Evaluation criterion tutorial

1. The Scenario

Research Question: How does the acceleration of a cart depend on the pulling force applied to it, keeping the total mass constant?

Method:

"I placed a 0.50 kg cart on a metal track. To keep the **Total System Mass** constant, I loaded the cart with **five** 0.05 kg slotted masses, and then transferred one mass at a time from the cart to the hanger. I calculated the Independent Variable (Pulling Force) as the weight of the hanging mass ($F_{pull} = m_{hanger}g$).

Measurement: I fixed two photogates exactly **50 cm** apart (*s* = 0.50 m). I released the cart from rest at the first gate and measured the time interval (*t*). I repeated this **5 times** for each mass setting and calculated the average time. I weighed the cart with all five slotted masses on a digital balance three times; every reading was 0.750 kg, so the total mass is (0.750 ± 0.001) kg.

Since the total system mass is constant,

$$F_{pull} = M_{total}a$$

and

$$a = \frac{1}{0.75\ kg}F_{pull}$$

and thus the graph of *a* vs. F_{pull} should be a straight line through the origin with slope $\approx 1.33\ kg^{-1}$."

Qualitative Observations (Student Notes):

- **Obs 1:** "As the cart moved, I heard the wheels rattling against the metal grooves."
- **Obs 2:** "The pulley was clamped to the end of the track, but its angle was not quite right. I noticed the string was not horizontal; it **slanted upwards** from the cart to the pulley."
- **Obs 3:** "The cart did not move immediately when the first small mass was added. I had to give it a tiny nudge to overcome the initial stickiness."
- **Obs 4:** "It was difficult to release the cart smoothly. My releases were **inconsistent**: sometimes I accidentally gave the cart a slight forward shove, and other times my finger dragged on it slightly."

2. The Data Processing Task

1. Calculation Method: The student calculated acceleration from the average time using the equation:

$$a = \frac{2s}{t^2}$$

2. The Processed Data: *You must plot the data below in Vernier Graphical Analysis.*

Hanging mass (kg)	Pulling force F_{pull} (N)	Average acceleration *a* ($m\ s^{-2}$)	Uncertainty Δa ($m\ s^{-2}$)
0.05	0.49	0.37	±0.10
0.10	0.98	0.91	±0.15
0.15	1.47	1.52	±0.12

Hanging mass (kg)	Pulling force F_{pull} (N)	Average acceleration a ($m\ s^{-2}$)	Uncertainty Δa ($m\ s^{-2}$)
0.20	1.96	2.03	±0.14
0.25	2.45	2.64	±0.18

Note: The uncertainty in average acceleration was calculated using half the range of acceleration values from the 5 trials.

Plotting Instructions:

1. Plot **Pulling force** on the x-axis and **Average acceleration** on the y-axis.
2. Add **Uncertainty Bars** for the acceleration data.
3. Add a **Linear Line of Best Fit**. Record the Slope and Y-intercept.
4. Manually add lines of **Max and Min Slope** through the uncertainty bars to determine the gradient and y-intercept uncertainties. If you've forgotten how to do this, review this tutorial.

Export your graph from Vernier Graphical Analysis and paste it here:

Model Conclusion (given)

The Evaluation picks up where the Conclusion stops, so here is the Conclusion for this data set, written the Part 3 way from the teacher's graph. Your best-fit gradient and intercept should be the same as these; your max/min lines, and so your uncertainties, will differ slightly.

Acceleration increased linearly with the pulling force over the range tested, 0.49 N to 2.45 N, but the relationship was not proportional, because the line of best fit did not pass through the origin. This is the trend Newton's second law predicts for a system of constant total mass: the larger the net force, the larger the acceleration, with $a = \frac{F_{pull}}{M_{total}}$, so a graph of a against F_{pull} should be a straight line through the origin with gradient $\frac{1}{M_{total}}$. The method was valid for the question, because transferring masses from the cart to the hanger kept the total mass constant, so the pulling force was the only variable changed. Since the model already predicts a straight line, no linearisation was needed, and the raw data graph is also the graph analysed below.

A straight line of best fit passed through all five uncertainty bars, and the points were scattered evenly on both sides of it, so the data are consistent with a linear relationship. The gradient of the best-fit line was 1.155 kg^{-1}; maximum and minimum gradient lines drawn through the uncertainty bars gave 1.31 kg^{-1} and 1.02 kg^{-1}, so the gradient was (1.16 ± 0.15) kg^{-1}. The y-intercept of the best-fit line was −0.204 m s^{-2}, with a range from the max/min lines of −0.41 m s^{-2} to −0.02 m s^{-2}. The model predicts a proportional relationship, so the test is whether this range includes zero. It does not, so proportionality was not supported: the intercept is negative. Compared with the measured accelerations, 0.37 m s^{-2} to 2.64 m s^{-2}, an intercept of −0.20 m s^{-2} is not negligible (it is more than half of the smallest acceleration), which points to a systematic error.

The only quantity in the theoretical gradient is the total mass, measured as (0.750 ± 0.001) kg, so the theoretical gradient is $\frac{1}{M_{total}} = (1.333 \pm 0.002)\ kg^{-1}$. Its uncertainty is so much smaller than the experimental one that the comparison is effectively between the experimental range and a single value. The theoretical gradient does not lie within the uncertainty range of my experimental gradient: it is 0.02 kg^{-1} above the upper limit of 1.31 kg^{-1}, about 0.07 of the width of the range (0.29 kg^{-1}). The percentage error in the gradient is $\frac{1.16 - 1.33}{1.33} \times 100\% = -13\%$.

