ECT Lesson Plan: Area of a Circle

Lesson plan at a glance...

Core subject(s)	Mathematics
Subject area(s)	Geometry
Suggested age	13 to 17 years old
Prerequisites	Basic Python (Introduction to Python); Calculating area of rectangle and parallelogram; Perpendicular vs slanted height; Circumferences and percent differences
Time	**Preparation:** 27 to 37 minutes **Instruction:** 75 minutes
Standards	**Core Subject:** CCSS Math **CS:** CSTA

In this lesson plan…

Lesson Overview

Students will learn to derive the formula for the area of a circle, relate a circle to a parallelogram and rectangle, and understand the concept of infinity as they imagine slicing an circle into infinitesimally small slices, a concept key in higher mathematics. Students will improve their computational thinking by **recognizing patterns** in circles that are sliced into increasingly smaller pieces and rearranged into a parallelogram-like shape, **generalizing the relationship** between the circumference of the circle and the base of the parallelogram-like figure, and **developing an algorithm** for calculating the area of a circle.

Materials and Equipment

- For the teacher:
  - *Required:* Colored pencils/markers, or provide students with pre-shaded images
  - *Recommended:* Presentation set-up
    - Internet-connected computer
      - Chrome browser (https://www.google.com/chrome/browser/desktop) recommended
    - Projector and projection screen or other flat projection surface
- For the student:
  - *Required:* Internet-connected computers (1 computer per student recommended)
  - *Required:* Software Development Environment
    - Python 2.x (https://www.python.org/) OR a web-based Integrated Development Environment (IDE) such as Trinket (https://trinket.io/)

Preparation Tasks

	Confirm that all computers are turned on and logged-in	1 to 3 minutes
	Confirm that your projector is turned on and is projecting properly	1 to 4 minutes
	Install Python 2.x (https://www.python.org/downloads) or navigate to Trinket (https://trinket.io/)	5 to 10 minutes
	Pre-load Python Starter Files (or copy and paste in the code)	20 minutes

The Lesson

Warm-up Activity: Exploring the relationship between a circle and parallelogram	5 minutes
Activity 1: Deriving the area of a circle	10 minutes
Activity 2: Calculating the area of a circle	15 minutes
Activity 3: Programming Python to calculate the area of a circle	20 minutes
Wrap-up Activity: Building an algorithm for the formula of a circle	20 minutes

Warm-up Activity: Exploring the relationship between a circle and a parallelogram
(5 minutes)

Activity Overview: In this activity, students will look for patterns in the shape of an outstretched circle as it is cut into increasingly smaller pieces. Students will use the CT concepts of pattern recognition and decomposition.

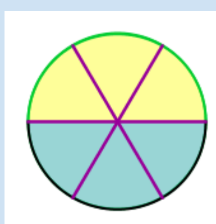
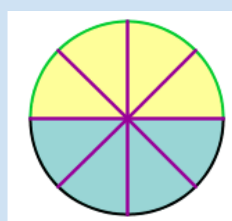
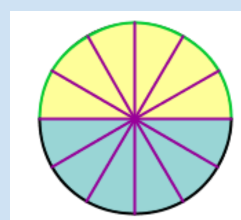
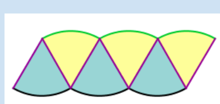
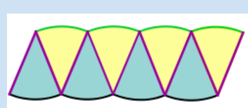
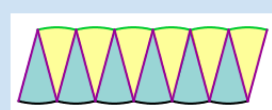
Activity:
Have students work through the following activity.

Below are three congruent circles that are sliced into six, eight, and twelve slices, respectively, and then unraveled and rearranged to form a new shape.

Q1: Describe any patterns you notice in the unraveled circles as the slices become smaller and smaller.

Q2: As the circle is cut into more and more slices, what shape does it begin to resemble?

Q3: Do you think it is possible to cut the circle into so many tiny pieces that when we unravel it, it becomes a perfect rectangle with four 90 degree angles and four straight edges? Explain why or why not.

	Six Slices	Eight Slices	Twelve Slices
Circle			
Unraveled Circle			

Notes to the Teacher:
After students have answered the questions, project the below chart onto the board and discuss the similarities and differences between the outstretched circles and the polygons that they resemble. Have students examine the "curvature" of each figure in the chart and determine which of the three outstretched figures' area is most similar to the area of a parallelogram.

