Graduate School of EDUCATION

Mathematical Agency Improvement Community

Abolishing the phrase "I'm not a math person."

Research Lesson Memorialization Document

Team Members

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Lesson Date:	Instructor:	Grade Level:
March 6th. 2020	Dr. Curtis Taylor	6th Grade

Summary Box # 1: Title of the Research Lesson

Algebratizing Numeric Expressions

Summary Box # 2: The Research Question

The problem of practice our team explored

How can we support students in trusting their own thinking, embracing the feeling of vulnerability in order to articulate their own and other's thinking?

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Resource adapted from:

The Lesson Study Group
at Mills College

Summary Box # 3: Your Team's Theory of Action

The long-term goals for our students and how we will get there

*If we as teachers pose questions with a **variety of methods** and **interpretations** and **we honor each method through discussion and analysis**, then students will see the value in diverse thinking resulting in students feeling more open to sharing their thinking and listening to others.*

Summary Box #4: The Research Lesson Topic

Algebraic Expressions

Summary Box #5: Background and Research on the Content Topic

Youcubed Border Problem

Cathy Humphreys Teaching the border problem

Jo Boaler teaching the border problem

NCTM Grade 6 Expressions & Equations

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Summary Box #6: Relationship of Unit Standards

Prior learning standards that unit builds on	Learning standards for this unit	Later standards for which this unit is a foundation
4.0A.B4 *Find all the factor pairs in the range 1-100..* *5.OA.A.2* *Write simple expressions that record calculations with numbers, and interpret numerical expressions with evaluating them.* *"Add 8 and 7 and multiply by 2 as (8+7) x 2* *5.NF.B.5* *Interpret multiplication as scaling…* *6.NS.B.4* **use the distributive property to express a sum of two whole numbers with a common factor* *Ex 36+8 as 4(9+2)*	**6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers. **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.	In grade 7, students will apply the properties of operations 7.EE.A.1 while evaluating algebraic expressions and solving equations and inequalities 7.EE.B.4. They will continue their work with rational numbers, and in grade 8, students will work with integer exponents 8.EE.A.1. Using linear equations to represent real world problems

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Summary Box #7: Goal of the Unit

Students will be transitioning from concrete arithmetic concepts to abstract algebraic concepts.

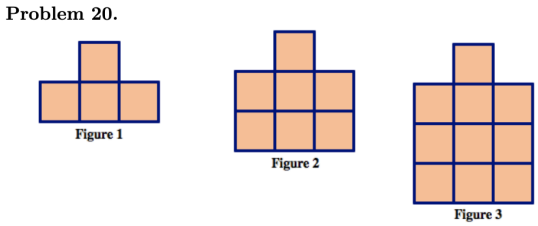
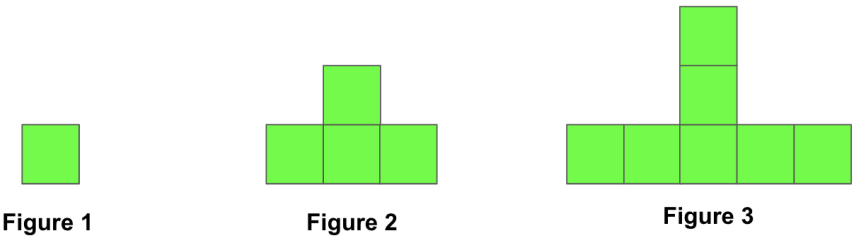
Summary Box #8: Flow of the Unit/Rationale for the Design of Instruction

This unit begins with students being introduced to various visual patterns to establish a visual anchor between the figure number and a visual pattern. They will be asked to find the total number of things (squares, circles, toothpicks) within these patterns as a way of getting students to record various numerical expressions. Later in the unit students will be asked to find the total number of things (circles, squares, etc) for any figure number thus moviding towards algebraic expressions. The patterns are picked strategically to have a variety of possible numeric expressions and algebraic expressions. The goal will then be for students to use properties they are familiar with using previously such as the distributive property to show how their various algebraic expressions are equivalent to each other.

