f(y)$.
- c) Check whether $y = \sqrt{x}$ is differentiable at $X=0$ or not.
- d) Find the absolute minimum and maximum values of $f(x) = 4 - x^2, -3 \leq x \leq 1$.
- e) Define the inflection point and concavity of a curve.
- f) Find the polar form of $x^2 + (y - 4)^2 = 16$.
- g) Find the asymptotes for the hyperbola $9x^2 - 25y^2 = 225$.
- h) What are the tangents to the curve $x^3 + y^3 = 3axy$ at the origin?
- i) What is the working rule to find the extreme values of a function $z = f(x, y)$.
- j) Define composite function with an example.

Section – B

(5 marks each)

Q2. Compute $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ for $\vec{F} = x^2y\vec{i} - (z^3 - 3x)\vec{j} + 4y^2\vec{k}$.

Q3. Show that every differentiable function is continuous.

Q4. Find the value of $\partial z / \partial x$ at the point (1,1,1) if the equation $xy + z^3x - 2yz = 0$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

Q5. Find the derivative of the function $g(x, y, z) = 3e^x \cos \cos yz$ at point $P(0,0,0)$ in the direction of $A = 2i + j - 2k$.

Q6. Solve the system $u = x - y, v = 2x + y$ for x and y in terms of u and v . Then find the value of the Jacobian $\partial(x, y) / \partial(u, v)$.

Section – C

(10 marks each)

Q7. Solve and prove Euler's theorem for homogenous function.

Q8. Find the tangent plane and normal line at the point $P(3, 5, -4)$ on the surface $x^2 + y^2 - z^2 = -18$.

Q9. Find the greatest and smallest values that the function $f(x, y) = -xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ using the method of Lagrange's multipliers.