PRACTICE 5.3 – The Product and Quotient Rules

* Full, worked solutions can be found in the folder linked on the Course Website ☺

Exercise 5H

1 Differentiate each function with respect to x.

a $y = x^2(2x-1)$

b $y = (2x-3)(x+3)^3$

c $y = x\sqrt{2-3x}$

d $y = (2x+1)(x^2-x+1)^2$

e $y = \sqrt{(x+2)}(2-3x)$

2 $y = \sqrt{x+1}(3-x)^2$

a Show that $y' = \dfrac{(x-3)(5x+1)}{2\sqrt{x+1}}$.

b Find the x-coordinates of all points on the graph of y where the tangent to the curve is parallel to the x-axis.

3 Find the equation of the normal to the curve $y = \dfrac{x}{1-2x}$, $x \neq \dfrac{1}{2}$ at $(0,0)$.

Exercise 5I

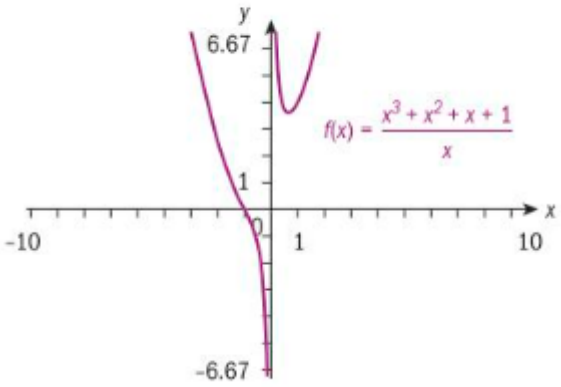
1 Differentiate each expression using the quotient rule.

a $y = \dfrac{1+3x}{5-x}$ **b** $y = \dfrac{\sqrt{x}}{2-x}$

c $y = \dfrac{1+2x}{\sqrt{1-x^2}}$ **d** $y = \dfrac{1+3x}{x^2+1}$

2 Find the equation of the normal to the curve $f(x) = \dfrac{3x-2}{x^2+1}$ at the point where $x = 0$.

3 Consider the "trident of Newton" curve below:



Find the value of x where the tangent to the curve is parallel to the line $y = x$.

Don't stop now.
More on back ☐

Reflect If you are asked to differentiate an expression in an examination, how could you determine which of the three differentiation rules (chain, product and quotient rules) to use?

Exercise 5J

1 Differentiate each function with respect to x.

a $y = (x-1)(x+3)^2$

b $y = (x+1)\sqrt{1-2x}$

c $y = \dfrac{x+1}{x-1}$

d $y = \dfrac{2}{x^4 - 2x + 1}$

2 Find the equations of the tangent and the normal to the curve $f(x) = \dfrac{1+\sqrt{x}}{x-1}, x \neq 1$ at the point $(9, \frac{1}{2})$.

Did you know?

The "trident of Newton", which you encountered in exercise 5I question **3**, was the 66th curve in Newton's classification of cubics. Can you identify why the curve has this name? Research how this, and other famous curves, have been used to model real-life situations.