Tensors and arithmetic circuits, from complexity theory to machine learning

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Arithmetic circuits provide a natural model for the computation of polynomials. These circuits are made of additions and multiplication gates, instead of the Boolean gates which form the building blocks of Boolean circuits. The arithmetic circuit model gives rise to an algebraic version of classical (Boolean) complexity theory, with specific results and proof techniques. In particular,

1) One can define analogues of the main complexity classes such as P and NP, and prove completeness results for the determinant and permanent.
2) One can prove lower bounds on the circuit size of certain polynomials (obtaining a superpolynomial lower bound would be an analogue of "P<>NP" in the Boolean world).
3) One can design algorithms that take arithmetic circuits as inputs. For instance, one can divide, factorize and differentiate arithmetic circuits. Efficient differentiation is a key element in gradient-based methods for "deep learning".

These 3 topics will be covered in this course.

Tensors can be viewed as multidimensional arrays. In 2 dimensions these are just the matrices that are well known from linear algebra, but tensors can also be three-dimensional, four-dimensional...They have become important in diverse areas such as algebraic complexity (in particular, for the study of matrix multiplication), machine learning and signal processing. They are naturally connected to arithmetic circuits since a tensor can be viewed as the array of coefficients of a multivariate polynomial; moreover, the key notion of "tensor rank" is nothing but the arithmetic circuit size of the corresponding polynomial (for a very simple class of arithmetic circuits). In applications, the main algorithmic task is often to compute the corresponding arithmetic circuit from an input tensor. This is called "tensor decomposition", or "arithmetic circuit reconstruction" in the language of arithmetic circuits.

In this part of the course we will cover basic notions on tensors, prove lower bounds on tensor rank, study tensor decomposition algorithms and some of their applications.

Bibliographic References:

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