Summer Math Practice

Review of Math 8

West Rocks School is pleased to provide you with a variety of suggested summer math activities, designed for students to practice and review skills that are necessary to be ready for the next grade level in math. These activities are not a school requirement, but are offered as a way to help students who want to ensure they have a solid grasp of the math concepts they learned this school year.

Enclosed you will find a variety of skill-based math problems that review the main concepts from your child's math course this past school year, along with an answer key. Each concept has been paired with sections from IXL that students can use for additional practice. As an added resource, IXL provides a worked out solution for every problem with explanations to aid students in correcting their mistakes.

IXL: https://www.ixl.com/signin/norwalkps or go directly to https://www.ixl.com/math/grade-8

> *The username for IXL is the same as the students' username for school with @norwalkps added to the end. The password for IXL is the students' ID number, which is also their lunch pin number.*

A wealth of additional resources and activities can be found on the West Rocks Mathematics website, which is located on the West Rocks website under *Academics–Summer Learning-Mathematics*. You do not need to be logged into the school website to have access to these resources.

Volume and Surface Area of Solids

For this review, *r* represents radius and *h* represents height. You may use a calculator and use 3.14 as an approximation for π. Round your answers to the nearest tenth unless otherwise stated.

1 Find the volume of each cylinder to the nearest unit. Use the given dimensions.

a) $r = 4.2$ inches; $h = 14$ inches

b) $r = 7$ centimeters; $h = 12$ centimeters

2 Find the volume of each cone to the nearest unit. Use the given dimensions.

a) $r = 3$ centimeters; $h = 8$ centimeters

b) $r = 8$ inches; $h = 15$ inches

3 Find the volume of each pyramid. Use the given dimensions.

a) A square base with edge length of 6 centimeters; $h = 4$ centimeters.

b) A rectangular base with length of 6 inches and width = 3.3 inches; $h = 7$ inches.

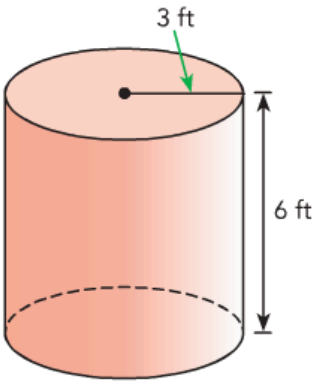
4 Find the volume of each sphere to the nearest unit. Use the given dimensions.

a) $r = 9.6$ centimeters

b) $d = 26$ centimeters

5 Find the exact surface area of each solid.

a)



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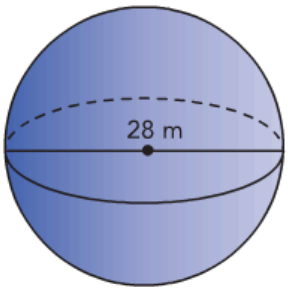

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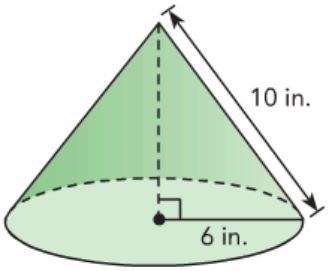


6 Find the volume and surface area of each solid. Round to the nearest tenth.

a) A solid cone with a diameter of 5 feet, a slant height of 7 feet, and a height of 6.5 feet.

b) A sphere with a radius of 28 millimeters.

c) A solid cylinder with a radius of 1.4 inches and a height of 4.2 inches.

Solve. Show your work.

7 The volume of a cone is 450 cubic centimeters and the radius of the base is 5 centimeters. What is the height of the cone to the nearest tenth?

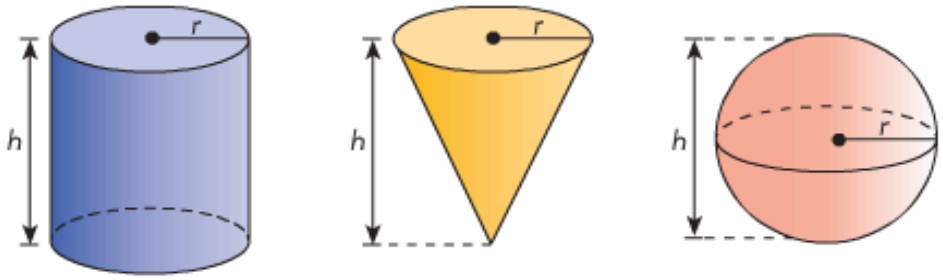
8 The surface area of a sphere is 498.96 square centimeters. What is the radius of the sphere to the nearest tenth?

9 A cone with a height of 6 inches and a slant height of 7.5 inches has a lateral surface with an area of approximately 106 square inches.

a) What is the radius? Round to the nearest tenth.

b) What is the volume of the cone? Round to the nearest tenth.

10 A cylinder, a cone, and a sphere are shown below. Each solid has a radius of 1 inch and a height of 2 inches. Which of them has the greatest volume? Justify your answer.



IXL Strands:
Level J – Geometry – Skills Q.14 - Q.16 and Q.24 - Q.30

Find the range, the three quartiles, and the interquartile range.

1. 2, 4, 1, 7, 3, 3, 9, 10, 1, 0, 6, 8, 5, 5, 9

2. 34, 66, 90, 25, 46, 81, 40, 67, 95, 104, 36, 49

3. 1.23, 1.45, 1.09, 1.78, 1.55, 1.67, 1.37, 1.05, 1.23, 1.11

4. 162.5, 248.6, 130.7, 344.9, 322.0, 234.2, 150.8, 304.7, 326.4

Use the information below to answer the following.

Tara tossed two number dice 24 times. She found the sum of the values for each throw and displayed the sums in a dot plot.

Sum of Two Number Dice

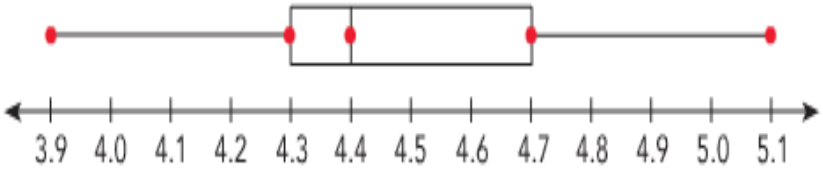
5. Find the range of the data.

6. Find the 3 quartiles of the data.

7. Find the interquartile range.

Refer to the box plot to answer the following.