The repeats were reliable at the larger forces, where half the range was about 7% of the mean acceleration (±0.14 m s^{-2} on 2.03 m s^{-2}, ±0.18 m s^{-2} on 2.64 m s^{-2}), but much less so at the smallest force, where it was 27% (±0.10 m s^{-2} on 0.37 m s^{-2}), which is where an inconsistent release has the largest effect on a short timing. The precision of the result was moderate: the percentage uncertainty in the gradient was 13% ($\frac{0.15}{1.16} \times 100\% = 13\%$), set by the size of the uncertainty bars, which happens to be the same size as the percentage error (a coincidence of this data set: the two are different quantities, and there is no reason for them to be equal); the scatter about the line was small (r = 0.9997, which supports the small scatter but says nothing about accuracy). The accuracy was no better than the precision: the percentage error of 13% is the same size as the percentage uncertainty, and the theoretical value lies just outside the max/min gradient range. Both shifts were in the same direction: the gradient was lower than the theoretical gradient and the intercept was below the expected value of zero, so the measured accelerations were consistently lower than the model predicts.

Because the theoretical gradient lies outside the max/min range, the data did not support the model as derived, although only marginally (the theoretical gradient is 0.02 kg^{-1} beyond the upper limit of the max/min gradient range, less than a tenth of its width, and the y-intercept range of −0.41 m s^{-2} to −0.02 m s^{-2} misses zero by 0.02 m s^{-2}, about a twentieth of its width). Neither gap on its own would be convincing; what points to a real effect is that both miss in the same direction, towards lower accelerations. This could be due to either a systematic error in the method or an invalid assumption in the model, such as neglecting friction.

3. The Evaluation (Analysis)

Part A: Random Errors (The Scatter)

*Look at the **uncertainty bars** and the scatter of points around your line.*

1. **Source 1 (Apparatus):** The student noted "rattling" (Obs 1). Explain how loose mechanical contact causes vibrations that lead to random fluctuations in the time measured.
2. **Improvement:** Suggest a specific equipment change or modification to the wheels/track to stop the rattling.
3. **Source 2 (Method):** The student noted "inconsistent releases" (Obs 4). Explain how varying the initial velocity introduces random noise into the data.
4. **Improvement:** Suggest a specific equipment change to remove the inconsistency in the release.

Classify: Before you write, state whether each of the two sources is a weakness or a limitation of the methodology (see Part 4 of the style guide).

Draft your evaluation of Random Errors here:

Part B: Systematic Error 1 (The Intercept)

Look at your Y-intercept. Theory predicts it should be zero.

1. **Observation:** Record your y-intercept value. Is it positive or negative?
2. **Diagnosis:** Use **Observation 3** (the "nudge") to explain this. What force opposes motion?
3. **Graph Impact:** Explain mathematically why subtracting a constant force shifts the line **vertically** but keeps it parallel (constant slope).
4. **Improvement:** Suggest how the resistive force could be measured, or built into the model, so that it no longer shifts the intercept.

Classify: Before you write, state whether neglecting friction in the model is a weakness or a limitation of the methodology.

Draft your evaluation of the Intercept here:

Part C: Systematic Error 2 (The Slope)

Compare your Best Fit Slope to the Theoretical Slope (1.33 kg^{-1}).

1. **Observation:** Is your slope steeper or shallower than expected?
2. **Diagnosis:** Use **Observation 2** (The Angled String). The string pulls at an angle θ.
3. **The Physics:** Only the **horizontal component** ($F_{pull}\ cos\ \theta$) accelerates the cart.
4. **The Logic:** Although the angle changes slightly during a run, the cart travels the **same fixed path** (50 cm) every time. Therefore, the **average angle** is consistent for every trial.
5. **Graph Impact:** Explain why this acts as a "Scaling Error" (multiplying by a percentage) rather than an Intercept Shift.
6. **Improvement:** Suggest a change to the setup that would make the string horizontal.

Classify: Before you write, state whether the angled string is a weakness or a limitation of the methodology, and why it produced a systematic rather than a random error.

Draft your evaluation of the Slope here:

<u>Extension</u>

The Scenario:

In this experiment, the student kept the total mass constant by transferring masses.

Imagine a different student performed the experiment by keeping the 0.5 kg cart empty and simply **adding new masses** from the table to the hanger to increase the force.

The Physics Flaw:

In this second method, the **Total Mass** of the system is not constant. It increases every time a mass is added to the hanger. (In the InThinking "Mass of a cart" lab we did in class, the same simplification was made on purpose: the tension was taken as $m_{hanger}g$ and the hanging mass was left out of the accelerating mass. There it was small enough to ignore; here it is not, so the model has to include it.)

The Task:

1. Write Newton's Second Law for the whole system: $F_{net} = M_{total}a$
2. Substitute the specific masses (M_{cart} and m_{hanger}) and force ($m_{hanger}g$).
3. Rearrange to get an equation for *a* in terms of m_{hanger}.
4. Using this equation, explain mathematically why the graph of *a* vs. F_{pull} would be a curve, not a straight line.

Write your derivation and explanation here: