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Total No. of Printed Pages: 1

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B.Sc. (Non-Medical) (Semester – 4th)

ALGEBRA-II

Subject Code: BSNMS1-407

Paper ID: [22131423]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a. Prove that every division ring is an integral domain but not conversely.
- b. Prove that sum of two units of a ring need not be a unit.
- c. Let R be a commutative ring with 1. Then R is a field $\Leftrightarrow (0)$ and R are the only ideals of R .
- d. Prove that $S_n (n \geq 3)$ has trivial centre.
- e. Prove that product of two even permutations is again an even permutation.
- f. Prove that $W^\perp$ is a subspace of V .
- g. Define Inner product spaces.
- h. Prove that element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in V , $vT = \lambda v$
- i. Show that the mapping $T: R^2 \rightarrow R$ defined by $T(x, y) = |2x - 3y|$ is not a linear transformation.
- j. Show that there is no non-singular linear transformation from R^4 to R^3 .

Section – B

(5 marks each)

Q2. If R is a commutative ring with 1, then $A \in M_n(R)$ is a unit $\Leftrightarrow \det(A)$ is a unit in R .

Q3. Let $f: R \rightarrow S$ be a homomorphism with $\ker f = N$. Then $\frac{R}{N} \cong \text{Im} f$

Q4. Prove that Z_n is an integral domain $\Leftrightarrow n$ is a prime $\Leftrightarrow Z_n$ is a field.

Q5. Prove that the matrix $A = [1 \ 1 \ 1 \ -1 \ -1 \ -1 \ 1 \ 1 \ 0]$ is nilpotent and find its Jordan form.

Q6. Let V be a finite dimensional inner product space; then V has an orthonormal set as a basis.

Section – C

(10 marks each)

Q7. If $u, v \in V$ then $|(u, v)| \leq \|u\| \|v\|$

Q8. Prove that the range of T and $U = \{v \in V: vT = 0\}$ are subspaces of V . Further if $n(T)$ is the dimension of the U , prove that $r(T) + n(T) = \dim V$

Q9. State and prove Sylow's third theorem. Also show that group of order 72 is not simple.