The box plot summarizes the heights of bean sprouts, in centimeters.



Heights (in centimeters) of Bean Sprouts

10. Find the lower quartile, the median, and the upper quartile.

11. Calculate the range and the interquartile range.

Solve. Show your work.

8 The map shows the maximum temperature, in degrees Fahrenheit, recorded in 20 cities across the United States in a certain year. Display the data in a stem-and-leaf plot.

9 The table shows the weights of Labrador dogs, in pounds.

72	73	79	68	101
88	78	71	85	94
93	77	98	95	75
81	56	51	62	70

a) Draw a stem-and-leaf plot for the data.

b) How many Labrador dogs are there?

c) What is the range?

d) What is the mode of the data?

e) What is the median weight?

Probability

Solve.

1. You select a card at random from 50 cards numbered from 1 to 50. What are the possible outcomes for the event of choosing a number that is a multiple of 6?

2. Three fair coins are tossed together once. List the outcomes that are favorable for the event of only two of the coins landing on heads.

3. Daniel wants to write all the 2-digit numbers with no repeating digits that can be formed using the digits 5, 6, and 7.

   a) List all the possible outcomes.

   b) *X* is the event that the 2-digit number is divisible by 5. How many of the outcomes are favorable to event *X*?

4. Amy writes a computer program that will choose two letters from her own name to make a two-letter "string." The order of the letters matters. For example, AM and MA are different strings.

   a) List all the possible outcomes for forming a two-letter string.

   b) What is the probability that Amy forms a two-letter string with the letter M in it?

5. Two-digit numbers are formed using digits 2, 3, and 4, with no repeating digits.

   a) List all the possible outcomes.

   b) What is the probability of forming a number greater than 32?

6. Tim has three DVDs. One is a science fiction movie, one is an action movie, and the other is a documentary. If he stacks the DVDs randomly, what is the probability that the science fiction movie is on top, the action movie is in the middle, and the documentary is on the bottom?

7 A ribbon is selected at random out of a total of 4 orange ribbons, 5 yellow ribbons, and 3 red ribbons. What is the probability of selecting an orange ribbon?

8 Use the spinner shown.

a) What is the probability of landing on an even number?

b) What is the probability of landing on a number less than 4?

Solve.

9 Olivia and Jackie played a game with the spinner shown.

Olivia spun a 2 on 12 spins out of 50, while Jackie spun a 2 on 19 spins out of 100.

a) Find each person's experimental probability of spinning a 2. Express your answers as decimals.

b) Suppose the spinner is fair, meaning that it is equally likely to land on any of the numbers. What is the theoretical probability of spinning a 2?

c) Assuming the spinner is fair, what do you predict will happen to the experimental probability of getting a 2 if the spinner is spun 500 times?

IXL Strands:
Level J – Probability – Skills BB.1 - BB.4

Exponent

Identify the base and exponent in each expression.

1. $\left(-\frac{1}{5}\right)^{-3}$
2. -0.92^4

Tell whether each statement is correct. If it is incorrect, state the reason.

3. $-0.7^3 = -0.7 \cdot 0.7 \cdot 0.7$
4. $5^{-4} = (-5) \cdot (-5) \cdot (-5) \cdot (-5)$

Write in exponential notation.

5. $2 \cdot 2 \cdot 2 \cdot 2$
6. $4.8 \cdot 4.8$
7. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
8. $c \cdot c \cdot c \cdot c \cdot c \cdot c$
9. $\frac{3}{4}k \cdot \frac{3}{4}k \cdot \frac{3}{4}k \cdot \frac{3}{4}k$
10. $(-1.2)(-1.2)(-1.2)(-1.2)$

Write the prime factorization of each number in exponential notation.

11. 3,780
12. 27,720

Expand and evaluate each expressions.

13. $(-6)^2$
14. 1.1^2
15. 10^5
16. $\left(\frac{2}{3}\right)^3$

Simplify each expression. Write your answer using a positive exponent.

17. $(-3)^{-1} \cdot (-3)^0$
18. $\left(\frac{5}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^3$
19. $5m^3n^4 \cdot 4m^5n^2$
20. $\left(\frac{7}{8}\right) \div \left(\frac{7}{8}\right)^3$

21 $(-h)^9 \div (-h)^{15}$

22 $x^8z^5 \div x^3z^9$

23 $25p^6q^9 \div 45p^8q^4$

24 $\left[\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^{-1}\right]^3$

25 $40c^5d^3 \div 10c^9d^2$

26 $\left(\frac{72b^{-1}}{32c^{-1}}\right)^{-2}$

27 $\frac{(9^{-2})^{-2} \cdot 2^2}{9^2}$

28 $\frac{6^8 \cdot 56^{-3}}{6^5 \cdot 7^{-3}}$

29 $\frac{42^{-1}}{(2^0)^{12} \cdot 21^{-1}}$

30 $\frac{(3^5 \cdot 3^4)^2}{(3^3)^6}$

Solve each equation involving a variable that is squared.

31 $r^2 = 256$

32 $c^2 = \frac{121}{169}$

Solve each equation involving a variable that is cubed.

33 $x^3 = 32.768$

34 $t^3 = -\frac{27}{343}$

35 The expanded form of a number is $5 \cdot 10^1 + 8 \cdot 10^0 + 1 \cdot 10^{-1} + 9 \cdot 10^{-2}$. What is this number in standard form?

IXL Strands:
Level J – Exponents and roots – Skills F.1 - F.19

Scientific Notation

Tell whether each number is written correctly in scientific notation. If incorrectly written, state the reason.

1. $10 \cdot 10^2$
2. $0.99 \cdot 10^{12}$
3. $1.4 \cdot 10^2$
4. $0.4 \cdot 10^{25}$

Write each number in scientific notation.

5. 714,000
6. 0.00087

Write each number in standard form.

7. $3.46 \cdot 10^2$
8. $5.4 \cdot 10^4$

Identify the greater number in each pair of numbers.

9. $7.8 \cdot 10^{-5}$ and $5.4 \cdot 10^{-7}$
10. $1.4 \cdot 10^{-5}$ and $6 \cdot 10^{-4}$
11. $6.5 \cdot 10^{-15}$ and $9.3 \cdot 10^{-12}$
12. $3.5 \cdot 10^{-2}$ and $4 \cdot 10^{-3}$

Evaluate. Write your answer in scientific notation. Round the coefficient to the nearest tenth.