Summary Box #9: Unit Plan

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The lesson sequence of the unit, with the task and learning goal of each lesson. The asterisk (*) shows the research lesson

Lesson	Learning goal(s) and tasks
March 2	*Lesson Goal: For students to connect the figure number with the structure of the pattern AND to be able to write a numerical expression that represents the total number of squares* *Task: Cupcake Pattern*  1.
March 4	*Lesson Goal:* For students to connect the figure number with the structure of the patter and have students validate that their numerical expressions are accurate *Task: Upside Down T-Pattern* 

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*March 6	Lesson Goal :Students will look for structure in the Border Problem and use this structure to figure out the total number of border tiles in various figures. They will construct numerical expressions for the various figure numbers and look at the structure within those expression in order to begin to generalize how to figure out the total number of squares for **any figure** in this pattern. *Task:* Youcubed Border Problem

Summary Box #10: Mathematical Understanding Goal

*Students will understand how a numerical expression can model a visual representation and use the **structure** within these expressions to generalize their expression in order to find the total number of squares for any figure number.*
i.e.
10 + 10 + 8 +8 (for figure 10)
3+3+1+1 (for figure 3)
123+123+ 121+121 (for figure 123)
Figure number +Figure Number+ (figure number -2) + (figure number - 2) (any figure number)

Summary Box #11: The Equity Goal

Through turn-and talks and whole class share outs, students will develop math flexibility by valuing each others' ideas. Students will demonstrate value by:

- listening to,
- questioning,
- and/or applying math strategies of their peers

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Summary Box #12: Scripting the Lesson

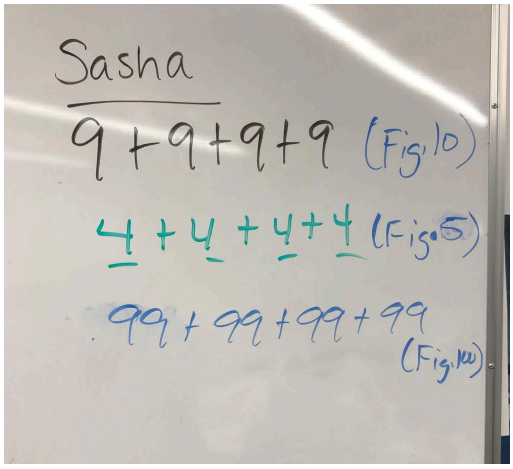
Learning task and activities, anticipated student responses, key questions or comparisons that will build insights	Anticipated student responses	Assessment (Points to Notice)
Good afternoon, mathematicians! We have been surfacing some of our understanding of algebra in class this past week, and now we are going to do some deeper digging. Are you ready? Show visual. How many red squares are in figure 10? "why do you think this figure is called figure 10?" Help students make the connection that figure 10 is a 10 x 10 grid.	40, 38, 36. (possible answers for figure 10)	The various numerical expression students are using to find the total number of red squares for figure 10.
"I am more interested in the method you used than the answer of how many red squares there are." Students will work individually and share with a partner. Curtis will ask for students to share. Students will share out numerical expressions "How did you know that it would be 5 + 5 + 3 + 3?"		Underline the matching sections so students can begin to identify the repeated structure. Label each section as top, bottom, sides, etc and write in parenthesis next to the strategy (figure #). Have students point

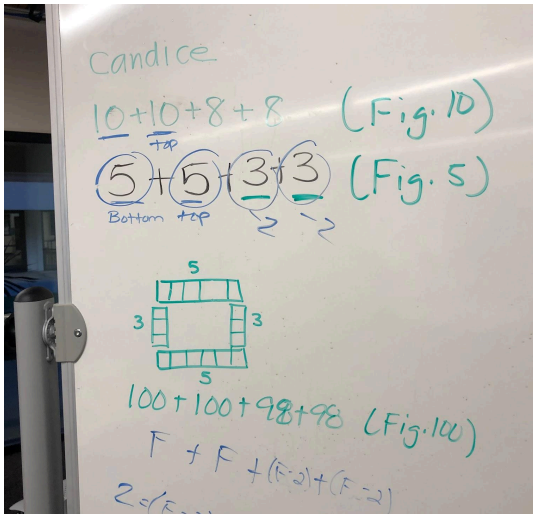
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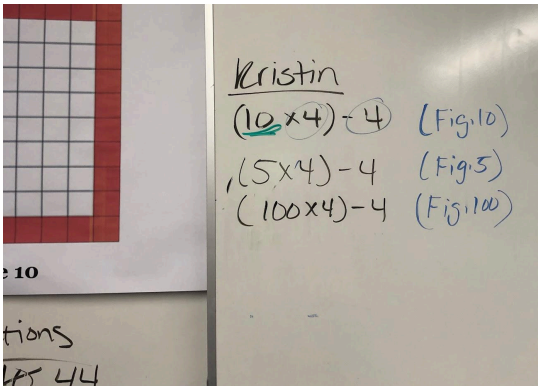
		to explain how they knew how many squares were in each section.
"Choose a strategy. Imagine figure 100 in your head. What numbers would you use to find the red squares for figure 100? I am less interested in the answer and more interested in what numbers you think would be used in the strategy you chose." Begin to generalizing "From these expressions, what's staying the same? What's changing?"	We anticipate that students will say that figure number changes but there are certain values that stay such as the -4 for the corners. This will lead up to the question of finding the number of red squares for any size figure.	Curtis will select students by writing their names on the board. Those students will share. Always ask, "how do you know that? where do you see this number?"

Resource adapted from:
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at Mills College

Summary Box #13: Boardwork Plan







Summary Box #14: Data Collection Plan

During the lesson the team will tally the number of times the focus students….

- sharing their ideas with (whole class and partner)

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- asking questions (whole class and partner)
- has their idea used by peers

After class students will be given an exit ticket asking if they feel that their ideas were being valued by their peers today.

Summary Box #15: End of Cycle Reflection

Include notes from the following debrief activities. The first two prompts are done publicly, followed by a discussion of one or more of the whole team reflection questions, and comments from the equity and content commentators. Any team reflection questions not addressed in the public debrief should be addressed in a private team debrief afterward.

After the lesson the presenting teacher has an opportunity to reflect and the team shares observational data about their focus students:

- *Presenting teacher reflection -*
  -
- *Observing teacher data (what focus students said/did during the lesson)*

Team discussion of the following prompts. What did the team learn about:

- *The mathematical concept? Moving from numeric expressions to algebraic ones*
  - *Dr. Curtis*
    - *We saw students who were able to see the structure and generalize. Post lesson study students looked at their numeric expressions and the algebraic and matched them to the visual.*
  - *Kristin*
    - *Students naturally recognize the pattern. They didn't think of their thinking as algebraic thinking. They seperated their thinking as pattern recognition.*
  - *Dan*

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        - *Thinking about my focus students. She bounced between two methods. When Curtis wrote down another method to consider his own, I didn't influence them at all.*
    - *Candice*
    - *Katerina how do students conceptualize the concept of "any size?"*
    - *Dr. Butler*
        - *Students gravitated towards two different methods. They moved away from their initial thinking. It would be important to have a conversation around the value of all the different methods. Surfacing and honoring of all of these different methods. Asking students to hold on or intentionally taking on others. Need to intentionally honor ALL the methods equally. For those who were using a, b on their paper. If students were pressed to explain what the a,b, f, and figure meant. I am certain that students would be able to give a contextualized definition of what a variable mean...50 percent.*
- *Student thinking?*
    - *See above comments*
- *Teaching & pedagogy?*
    - *Dr. Curtis*
        - *From the commentary my takeaway was from Dr. Butler and really attending to the students thinking and listening to them. One of the students came up to me to re clarify his method. To slow myself down and listen. From Bryan assigning competence and being explicit about what I am seeing.*

    - *Dan*
        - *Assigning competence to students and highlighting their genius. The power of Anticipatory Planning. My challenge is how I can bring aspects of this process to my own school. I was also struck when Bryan made the comment of "whose writing is on the wall" and*

"who's voice is being heard".

Kristin: Daisy pointed out that math understanding comes from unpacking the small details of the lesson. When Curtis continued to ask the questions 'where do you see that?" I saw a moment of understanding for the students. From the commentary about valuing ideas one part is making sure everyone is listening by the other half is people see the value in what

Katerina: Teachers can use their mathematical authority to help make clear to students how their ideas are valuable and helpful. We need to look for these opportunities. Teachers can improve their content knowledge so they can anticipate and leverage moments of student value.
"The struggle is real." - There is a tension in decision-making within the moment. There are moments that we try to prepare for and yet, we still need to make those split-second choices.
The social impact that gives value to who is sharing and what weight it carries in the classroom- how can we be more aware of all the social moments (discretionary moments)?
Teaching mathematics is a complex job. It is an artform.

Dr. Butler 2 things...
tangible: Having teachers think about how they can revisit ideas from previous units and revisit them in the future units. It is important for students to see how their ideas build on each other

Non tangible: Curtis you are such a gifted math instructor. There is research that suggests that multitasking is nearly impossible. Our brain cannot...we are asking teachers to do something that is nearly impossible! How do we ease the load of all the things we are hoping teachers notice?

If we can't do the impossible-then what can we do? - Dan
I haven't given up on the impossible. - Libby

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How can we train our brains to look for status, keep the flow going in terms of what you've anticipated, and meet them where they are at...but I believe in the impossible.
Using purposeful pauses and getting students used to the moments when we are processing so they see you as a learner.

- *Our research question: How can we support students in trusting their own thinking, embracing the feeling of vulnerability in order to articulate their own and other's thinking?*
    - *We need to create the space and honor where students are in their thinking while still keeping our goals in mind.*
    - *Being explicit in how their ideas are valuable*
    - *Showing our own vulnerability by using purposeful pauses*

- *Our theory of action: If we as teachers pose questions with a **variety of methods** and **interpretations** and **we honor each method through discussion and analysis**, then students will see the value in diverse thinking resulting in students feeling more open to sharing their thinking and listening to others.*

    - *Students not only need to see the variety of methods but we also need to take the steps in honoring and valuing each method by showing students the mathematical value in their process. Use our mathematical authority to assign competency and to change perceptions. In the Anticipatory Planning think about how the methods connect.*

- *What do individual team members want to implement in their own practice?*
    - *Curtis: I want to learn to how to do the impossible...having the candor to talk to the students when there is a curveball*
    - *Dan: being thoughtful about opportunities to assign competence. Keep my eyes open and look for those opportunities.*
    - *Kristin: Same as Dan. Thinking about my own content knowledge. How can we leverage the value of each of the students' ideas?*

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- *Katerina: Use a purposeful pause to slow down.*

Notes from the Expert Commentary - Content (Dr. Libby Butler)

- *Lots of great questioning strategies*
- *The use of the board work to help students see the structure of the pattern*

Posing of the question "what is changing vs what is staying the same" pushed students to use the variable

- *Going slow to set the strong foundation will pay off in the long run*

Missed Opportunities

- *Meeting students where they are with their thinking. Not what the standards call for, not what we had planned, but really tuning it to where they are at that moment.*
- *When asking the question of "why is figure 10" use students' thinking and play the skeptic "well, what about this 9 here, what does it have to do with figure 10?*

Notes from the Expert Commentary - Equity (Bryan Mayer)

- *Who is writing on the board?-the students!*
- *"Good afternoon mathematicians" -setting the expectations and positing the students as the "doers of mathematics"*
- *Curtis created a safe atmosphere for students to share ideas (right or wrong ones)*
- *Almost all students were up on the board.*

Next Steps

- *What happens after the students present?*
  - *Who should be the one to respond to them? Right now it is the teacher but the goal should be the students*

- *While we hold the assumption "everyone is mathematically briallint" how do we make this explicit and public to students?*
  - **Emma's diagram*
  - **Joanna's question about the corner (skepticism)*
  - **Gisselle and Deliah color coding systems*

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- *Huan's "what if questions"*
- *Will and Amedius generalization*

**above are examples of ways to assign mathematical competence to students*

- *Who's contributions are perceived as valuable?*

Images of Board Work
Poster from the audience
Data

Exit Ticket Data:

"Do you feel that you ideas were valued today? Explain how you know they were valued or not valued.

Yes	No	Other
I I I I I I I I I I I I (12) I (Avery) "I believe they were very valued. I saw many in the crowd, and the person at my table writing down. My classmates were attentive and could repeat what I said. People in the crowd seemed interested, and a camera in my face haha :)" Many comments spoke either about peers being		I (Gabriel) " I don't know but the idea that says Gabriel on the board wasn't my idea it was Kyla's, my idea was the same as hans" I (Giselle) "Today I kind of do and kind of don't think they were valued. I say they were because I saw people were actually listening and some did use my strategy (I think). Also I say they weren't because no one used it and

attentive or using their strategy but on the flip side some noted the camera and audience as the marker for them feeling valued.		my board was in the corner barely shown :(" I (William) " a little bit because one person when I was walkng balk said your wrong…" I (Ruby) Maybe, I mean i just said what I thought. Han's thinking was in my perspective.

Mathematical Goal

Focus student 1: Wrote algebraic expression on their exit ticket

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