Assessment:
A1: Answers may vary, however students should notice that as the slices become smaller, bases of the unraveled circle become flatter and flatter.

A2: A parallelogram or a rectangle.

A3: Answers may vary; the explanations are more important than the actual answers.

Activity 1: Deriving the area of a circle (10 minutes)

Activity Overview: In this activity, students will look for patterns between the circumference of a circle and the length of the base of the unravelled circle. They should also look for trends in the percent change between the slanted height and perpendicular height as the number or slices increases. They will use pattern recognition and decomposition to find patterns in the circumference of a circle.

Notes to the Teacher:
After students answer the questions, project the chart onto the board and discuss the concept of infinite slices. Have students consider what the figure would look like if we cut the circle into infinitesimally small pieces. Discuss the concept of a “theoretical activity” vs. a “realistic activity”, and the idea that although many ideas in higher-level mathematics are based on theory, they actually work out in reality.

Point out that in spite of the fact that our figures appear to be an approximation of a rectangle, the formula A = pi*r * r, or A = pi*r^2, is not an approximation; it is exact!

Activity:
Have students work through the following activity.

1. **The figures below illustrate the relationship between the perpendicular height of the unraveled figures and the radii of the circles.**

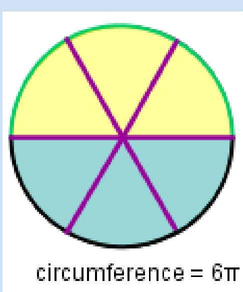
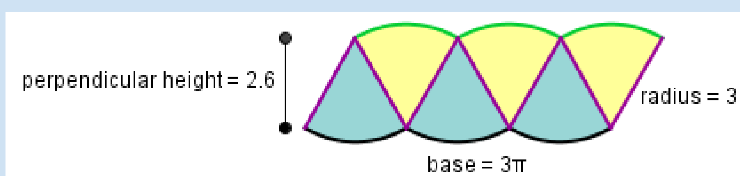
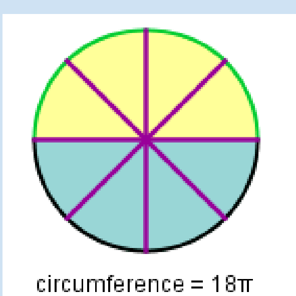
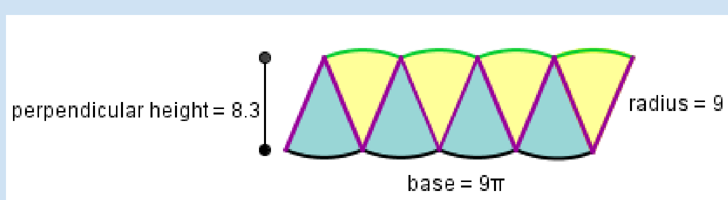
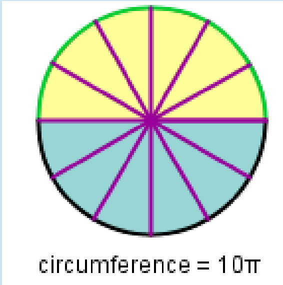
Figure 1		
Figure 2		

Figure 3



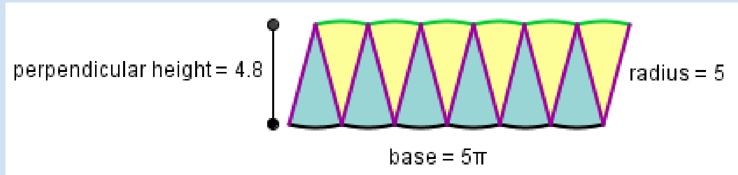
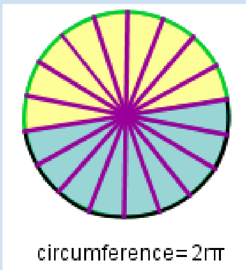
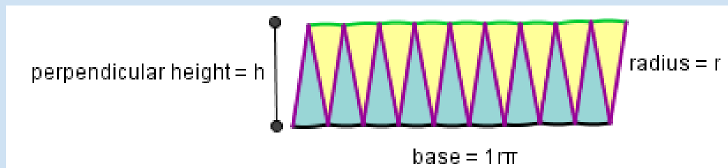