13. $2.44 \cdot 10^3 + 1.9 \cdot 10^5$
14. $3.12 \cdot 10^{-3} - 3 \cdot 10^{-3}$
15. $2.4 \cdot 10^{-2} \cdot 5 \cdot 10^{-1}$
16. $3.2 \cdot 10^8 \div (1.6 \cdot 10^4)$

IXL Strands:
Level J – Scientific notation – Skills G.1 - G.4

Linear Equations

Solve each linear equation. Show your work.

1. $2(x - 5) - 8 = 20$
2. $2x - (5 - x) = \frac{5}{2}$
3. $\frac{1}{4}(x + 2) - 2 = 0.5$
4. $4x - \frac{5 - 2x}{5} = \frac{3}{5}$

Write each repeating decimal as a fraction. Show your work.

5. $0.\overline{2}$
6. $0.9\overline{3}$
7. $0.2\overline{6}$
8. $0.3\overline{16}$

Tell whether each equation has one solution, no solution, or an infinite number of solutions. Show your work.

9. $2x + 4 = -2\left(\frac{1}{2} - x\right)$
10. $6y + (16 - 2y) = 4(4 + y)$
11. $4x + 5 = 2x - 7$
12. $2x + 5 = -4\left(-\frac{5}{4} - \frac{1}{2}x\right)$

Find the value of y when $x = 4$.

13. $4x - 2 = y + 5$
14. $x - 4y = 2$
15. $y - x = \frac{1}{3}(x + 14)$
16. $\frac{3x - 7}{y} = \frac{1}{3}$
17. $\frac{1}{7}(3x + y) = x$
18. $\frac{3y + 1}{4} = 2x$

Express x in terms of y. Find the value of x when $y = -2$.

19 $4x + y = 2(x + 3y)$

20 $3(x - 2y) = 4x + 5y$

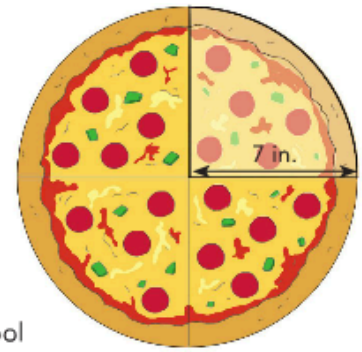
21 $\frac{1}{3}x + \frac{5}{6}y = 2$

22 $\frac{0.5(x - 3)}{y} = 10$

23 $0.25(x + y) = 15 - x$

24 $\frac{y}{2} - \frac{2x + y}{5} = 7$

Solve. Show your work.



25 A circular pizza with a radius of 7 inches is cut into four quadrants. The perimeter Q of each quadrant can be found using the formula $Q = d\left(1 + \frac{\pi}{4}\right)$, where d is the diameter of the pizza. Find the perimeter of each quadrant of the pizza. Use $\frac{22}{7}$ as an approximation for π.

26 Some students painted a design on the wall of the cafeteria using the school colors. The middle section of the design is 4.2 feet tall, and is painted white. The top section is red, and the bottom section is blue. The ratio of the height of the blue section to the height of the red section is 1 : 2. The total height of the design is 10.5 feet. Find the height of the red section of the design.

27 In a grocery store, each pound of green beans costs one and a quarter times the price of each pound of potatoes. Mrs. Gomez bought 4 pounds of green beans and 5 pounds of potatoes. Miss Jacobs bought 10 pounds of potatoes. They paid the same amount.

a) Write a linear equation to find the cost of each pound of potatoes, p dollars.

b) Tell whether the equation has one solution, is inconsistent, or is an identity. Explain your reasoning.

IXL Strands:
Level J – Variable expressions – Skills T.1 - T.7
Level J – Single-variable equations – Skills U.1 - U.9

Additional IXL Topics

These are other topics that were either touched on or reviewed in Math 8 that students may find helpful to review to improve their math skills before starting 9th grade.

All strands are from Level J

Number theory – Skills A.1 - A.8

Integers – Skills B.1 - B.5

Operations with integers – Skills C.1 - C.9

Rational numbers – Skills D.1 - D.8

Operations with rational numbers – Skills E.1 - E.10

Ratios and proportions – Skills H.1 - H.9

Percents – Skills J.1 - J.7

Pythagorean Theorem – Skills O.1 - O.3

Answer Key

Volume and Surface Area of Solids

1. a) Volume of cylinder:
$\pi r^2 h \sim 3.14 \cdot 4.2 \cdot 4.2 \cdot 14$
$\sim 775 \text{ in}^3$

b) Volume of cylinder:
$\pi r^2 h \sim 3.14 \cdot 7 \cdot 7 \cdot 12$
$= 1{,}846.32$
$\sim 1{,}846 \text{ cm}^3$

2. a) Volume of cone:
$\frac{1}{3} \pi r^2 h \sim \frac{1}{3} \cdot 3.14 \cdot 3 \cdot 3 \cdot 8$
$= 75.36$
$\sim 75 \text{ cm}^3$

b) Volume of cone:
$\frac{1}{3} \pi r^2 h \sim \frac{1}{3} \cdot 3.14 \cdot 8 \cdot 8 \cdot 15$
$= 1{,}004.8$
$\sim 1{,}005 \text{ in}^3$

3. a) Volume of pyramid:
$\frac{1}{3} Bh = \frac{1}{3} \cdot (6 \cdot 6) \cdot 4$
$= 48 \text{ cm}^3$

b) Volume of pyramid:
$\frac{1}{3} Bh = \frac{1}{3} \cdot (6 \cdot 3.3) \cdot 7$
$= 46.2 \text{ in}^3$

4. a) Volume of sphere:
$\frac{4}{3} \pi r^3 \sim \frac{4}{3} \cdot 3.14 \cdot 9.6 \cdot 9.6 \cdot 9.6$
$\sim 3{,}704 \text{ cm}^3$

b) Radius = Diameter ÷ 2
$= 26 \div 2$
$= 13 \text{ cm}$
Volume of sphere:
$\frac{4}{3} \pi r^3 \sim \frac{4}{3} \cdot 3.14 \cdot 13 \cdot 13 \cdot 13$
$\sim 9{,}198 \text{ cm}^3$

5. a) Surface area of cylinder:
$2\pi rh + 2\pi r^2 = (2 \cdot \pi \cdot 3 \cdot 6) + (2 \cdot \pi \cdot 3 \cdot 3)$
$= 54\pi \text{ ft}^2$

b) Radius = Diameter ÷ 2
$= 28 \div 2$
$= 14 \text{ m}$
Surface area of sphere:
$4\pi r^2 = 4 \cdot \pi \cdot 14 \cdot 14$
$= 784\pi \text{ m}^2$

c) Surface area of cone:
$\pi r^2 + \pi r l = (\pi \cdot 6 \cdot 6) + (\pi \cdot 6 \cdot 10)$
$= 36\pi + 60\pi$
$= 96\pi \text{ in}^2$

6. a) Radius = Diameter ÷ 2
$= 5 \div 2$
$= 2.5 \text{ ft}$
Volume of cone:
$\frac{1}{3} \pi r^2 h \sim \frac{1}{3} \cdot 3.14 \cdot 2.5 \cdot 2.5 \cdot 6.5$
$\sim 42.5 \text{ ft}^3$
Surface area of cone:
$\pi r^2 + \pi r \ell \sim (3.14 \cdot 2.5 \cdot 2.5) + (3.14 \cdot 2.5 \cdot$
$= 19.625 + 54.95$
$= 74.575$
$\sim 74.6 \text{ ft}^2$

b) Surface area of sphere:
$4\pi r^2 \sim 4 \cdot 3.14 \cdot 28 \cdot 28$
$= 9{,}847.04$
$\sim 9{,}847.0 \text{ mm}^2$
Volume of sphere:
$\frac{4}{3} \pi r^3 \sim \frac{4}{3} \cdot 3.14 \cdot 28 \cdot 28 \cdot 28$
$\sim 91{,}905.71$
$\sim 91{,}905.7 \text{ mm}^3$

c) Surface area of cylinder
$2\pi rh + 2\pi r^2$
$\sim (2 \cdot 3.14 \cdot 1.4 \cdot 4.2) + (2 \cdot 3.14 \cdot 1.4 \cdot 1.4)$
$= 36.9264 + 12.3088$
$= 49.2352$
$\sim 49.2 \text{ in}^2$
Volume of cylinder:
$\pi r^2 h \sim 3.14 \cdot 1.4 \cdot 1.4 \cdot 4.2$
~ 25.84
$\sim 25.8 \text{ in}^3$

7. Volume of cone $= \frac{1}{3} \cdot \pi r^2 \cdot h$

$450 \approx \frac{1}{3} \cdot 3.14 \cdot 5 \cdot 5 \cdot h$

$450 = \frac{78.5}{3} h$

$3 \cdot 450 = \frac{78.5h}{3} \cdot 3$

$1{,}350 = 78.5h$

$\frac{1{,}350}{78.5} = \frac{78.5h}{78.5}$

$17.2 \approx h$

The height of the cone is about 17.2 centimeters.

8. Surface area of sphere $= 4\pi r^2$

$498.96 \approx 4 \cdot 3.14 \cdot r^2$

$498.96 = 12.56r^2$

$\frac{498.96}{12.56} = \frac{12.56 \cdot r^2}{12.56}$

$39.73 \approx r^2$

$\sqrt{39.73} = r$

$6.3 \approx r$

The radius of the sphere is about 6.3 centimeters.

9. a) Lateral surface area $= \pi r \ell$

$106 \approx 3.14 \cdot r \cdot 7.5$

$106 = 23.55r$

$\frac{106}{23.55} = \frac{23.55\, r}{23.55}$

$4.5 \approx r$

The radius of the cone is about 4.5 inches.

b) Volume of cone:

$\frac{1}{3}\pi r^2 h \approx \frac{1}{3} \cdot 3.14 \cdot 4.5 \cdot 4.5 \cdot 6$

$\approx 127.2 \text{ in}^3$

The volume of the cone is about 127.2 cubic inches.

10. The cylinder has the greatest volume; Volumes are (from left to right) 2π, $\frac{2}{3}\pi$, and $\frac{4}{3}\pi$. Since $\pi > 1$, 2π is the greatest value.

Statistics

1. First, arrange the numbers in ascending order:
0, 1, 1, 2, 3, 3, 4, 5, 5, 6, 7, 8, 9, 9, 10

Range $= 10 - 0$

$= 10$

Q_2 is the median of the data.

So, $Q_2 = 5$

Q_1 is the median of the lower half of the data:
0, 1, 1, 2, 3, 3, 4

So, $Q_1 = 2$

Q_3 is the median of the upper half of the data:
5, 6, 7, 8, 9, 9, 10

So, $Q_3 = 8$

Interquartile range $= Q_3 - Q_1$

$= 8 - 2$

$= 6$

2. First, arrange the numbers in ascending order:
25, 34, 36, 40, 46, 49, 66, 67, 81, 90, 95, 104

Range $= 104 - 25$

$= 79$

Q_2 is the median of the data.

So, $Q_2 = \frac{49 + 66}{2}$

$= 57.5$

Q_1 is the median of the lower half of the data:
25, 34, 36, 40, 46, 49

So, $Q_1 = \frac{36 + 40}{2}$

$= 38$

Q_3 is the median of the upper half of the data:
66, 67, 81, 90, 95, 104

So, $Q_3 = \frac{81 + 90}{2}$

$= 85.5$

Interquartile range $= Q_3 - Q_1$

$= 85.5 - 38$

$= 47.5$

3. First arrange the numbers in ascending order:
1.05, 1.09, 1.11, 1.23, 1.23, 1.37, 1.45, 1.55, 1.67, 1.78

Range = 1.78 − 1.05
= 0.73

Q_2 is the median of the data.

So, $Q_2 = \frac{1.23 + 1.37}{2}$
= 1.3

Q_1 is the median of the lower half of the data:
1.05, 1.09, 1.11, 1.23, 1.23

So, Q_1 = 1.11

Q_3 is the median of the upper half of the data:
1.37, 1.45, 1.55, 1.67, 1.78

So, Q_3 = 1.55

Interquartile range = $Q_3 - Q_1$
= 1.55 − 1.11
= 0.44

4. First arrange the numbers in ascending order:
130.7, 150.8, 162.5, 234.2, 248.6, 304.7, 322.0, 326.4, 344.9

Range = 344.9 − 130.7
= 214.2

Q_2 is the median of the data.

