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### **Learning Goals**

- 1. Let's explore how students learn math in a problem-based classroom.
- 2. Let's see how the lesson structure ensures students make their thinking visible.
- 3. Let's reflect on teacher moves that support problem-based learning.
- 4. Let's examine elements of a problem-based classroom community.

## **Teach and Learn Virtual Note Page Links**

Part 1 | Part 2 | Part 3 | Part 4



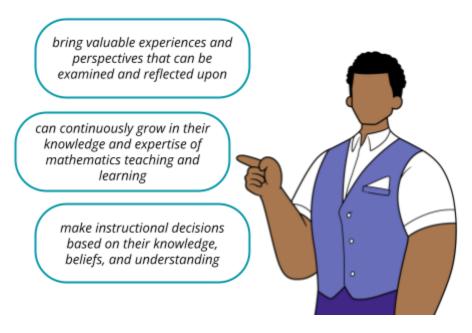
# Doing Math as a Mathematical Community

Mathematical Community	
Doing Math sounds like	Doing Math looks like
<ul> <li>Exciting</li> <li>Difficult</li> <li>A bunch of thoughts</li> <li>Specific Vocabulary</li> <li>Asking questions (why? How did you get it? Can we use this? etc.)</li> <li>exciting</li> <li>difficult</li> </ul>	<ul> <li>Shapes</li> <li>Numbers</li> <li>Team work</li> <li>Symbolic representations</li> <li>White boards</li> <li>Problem sets/ challenge problems</li> <li>Numbers</li> <li>shapes.</li> </ul>

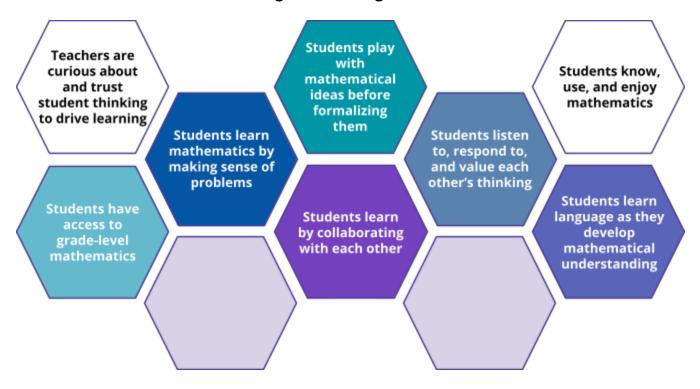


### Problem-Based Teaching and Learning Anchors

# **i V** believes all teachers . . .



### **Elements of Problem-Based Teaching and Learning**





### **HANDOUT 4b**

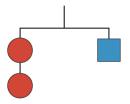
# Solving Systems by Elimination (Part 1)

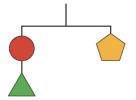
# Algebra 1 • Unit 2 • Lesson 14

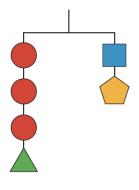
Let's investigate how adding or subtracting equations can help us solve systems of linear equations.

# Warm-up 14.1: Notice and Wonder: Hanger Diagrams

What do you notice? What do you wonder?







l wonder
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•
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### **Activity 14.2: Adding Equations**

Diego is solving this system of equations:

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$$

Here is his work:

$$4x + 3y = 10$$

$$-4x + 5y = 6 +$$

$$0 + 8y = 16$$

$$y = 2$$

$$4x + 3(2) = 10$$
$$4x + 6 = 10$$
$$4x = 4$$
$$x = 1$$

- Make sense of Diego's work and discuss with a partner:
  - a. What did Diego do to solve the system?
  - b. Is the pair of and values that Diego found actually a solution to the system? How do you know?
- 2. Does Diego's method work for solving these systems? Be prepared to explain or show your reasoning.

a. 
$$\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$$

b. 
$$\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$$

# **Activity 14.3: Adding and Subtracting Equations to Solve Systems**

Here are three systems of equations you saw earlier.

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases} \begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases} \begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$$

$$\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$$

$$\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$$

For each system:

- 1. Use graphing technology to graph the original two equations in the system. Then, identify the coordinates of the solution.
- 2. Find the sum or difference of the two original equations that would enable the system to be solved.
- 3. Graph the third equation on the same coordinate plane. Make an observation about the graph.