Figure 4





2. **Type the following program into the Python editor and run it to calculate the percent difference between the perpendicular height and the radii of all four circles. Record the results in the table.**

```
radius = input('Enter the radius: ')
height = float(input('Enter the perpendicular height: '))

percent_difference = 100 * (radius - height) / radius

print 'Percent difference:', percent_difference, '%'
```

	Figure 1	Figure 2	Figure 3	Figure 4
Circumference				
Length of Base				
Radius				
Perpendicular Height				
Percent Difference between Radius and Height				

Teaching Tips:
Although students do not contribute to the writing of the program above, take a minute to explain the variables and their values. Make sure students understand that whatever value is entered when asked for the radius will be stored in the variable `radius`, and whatever number is entered when asked for the perpendicular height will be stored in `height`. They will need to understand this concept in order to complete the programs in the following activities.

- Instruct students to keep their answers in terms of pi in order to identify patterns in the chart.
- The answers are provided below.

	Figure 1	Figure 2	Figure 3	Figure 4
Circumference	6*pi	18*pi	10*pi	2*r*pi
Length of Base	3*pi	9*pi	5*pi	1*r*pi
Radius	3	9	5	r
Perpendicular Height	2.6	8.3	4.8	h
Percent Difference between Radius and Height	13.33%	7.78%	4%	100*(r - h)/r *(Hint: Try it yourself; the answer should be in terms of h and r.)*

Activity:
Have students answer the following questions.

Q1: Describe any pattern you see between the length of the circle's circumference and the base of the unraveled circle.

Q2: Compare the radius to the perpendicular height of each circle. Which one is longer and why?

Q3: Does the percent difference between the radius and the perpendicular height increase or decrease as we use more slices?

Q4: Imagine slicing the circle into 10,000 pieces and unraveling them. Based on the patterns you see in the figures above, would the percent change between the perpendicular height and the radius be really big or really small? Could we use the radius as an approximation of the perpendicular height? Why or why not?

Q5: Do you think we could have slices so small that the radius equals the perpendicular height?

Q6: The base of the outstretched circle is half of the circumference and the height is approximately equal to the radius. Recall, circumference equals 2(pi)(r). Using this information, write a formula for the area of the outstretched circle. The radius, r, should be the only variable in your answer.

Assessment:
A1: The length of the base is half of the circumference.

A2: The radius is always longer because it stretches between the same two bases at a slant, whereas the perpendicular height does not slant.

A3: The percent difference decreases as the length of the slant height approaches the length of the perpendicular height.

A4: Based on the patterns in the chart above, the percent difference would be very small, so yes, it would be a good approximation of the perpendicular height.

A5: Answers may vary; the explanations are more important than the answers.

A6: Area = pi*r*r

Activity 2: Calculating the area of a circle (15 minutes)

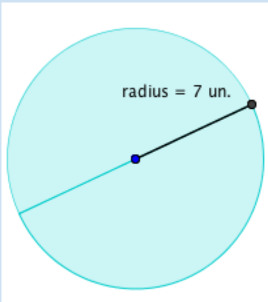
Activity Overview: In this activity, students will use the formula A = pi*radius^2 to complete the following code. Students may use algorithm design to create a procedure to calculate the area of a circle.

Student Activity:
Give the following code to students to use to calculate the area of a circle and instruct them as follows.

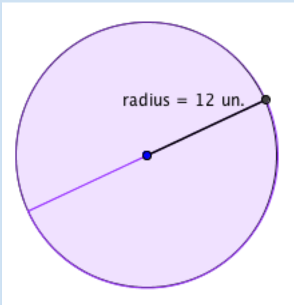
1. **Use the code below to calculate the area of a circle.**

```
r = input('Enter the radius: ')
area = 3.14 * r**2
print 'Area:', area, 'square units'
```

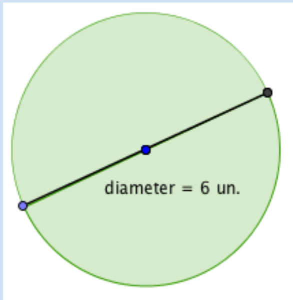
2. **Run the code to calculate the areas of the following three circles, to the nearest *ten thousandth*.**



Area = 153.938 square units



Area = 452.389 square units

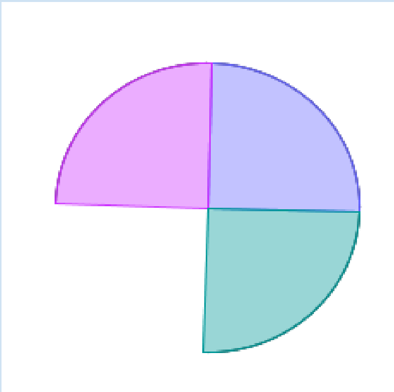
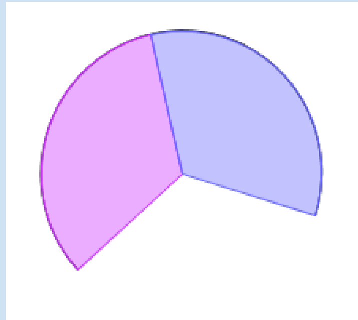
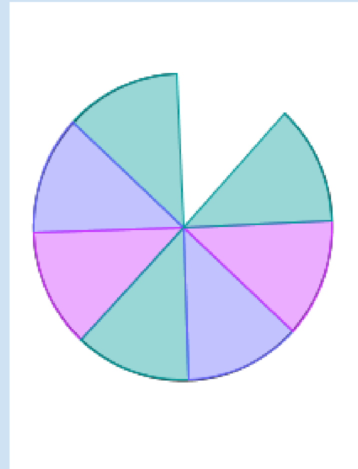


Area = 28.274 square units

3. **Notice that each of the circles below is missing a piece. In the chart below, explain how you would calculate the area of each circle with a missing piece (sector):**

Chart D

Radius = 7 cm.	Radius = 9 cm.	Radius = 12 cm.

		
Possible answer: Find the area of the whole circle, divide it by 4 to get the area of one sector, and then multiply the result by 3.	Possible answer: Find the area of the whole circle, divide it by three to get the area of one sector, and subtract the area of the missing sector from the area of the whole circle.	Possible answer: Find the area of the whole circle, divide it by 8 to get the area of one sector, and then multiply the result by 7.

Activity 3: Programming Python to calculate the area of a circle (20 minutes)

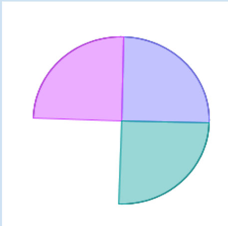
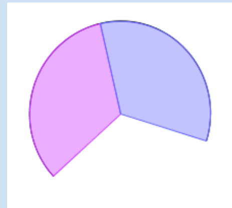
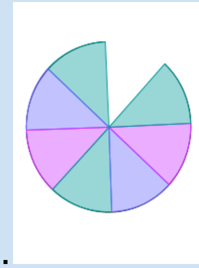
Activity Overview: In this activity, students will develop one of two common algorithms for finding the area of a circle with a sector removed: (1) Calculate the area of the missing sector and subtract it from the total area, or (2) Multiply the fraction of the circle remaining by the entire area. In the second half of this activity, they will generalize this algorithm so that the code will calculate the area of a partial circle with any size sector removed. Students may use **algorithm design** to design a procedure to calculate the area of a circle.

Notes to the Teacher:
Students can translate the algorithms they wrote in the previous activity to Python code. Present them with the same three partial circles, and ask them to fill in each blank with a formula that Python can follow to calculate the partial circle's area.

There is more than one correct way to solve these problems. The solutions below represent one of several correct answers.

Activity:
Have students work through the following activity.

1. **Use the method you described for each partial circle in the last activity to complete the Python programs below. Then run each program to calculate the area of each partial circle.**

Radius = 7 cm.	Radius = 9 cm.	Radius = 12 cm.
		

`r = input('Radius: ')`
 	
`pi = 3.14`
`a = 3 * (pi * r**2) / 4`
 	
`print 'Area:', a, 'sq cm'`
	`r = input('Radius: ')`
	
	`pi = 3.14`
	`a = 2 * (pi * r**2) / 3`
	
	`print 'Area:', a, 'sq cm'`
		`r = input('Radius: ')`
		
		`pi = 3.14`
		`a = 7 * (pi * r**2) / 8`
		
		`print 'Area:', a, 'sq cm'`

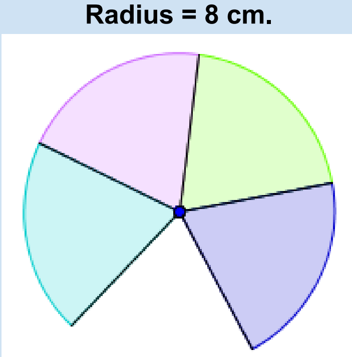
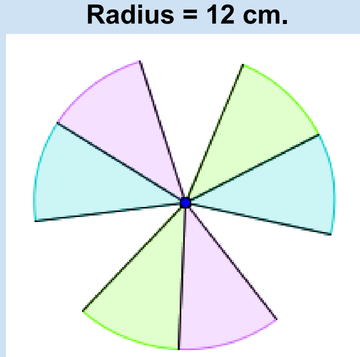
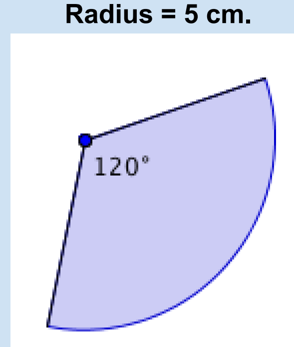
`Area: 115.454 square cm`	`Area: 169.646 square cm`	`Area: 395.841 square cm`

```
r = float(input('Enter the radius: '))
total_slices = float(input('Enter the total number of slices: '))
remaining_slices = input('Enter the number of remaining slices: ')

one_slice = (pi * r**2) / total_slices
area = one_slice * remaining_slices

print 'Area remaining: ', area, 'sq cm'
```

2. **Use the program above to calculate the area of the following partial circles** (with answers below)**.**

Radius = 8 cm.	Radius = 12 cm.	Radius = 5 cm.
		
Area remaining: 160.850 sq cm	Area remaining: 301.593 sq cm	Area remaining: 26.180 sq cm

Q1: It is repetitive to change the program for each different sized slice. In this last program, Python asks for the radius, the total number of slices, and the number of slices remaining. In your own words, explain how it will calculate the remaining area of any circle with any number of congruent slices removed.

Assessment:
A1: Answers may vary, students should understand that in this algorithm, Python calculates the area of each slice and then multiplies that area by the number of slices remaining in the picture.

Wrap-up Activity: Building an algorithm for the formula of a circle (20 minutes)

Activity Overview: In this activity, students will be assessed on their understanding of the derivation of the area of a circle from the area of a parallelogram. They should describe the process for coming up with the formula for the area of a circle using **algorithm design**.

Activity:
The final question serves as an evaluation of students' understanding of the derivation of the area of a circle from the area of a parallelogram. Emphasize that students are not being asked to write down the formula for the area of a circle, rather they should describe the process for coming up with that formula.

Q: Imagine that you were living in the 1600s. Back then, people knew how to calculate the area of rectangles and parallelograms, however no one had figured out exactly how calculate the area of a circle. Write an algorithm that people from that time could follow to derive the formula for the area of a circle.

Learning Objectives and Standards

Learning Objectives	Standards
LO1: Find arc lengths and areas of sectors of circles.	*Common Core* CCSS MATH.CONTENT.HSG.C.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. *Computer Science* CSTA L2.CT14: Examine connections between elements of mathematics and computer science including binary numbers, logic, sets and functions.

Additional Information and Resources

Lesson Vocabulary

Term	Definition	For Additional Information
Parallelogram	A (non self-intersecting) quadrilateral with two pairs of parallel sides.	http://en.wikipedia.org/wiki/Parallelogram
Radius	The length of a line segment from a circle's center to its perimeter.	http://en.wikipedia.org/wiki/Radius
Circumference	The linear distance around the edge of a closed curve or circular object.	http://en.wikipedia.org/wiki/Circumference

Computational Thinking Concepts

Concept	Definition
Pattern Recognition	The ability to find commonality in form, format, structure, method or plan.
Decomposition	To break down or regroup data to facilitate analysis.
Algorithm Development	Building a reusable description of generalized behavior.
Data Representation	Depicting and organizing data in appropriate graphs, charts, words or images

Administrative Details

Contact info	For more info about Exploring Computational Thinking (ECT), visit the ECT website (g.co/exploringCT)
Credits	Developed by the Exploring Computational Thinking team at Google and reviewed by K-12 educators from around the world.
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