So, Q_2 = 248.6

Q_1 is the median of the lower half of the data:
130.7, 150.8, 162.5, 234.2

So, $Q_1 = \frac{150.8 + 162.5}{2}$
= 156.65

Q_3 is the median of the upper half of the data:
304.7, 322.0, 326.4, 344.9

So, $Q_3 = \frac{322.0 + 326.4}{2}$
= 324.2

Interquartile range = 324.2 − 156.65
= 167.55

5. Range = 12 − 2
= 10

6. $Q_2 = \frac{7 + 8}{2}$
= 7.5

$Q_1 = \frac{5 + 5}{2}$
= 5

$Q_3 = \frac{10 + 10}{2}$
= 10

7. Interquartile range = $Q_3 - Q_1$
= 10 − 5
= 5

9. a) Weights (in pounds) of Labrador Dogs

Stem	Leaf
5	1 6
6	2 8
7	0 1 2 3 5 7 8 9
8	1 5 8
9	3 4 5 8
10	1

5 | 1 represents 51 pounds.

b) Number of Labrador dogs = 20

c) Range = 101 − 51
= 50

d) There is no modal weight.

e) Median weight = $\frac{77 + 78}{2}$
= 77.5 lb

10. Q_1 = 4.3 cm; Q_2 = 4.4 cm; Q_3 = 4.7 cm

11. Range = 5.1 − 3.9
= 1.2

Interquartile range = $Q_3 - Q_1$
= 4.7 − 4.3
= 0.4

Probability

1. The possible outcomes are 6, 12, 18, 24, 30, 36, 42, and 48.
2. The outcomes that are favorable are HHT, HTH, and THH.
3. a) The possible outcomes are 56, 57, 65, 67, 75, and 76.
   b) $X = \{65, 75\}$.
   The number of outcomes favorable to X is 2.
4. a) The possible outcomes are AM, AY, MA, MY, YA, and YM.
   b) $\frac{4}{6} = \frac{2}{3}$
   The probability that Amy forms a two-letter string with the letter M in it is $\frac{2}{3}$.
5. a) The possible outcomes are 23, 24, 32, 34, 42, and 43.
   b) $\frac{3}{6} = \frac{1}{2}$ or 0.5
   The probability of forming a number greater than 32 is $\frac{1}{2}$ or 0.5.
6. The possible outcomes are:

Science fiction	Science fiction	Action movie
Action movie	Documentary	Science fiction
Documentary	Action movie	Documentary

Action movie	Documentary	Documentary
Documentary	Science fiction	Action movie
Science fiction	Action movie	Science fiction

There are 6 possible outcomes.
The probability that the science fiction movie is on top, action movie is in the middle, and the documentary is on the bottom is $\frac{1}{6}$.

7. The sample space has 4 orange + 5 yellow + 3 red = 12 outcomes.
   $\frac{4}{12} = \frac{1}{3}$
   The probability of selecting an orange ribbon is $\frac{1}{3}$.
8. a) $\frac{2}{4} = \frac{1}{2}$
   The probability of landing on an even number is $\frac{1}{2}$.
   b) The probability of landing on a number less than 4 is $\frac{3}{4}$.
9. a) Olivia's experimental probability of spinning a 2 is $\frac{12}{50} = 0.24$
   Jackie's experimental probability of spinning a 2 is $\frac{19}{100} = 0.19$
   b) Since each of the 6 numbers has an equal probability, the theoretical probability of spinning a 2 is $\frac{1}{6}$ or $0.1\overline{6}$.
   c) It will be closer to the theoretical probability.

Exponents

1. The base is $-\frac{1}{5}$ and the exponent is -3.
2. The base is 0.92 and the exponent is 4.
3. Correct.
4. Incorrect. The base is 5, not -5, and the exponent is -4, not 4.
5. $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
6. $4.8 \cdot 4.8 = (4.8)^2$
7. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^3$
8. $c \cdot c \cdot c \cdot c \cdot c \cdot c = c^6$
9. $\frac{3}{4}k \cdot \frac{3}{4}k \cdot \frac{3}{4}k \cdot \frac{3}{4}k = \left(\frac{3}{4}k\right)^4 = (-1 \cdot 2)^4$
10. $(-1.2)(-1.2)(-1.2)(-1.2) = (-1.2)^4 = (1.2)^4$

11. 3,780
$= 2 \cdot 1{,}890$
$= 2 \cdot 2 \cdot 945$
$= 2 \cdot 2 \cdot 3 \cdot 315$
$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 105$
$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 35$
$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 7$
$= 2^2 \cdot 3^3 \cdot 5 \cdot 7$

2	3,780
2	1,890
3	945
3	315
3	105
5	35
7	7
	1

12. 27,720
$= 2 \cdot 13{,}860$
$= 2 \cdot 2 \cdot 6{,}930$
$= 2 \cdot 2 \cdot 2 \cdot 3{,}465$
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 1{,}155$
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 385$
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 77$
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 11$
$= 2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$

2	27,720
2	13,860
2	6,930
3	3,465
3	1,155
5	385
7	77
11	11
	1

13. $(-6)^2 = (-6) \cdot (-6)$
$= 36$

14. $1.1^2 = (1.1) \cdot (1.1)$
$= 1.21$

15. $10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
$= 100{,}000$

16. $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)$
$= \frac{8}{27}$

17. $(-3)^{-1} \cdot (-3)^0 = \frac{1}{-3} \cdot 1$
$= -\frac{1}{3}$

18. $\left(\frac{5}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^3 = \left(\frac{5}{6}\right)^{4+3}$
$= \left(\frac{5}{6}\right)^7$

19. $5m^3n^4 \cdot 4m^5n^2$
$= 5 \cdot m^3 \cdot n^4 \cdot 4 \cdot m^5 \cdot n^2$
$= 5 \cdot 4 \cdot m^3 \cdot m^5 \cdot n^4 \cdot n^2$
$= 20 \cdot m^{3+5} \cdot n^{4+2}$
$= 20m^8n^6$

20. $\left(\frac{7}{8}\right) \div \left(\frac{7}{8}\right)^3 = \left(\frac{7}{8}\right)^{1-3}$
$= \left(\frac{7}{8}\right)^{-2}$
$= \frac{1}{\left(\frac{7}{8}\right)^2}$
$= \frac{1}{\frac{7^2}{8^2}}$
$= \frac{8^2}{7^2}$
$= \left(\frac{8}{7}\right)^2$

21. $(-h)^9 \div (-h)^{15} = (-h)^{9-15}$
$= (-h)^{-6}$
$= \frac{1}{(-h)^6}$

22. $x^8z^5 \div x^3z^9 = \frac{x^8z^5}{x^3z^9}$
$= \frac{x^8}{x^3} \cdot \frac{z^5}{z^9}$
$= x^{8-3} \cdot z^{5-9}$
$= x^5 \cdot z^{-4}$
$= x^5 \cdot \frac{1}{z^4}$
$= \frac{x^5}{z^4}$

23. $25p^6q^9 \div 45p^8q^4 = \frac{25 \cdot p^6 \cdot q^9}{45 \cdot p^8 \cdot q^4}$
$= \frac{25}{45} \cdot \frac{p^6}{p^8} \cdot \frac{q^9}{q^4}$
$= \frac{5}{9} \cdot p^{6-8} \cdot q^{9-4}$
$= \frac{5}{9} \cdot p^{-2} \cdot q^5$
$= \frac{5q^5}{9p^2}$

24. $\left[\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^{-1}\right]^3 = \left[\left(\frac{2}{3}\right)^{2-1}\right]^3$

$= \left[\left(\frac{2}{3}\right)^1\right]^3$

$= \left(\frac{2}{3}\right)^{1\cdot 3}$

$= \left(\frac{2}{3}\right)^3$

25. $40c^5d^3 \div 10c^9d^2 = \frac{40c^5d^3}{10c^9d^2}$

$= \frac{40}{10} \cdot \frac{c^5}{c^9} \cdot \frac{d^3}{d^2}$

$= 4 \cdot c^{5-9} \cdot d^{3-2}$

$= 4 \cdot c^{-4} \cdot d^1$

$= 4 \cdot \frac{1}{c^4} \cdot d$

$= \frac{4d}{c^4}$

26. $\left(\frac{72b^{-1}}{32c^{-1}}\right)^{-2} = \left[\left(\frac{72}{32}\right) \cdot \left(\frac{b}{c}\right)^{-1}\right]^{-2}$

$= \left[\left(\frac{9}{4}\right) \cdot \left(\frac{b}{c}\right)^{-1}\right]^{-2}$

$= \left(\frac{9}{4}\right)^{-2} \cdot \left(\frac{b}{c}\right)^{-1\cdot(-2)}$

$= \frac{1}{\left(\frac{9}{4}\right)^2} \cdot \left(\frac{b}{c}\right)^2$

$= \left(\frac{4}{9}\right)^2 \cdot \left(\frac{b}{c}\right)^2$

$= \left(\frac{4b}{9c}\right)^2$

27. $\frac{(9^{-2})^{-2} \cdot 2^2}{9^2} = \frac{[9^{(-2)\cdot(-2)}] \cdot 2^2}{9^2}$

$= \frac{9^4 \cdot 4}{9^2}$

$= 9^{4-2} \cdot 4$

$= 9^2 \cdot 4$

$= 81 \cdot 4$

$= 324$

28. $\frac{6^8 \cdot 56^{-3}}{6^5 \cdot 7^{-3}} = \frac{6^8}{6^5} \cdot \frac{56^{-3}}{7^{-3}}$

$= 6^{8-5} \cdot \left(\frac{56}{7}\right)^{-3}$

$= 6^3 \cdot 8^{-3}$

$= 6^3 \cdot \frac{1}{8^3}$

$= \left(\frac{6}{8}\right)^3$

$= \left(\frac{3}{4}\right)^3$

$= \frac{3^3}{4^3}$

$= \frac{27}{64}$

29. $\frac{42^{-1}}{(2^0)^{12} \cdot 21^{-1}} = \frac{42^{-1}}{1^{12} \cdot 21^{-1}}$

$= \frac{42^{-1}}{1 \cdot 21^{-1}}$

$= \left(\frac{42}{21}\right)^{-1}$

$= 2^{-1}$

$= \frac{1}{2}$

30. $\frac{(3^5 \cdot 3^4)^2}{(3^3)^6} = \frac{(3^{5+4})^2}{3^{3\cdot 6}}$

$= \frac{(3^9)^2}{3^{18}}$

$= \frac{3^{9\cdot 2}}{3^{18}}$

$= \frac{3^{18}}{3^{18}}$

$= 3^{18-18}$

$= 3^0$

$= 1$

31. $r^2 = 256$

$r^2 = 16^2$ or $(-16)^2$

$r = 16$ or -16

32. $c^2 = \frac{121}{169}$

$c^2 = \left(\frac{11}{13}\right)^2$ or $\left(-\frac{11}{13}\right)^2$

$c = \frac{11}{13}$ or $-\frac{11}{13}$

33. $x^3 = 32.768$

$\sqrt[3]{x^3} = \sqrt[3]{32.768}$

$x = 3.2$

34. $t^3 = -\frac{27}{343}$

$t^3 = \left(-\frac{3}{7}\right)^3$

$\sqrt[3]{t^3} = \sqrt[3]{\left(-\frac{3}{7}\right)^3}$

$t = -\frac{3}{7}$

35. $5 \cdot 10^1 + 8 \cdot 10^0 + 1 \cdot 10^{-1} + 9 \cdot 10^{-2}$

$= 5 \cdot 10 + 8 \cdot 1 + 1 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100}$

$= 50 + 8 + 0.1 + 0.09$

$= 58.19$

Scientific Notation

1. Incorrect; The coefficient is 10. It needs to be less than 10.
2. Incorrect; The coefficient is less than 1. It needs to be greater than or equal to 1.
3. Correct
4. Incorrect; The coefficient is less than 1. It needs to be greater than or equal to 1.
5. $714{,}000 = 7.14 \cdot 100{,}000$
   $= 7.14 \cdot 10^5$
6. $0.00087 = 8.7 \cdot \frac{1}{10{,}000}$
   $= 8.7 \cdot 10^{-4}$
7. $3.46 \cdot 10^2 = 3.46 \cdot 100$
   $= 346$
8. $5.4 \cdot 10^4 = 5.4 \cdot 10{,}000$
   $= 54{,}000$
9. $7.8 \cdot 10^{-5}$; as $10^{-5} > 10^{-7}$
10. $6 \cdot 10^{-4}$; as $10^{-4} > 10^{-5}$
11. $9.3 \cdot 10^{-12}$; as $10^{-12} > 10^{-15}$
12. $3.5 \cdot 10^{-2}$; as $10^{-2} > 10^{-3}$
13. $2.44 \cdot 10^3 + 1.9 \cdot 10^5$
    $= 0.0244 \cdot 10^2 \cdot 10^3 + 1.9 \cdot 10^5$
    $= 0.0244 \cdot 10^{2+3} + 1.9 \cdot 10^5$
    $= 0.0244 \cdot 10^5 + 1.9 \cdot 10^5$
    $= (0.0244 + 1.9) \cdot 10^5$
    $= 1.9244 \cdot 10^5$
    $\sim 1.9 \cdot 10^5$
14. $3.12 \cdot 10^{-3} - 3 \cdot 10^{-3}$
    $= (3.12 - 3) \cdot 10^{-3}$
    $= 0.12 \cdot 10^{-3}$
    $= 1.2 \cdot 10^{-1} \cdot 10^{-3}$
    $= 1.2 \cdot 10^{-1+(-3)}$
    $= 1.2 \cdot 10^{-4}$
15. $2.4 \cdot 10^{-2} \cdot 5 \cdot 10^{-1}$
    $= 2.4 \cdot 5 \cdot 10^{-2} \cdot 10^{-1}$
    $= 12 \cdot 10^{-2+(-1)}$
    $= 12 \cdot 10^{-3}$
    $= 1.2 \cdot 10^1 \cdot 10^{-3}$
    $= 1.2 \cdot 10^{-2}$
16. $3.2 \cdot 10^8 \div (1.6 \cdot 10^4)$
    $= \frac{3.2 \cdot 10^8}{1.6 \cdot 10^4}$
    $= \frac{3.2}{1.6} \cdot \frac{10^8}{10^4}$
    $= 2 \cdot 10^{8-4}$
    $= 2 \cdot 10^4$

Linear Equations

1. $2(x-5)-8=20$

$2x-10-8=20$

$2x-18+18=20+18$

$\frac{2x}{2}=\frac{38}{2}$

$x=19$

2. $2x-(5-x)=\frac{5}{2}$

$2x-5+x=2.5$

$3x-5+5=2.5+5$

$\frac{3x}{3}=\frac{7.5}{3}$

$x=2.5$

3. $\frac{1}{4}(x+2)-2=0.5$

$0.25x+0.5-2=0.5$

$0.25x-1.5+1.5=0.5+1.5$

$\frac{0.25x}{0.25}=\frac{2}{0.25}$

$x=8$

4. $4x-\frac{5-2x}{5}=\frac{3}{5}$

$4x-1+\frac{2}{5}x=\frac{3}{5}$

$\frac{22}{5}x-1+1=\frac{3}{5}+1$

$\frac{22}{5}x=\frac{8}{5}$

$x=\frac{4}{11}$

5. Let $x=0.\overline{2}$.

$x=0.222\ldots$

$10x=2.222\ldots$

$10x-x=2.\overline{2}-0.\overline{2}$

$9x=2$

$\frac{9x}{9}=\frac{2}{9}$

$x=\frac{2}{9}$

6. Let $x=0.\overline{93}$.

$x=0.933\ldots$

$10x=9.333\ldots$

$10x-x=9.3\overline{3}-0.9\overline{3}$

$9x=8.4$

$\frac{9x}{9}=\frac{8.4}{9}$

$x=\frac{8.4\cdot 10}{9\cdot 10}$

$x=\frac{84}{90}$

$x=\frac{14}{15}$

7. Let $x=0.2\overline{6}$.

$x=0.266\ldots$

$10x=2.666\ldots$

$10x-x=2.6\overline{6}-0.2\overline{6}$

$9x=2.4$

$\frac{9x}{9}=\frac{2.4}{9}$

$x=\frac{2.4\cdot 10}{9\cdot 10}$

$x=\frac{24}{90}$

$x=\frac{4}{15}$

8. Let $x=0.3\overline{16}$.

$x=0.31616\ldots$

$100x=31.61616\ldots$

$100x-x=31.6\overline{16}-0.3\overline{16}$

$99x=31.3$

$\frac{99x}{99}=\frac{31.3}{99}$

$x=\frac{31.3\cdot 10}{99\cdot 10}$

$x=\frac{313}{990}$

9. $2x+4\overset{?}{=}-2\left(\frac{1}{2}-x\right)$

$2x+4\overset{?}{=}-1+2x$

$2x+4-2x\overset{?}{=}-1+2x-2x$

$4\neq -1$

Because $4\neq 1$, the equation has no solution. So, the equation is inconsistent.

10. $6y + (16 - 2y) \stackrel{?}{=} 4(4 + y)$

$6y + 16 - 2y \stackrel{?}{=} 16 + 4y$

$4y + 16 - 16 \stackrel{?}{=} 16 + 4y - 16$

$4y - 4y \stackrel{?}{=} 4y - 4y$

$0 = 0$

Since $0 = 0$ is always true, the equation is true for any value of y. So, the equation has infinitely many solutions, and it is an identity.

11. $4x + 5 \stackrel{?}{=} 2x - 7$

$4x + 5 - 5 \stackrel{?}{=} 2x - 7 - 5$

$4x - 2x \stackrel{?}{=} 2x - 12 - 2x$

$2x \stackrel{?}{=} -12$

$\frac{2x}{2} \stackrel{?}{=} \frac{-12}{2}$

$x = -6$

Because the equation has one solution, it is consistent.

12. $2x + 5 \stackrel{?}{=} -4\left(-\frac{5}{4} - \frac{1}{2}x\right)$

$2x + 5 \stackrel{?}{=} 5 + 2x$

$2x + 5 - 5 \stackrel{?}{=} 5 + 2x - 5$

$2x - 2x \stackrel{?}{=} 2x - 2x$

$0 = 0$

Since $0 = 0$ is always true, the equation is true for any value of x. So, the equation has infinitely many solutions, and it is an identity.

13. Substitute 4 for x into the equation:

$4(4) - 2 = y + 5$

$16 - 2 = y + 5$

$14 - 5 = y + 5 - 5$

$9 = y$

$y = 9$

14. Substitute 4 for x into the equation:

$4 - 4y = 2$

$4 - 4y - 4 = 2 - 4$

$-4y = -2$

$\frac{-4y}{-4} = \frac{-2}{-4}$

$y = 0.5$

15. Substitute 4 for x into the equation:

$y - 4 = \frac{1}{3}(4 + 14)$

$y - 4 = \frac{1}{3}(18)$

$y - 4 = 6$

$y - 4 + 4 = 6 + 4$

$y = 10$

16. Substitute 4 for x into the equation:

$\frac{3(4) - 7}{y} = \frac{1}{3}$

$\frac{12 - 7}{y} = \frac{1}{3}$

$\frac{5}{y} \cdot \frac{y}{5} = \frac{1}{3} \cdot \frac{y}{5}$

$1 \cdot 15 = \frac{y}{15} \cdot 15$

$15 = y$

17. Substitute 4 for x into the equation:

$\frac{1}{7}\left[3(4) + y\right] = 4$

$\frac{12}{7} + \frac{1}{7}y - \frac{12}{7} = 4 - \frac{12}{7}$

$\frac{1}{7}y = \frac{28 - 12}{7}$

$\frac{1}{7}y = \frac{16}{7}$

$\frac{1}{7}y \cdot 7 = \frac{16}{7} \cdot 7$

$y = 16$

18. Substitute 4 for x into the equation:

$\frac{3y + 1}{4} = 2(4)$

$\frac{3y + 1}{4} \cdot 4 = 8 \cdot 4$

$3y + 1 - 1 = 32 - 1$

$\frac{3y}{3} = \frac{31}{3}$

$y = 10\frac{1}{3}$

19. $4x + y = 2(x + 3y)$

$4x + y - 2x = 2x + 6y - 2x$

$2x + y = 6y$

$2x + y - y = 6y - y$

$\frac{2x}{2} = \frac{5y}{2}$

$x = \frac{5}{2}y$

20. $3(x - 2y) = 4x + 5y$

$3x - 6y + 6y = 4x + 5y + 6y$

$3x - 4x = 4x + 11y - 4x$

$-x = 11y$

$\frac{-x}{-1} = \frac{11y}{-1}$

$x = -11y$

Substitute -2 for y into the equation:

$x = -11(-2)$

$x = 22$

21. $\frac{1}{3}x + \frac{5}{6}y = 2$

$\frac{1}{3}x + \frac{5}{6}y - \frac{5}{6}y = 2 - \frac{5}{6}y$

$\frac{1}{3}x \cdot 3 = \left(2 - \frac{5}{6}y\right) \cdot 3$

$x = 6 - \frac{5}{2}y$

Substitute -2 for y into the equation:

$x = 6 - \frac{5}{2}(-2)$

$= 6 + 5$

$x = 11$

22. $\frac{0.5(x - 3)}{y} = 10$

$\frac{0.5(x - 3)}{y} \cdot y = 10 \cdot y$

$0.5x - 1.5 = 10y$

$0.5x - 1.5 + 1.5 = 10y + 1.5$

$\frac{0.5x}{0.5} = \frac{10y + 1.5}{0.5}$

$x = 20y + 3$

Substitute -2 for y into the equation:

$x = 20(-2) + 3$

$= -40 + 3$

$x = -37$

23. $0.25(x + y) = 15 - x$

$0.25x + 0.25y = 15 - x$

$0.25x + 0.25y - 0.25y = 15 - x - 0.25y$

$0.25x + x = 15 - x - 0.25y + x$

$1.25x = 15 - 0.25y$

$\frac{1.25x}{1.25} = \frac{15 - 0.25y}{1.25}$

$x = 12 - 0.2y$

Substitute -2 for y into the equation:

$x = 12 - 0.2(-2)$

$= 12 + 0.4$

$= 12.4$

24. $\frac{y}{2} - \frac{2x + y}{5} = 7$

$\frac{y \cdot 5}{2 \cdot 5} - \frac{(2x + y) \cdot 2}{5 \cdot 2} = 7$

$\frac{5y}{10} - \frac{4x + 2y}{10} = 7$

$\frac{5y - (4x + 2y)}{10} = 7$

$\frac{5y - 4x - 2y}{10} = 7$

$\frac{5y - 4x - 2y}{10} \cdot 10 = 7 \cdot 10$

$3y - 4x - 3y = 70 - 3y$

$-4x = 70 - 3y$

$\frac{-4x}{-4} = \frac{70 - 3y}{-4}$

$x = \frac{3y - 70}{4}$

Substitute -2 for y into the equation.

$x = \frac{3(-2) - 70}{4}$

$= \frac{-6 - 70}{4}$

$= \frac{-76}{4}$

$= -19$

25. $d = 2 \cdot \text{radius}$

$= 14$

Substitute 14 for d into the equation:

$Q = 14 \cdot \left(1 + \frac{\pi}{4}\right)$

$= 14 \cdot \left(1 + \frac{\frac{22}{7}}{4}\right)$

$= 14 \cdot \left(1 + \frac{22}{28}\right)$

$= 14 \cdot \left(1 + \frac{11}{14}\right)$

$= 14 \cdot \left(\frac{14 + 11}{14}\right)$

$= 14 \cdot \frac{25}{14}$

$= 25$

The perimeter of each quadrant of the pizza is 25 inches.

26. Let the height of the blue section be x feet.
So, the height of the red section is $2x$ feet.
Because the total height is 10.5 feet.

$$x + 4.2 + 2x = 10.5$$
$$3x + 4.2 = 10.5$$
$$3x + 4.2 - 4.2 = 10.5 - 4.2$$
$$\frac{3x}{3} = \frac{6.3}{3}$$
$$x = 2.1$$

Height of red section $= 2x$
$= 2(2.1)$
$= 4.2$

The height of the red section is 4.2 feet.

27. a) Let the cost of potatoes be p dollars.
So, the cost of green beans is $\frac{5}{4}p$ dollars.
Because 4 pounds of green beans and 5 pounds of potatoes cost the same as 10 pounds of potatoes,

$$4 \cdot \frac{5}{4}p + 5p = 10p$$

b)
$$4 \cdot \frac{5}{4}p + 5p = 10p$$
$$5p + 5p = 10p$$
$$10p - 10p = 10p - 10p$$
$$0 = 0$$

Since $0 = 0$ is always true, the linear equation is true for any value of p. So, this equation has infinitely many solutions, and it is an identity.