### Are you ready for more?

Mai wonders what would happen if we multiply equations. That is, we multiply the expressions on the left side of the two equations and set them equal to the expressions on the right side of the two equations.

- 1. In system B write out an equation that you would get if you multiply the two equations in this manner.
- 2. Does your original solution still work in this new equation?
- 3. Use graphing technology to graph this new equation on the same coordinate plane. Why is this approach not particularly helpful?

### Cool-down 14.4: What to Do with This System?

Here is a system of linear equations:

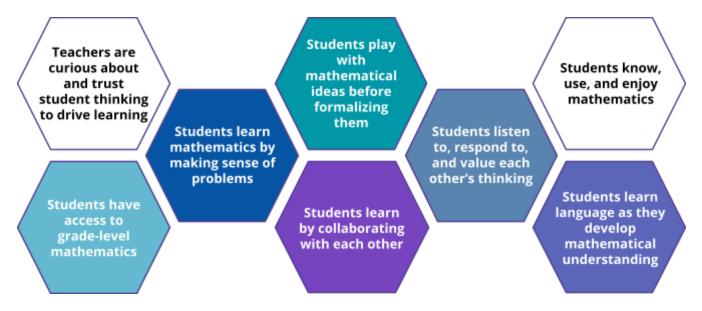
$$\begin{cases} 2x + \frac{1}{2}y = 7\\ 6x - \frac{1}{2}y = 5 \end{cases}$$

- 1. Which would be a more helpful for solving the system: adding the two equations or subtracting one from the other? Explain your reasoning.
- 2. Solve the system without graphing. Show your reasoning.



### Problem-Based Teaching and Learning Reflection

# **Elements of Problem-Based Teaching and Learning**



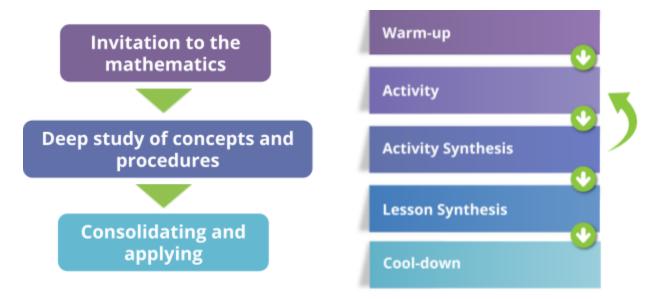
Element	Where in the lesson did you experience these?	What did your <b>teacher</b> do to create these opportunities for students?
Students play with mathematical ideas before formalizing them	<ul><li>Diego's work</li><li>•</li></ul>	<ul> <li>Asked for clarification</li> <li>Pulled in a comparison of other people's strategies</li> </ul>
Students learn mathematics by making sense of problems	<ul> <li>Comparing graphical representations using desmos</li> <li>Looking at Diego's work</li> </ul>	Demonstrated multiple representations of student's work



Students learn by collaborating with each other	<ul> <li>Working as a pair in breakout room</li> <li>Working in pairs gave me better clarification</li> </ul>	<ul> <li>Asking students to respond to other thoughts</li> <li>•</li> <li>•</li> <li>•</li> <li>•</li> <li>•</li> </ul>
Students listen to, respond to, and value each other's thinking	•	<ul> <li>Think-Pair-Share activities</li> <li>Independent Thinking</li> <li>•</li> <li>•</li> <li>•</li> </ul>
Students learn language as they develop mathematical understanding	•	•
Students have access to grade-level mathematics	•	•



### Problem-Based Lesson Structure



# Warm-up: The first event in every lesson is a ~5–10 minute warm-up.

### Purposes:

- Invite students to get ready for the day's lesson.
- Give students an opportunity to strengthen their number sense or procedural fluency.
- Strengthen students' skills in listening and speaking about mathematics.

# **Activity:** One to three classroom activities per lesson. The heart of the mathematical experience and the bulk of the time spent in class.

An activity can serve many purposes:

- Give students experience with a new context.
- Introduce a new concept and associated language.
- Introduce a new representation.
- Formalize a definition of a term for an idea they have encountered informally.
- Identify and resolve common mistakes and misconceptions that people make.
- Practice using mathematical language.
- Work toward mastery of a concept or procedure.
- Provide an opportunity to apply mathematics to a modeling or other application problem.

### **Are You Ready for More?:**

Some activities have associated optional extension problems, made available to all students.



# **Activity Synthesis:** The last portion of each lesson activity

- Students synthesize what they have learned in that particular activity.
- Each activity includes flexible options for teachers to support students in solidifying topics from the activity, often including questions to ask students.

# **Lesson Synthesis:** After the lesson activities, the teacher summarizes the day's activities for ~5–10 minutes.

- Teachers ask students questions about the day's activities that enable students to synthesize what they have learned.
- Each lesson includes synthesis questions that a teacher might pose.
- Teachers can use this time in any number of ways, including providing an opportunity for students to think about their learning using the learning targets.

# **Cool-down:** The last piece of every lesson is a ~5 minute cool-down.

- Students are given a brief formative assessment.
- Students work independently and turn in to the teacher.
- Responses can be used to guide further instruction.
- One cool-down is provided for every lesson, except the culminating lesson in the unit and the lessons in the culminating units (6–8) or modeling prompts (9–12).



### **Notes to Self**

What is the one most important thing you want to remember about the *Problem-Based Lesson Structure* as you prepare to teach?

- Give students an opportunity to strengthen their number sense or procedural fluency (background skills)
- Strengthen students' skills in listening and speaking about mathematics.
- Give students context to the lesson in the Warm-Up

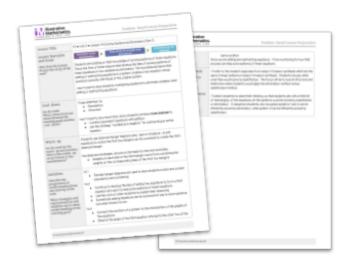
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# Problem-Based Lesson Preparation





As you compare what's in the teacher materials and the Problem-Based Lesson Preparation Sample, what do you notice? wonder?

<u>Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 | Group 7 | Group 8 | Group 9 | Group 10</u>

## **Group 1**

What do you notice?	What do you wonder?
•	•

## **Group 2**

What do you notice?	What do you wonder?
•	•
•	•
•	•

# **Group 3**

What do you notice?	What do you wonder?
•	•
•	•
•	•

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HANDOUT 8 Learning Targets

- 1. I can identify opportunities for students to learn mathematical ideas through the experience of solving problems.
- 2. I can describe the structure of a lesson and the purpose of each component.
- 3. I can recognize teacher moves that support problem-based learning.
- 4. I can describe teacher moves that build a positive inclusive mathematical community.

HANDOUT 9 My Reflections

Experiencing
<b>Problem-Based</b>
Instruction

The problem-based lesson structure engages students in learning math by solving problems because / by . . .

- •
- It allows students to interact with and explore mathematics and formalize after.
- The structure allows them build on the materials and develop better practices
- It allows them to solve a problem more than 1 way
- Giving them a problem based "hook" as a warm-up.



Are You Ready for More?

We value reflection and consolidation as an important part of learning. To that end, we offer the following set of carefully chosen activities to help deepen your understanding of Problem-Based Teaching and Learning.

### **Finding Opportunities to Build Mathematical Community**

In the Teacher Materials: <a href="https://im.kendallhunt.com/">https://im.kendallhunt.com/</a>

Navigate to a lesson (suggestions below) to explore how the teacher materials provide opportunities for students and teachers to build a positive and inclusive mathematical community for problem-based teaching and learning.

Where in the lesson do you see opportunities for the teacher and students to:

- be curious about and trust student thinking to drive learning
- learn by collaborating with each other
- learn math by making sense of problems
- listen to, respond to, and value each other's thinking?

Lessons from Unit 1	Lessons from Unit 2
<ul> <li>Algebra 1 Lesson 4: The Shape of Distributions</li> <li>Geometry Lesson 7: Construction Techniques 5: Squares</li> <li>Algebra 2 Lesson 1: A Towering Sequences</li> </ul>	<ul> <li>Algebra 1 Lesson 5: Equations and Their Graphs</li> <li>Geometry Lesson 2: Congruent Parts, Part 2</li> <li>Algebra 2 Lesson 1: Let's Make a Box</li> </ul>

### Take a tour of the online platform

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- Inviting Students to the Mathematics
- Concrete Representations that Give Student a Way to Get Started



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