Linear Algebra MAT313 Fall 2022
Professor Sormani
Lesson 2 Solving Linear Systems

Warning: do not start this lesson until you have completed Lesson 1 and submitted the classwork/homework (at least HW1-HW6) and received feedback from the professor and made necessary fixes.

Please be sure to mark down the date and time that you start this lesson. Carefully take notes on pencil and paper while watching the lesson videos. Pause the lesson to try classwork before watching the video going over that classwork. If you work with any classmates, be sure to write their names on the problems you completed together. Please wear masks when meeting with classmates even if you meet off campus.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT313F22-lesson2-lastname-firstname

and share editing of that document with me sormanic@gmail.com. You will also include your homework and any corrections to your homework in this doc.

If you have a question, type QUESTION in your googledoc next to the point in your notes that has a question and email me with the subject MAT313 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

****
This lesson has two parts each in its own playlist. Scroll down for the second playlist. Be sure to learn the methods taught in this lesson even if you already learned to solve a system in another course. I am teaching the method that leads to an algorithm that works with many equations and many unknowns. There are 10 HW problems.

****
Part 1: Using Row Actions to Solve a Linear System
Watch Playlist 313F21-2-1to7
****

Here we solve one particular system and explain what row actions are and introduce Echelon form.

Linear Algebra Lesson 2

Solving Linear Systems

- Let us review Lesson 1 ✓
- How to solve a linear system in a systematic way (not graphing) simplifying the system through a series of steps called row actions.

<u>Example</u>: Solve: $\boxed{\begin{aligned} x + 2y &= 3 \\ x + y &= 2 \end{aligned}}$ ← row 1 ρ_1, ← row 2 ρ_2

Greek letter "rho" ρ

Solution: (many ways to do this)

step 1 row action: $\rho_1 - \rho_2$

$$\begin{aligned} x + 2y &= 3 \\ \text{subtract} \quad x + y &= 2 \\ \hline 0x + y &= 1 \end{aligned}$$

If ρ_1 and ρ_2 are true then $\rho_1 - \rho_2$ is also true

So $0x + y = 1$ for every solution to our system

Thus $\boxed{y = 1}$

step 2 substitute $y=1$ into ρ_1

Solve for x:

$$\begin{aligned} x + 2y &= 3 \\ x + 2\cdot 1 &= 3 \\ x + 2 &= 3 \\ -2 & \quad -2 \\ \boxed{x = 1} \end{aligned}$$

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

$\boxed{\text{Hw1}}$ Use these two steps to solve

$$\boxed{\begin{aligned} x + 5y &= 6 \\ x - 3y &= 8 \end{aligned}}$$

Key Idea: To solve a linear system

Step 1: Do row actions to simplify the equations and find $x_m = \text{value}$ (if possible)

Step 2: Substitute upwards to find the values of $x_1, x_2 \ldots x_{m-1}$ (if possible)

Example Solve $\boxed{\begin{aligned} x + 2y &= 6 \\ 2x + y &= 8 \end{aligned}}$

Warning

$$\begin{array}{lc} \rho_1 & x + 2y = 6 \\ \rho_2 & 2x + y = 8 \\ \hline \rho_1 - \rho_2 & -x + y = -2 \end{array}$$

This does not give us $y = \sim$

Fails: cannot just subtract rows

Step 1

We want the first variable to disappear

$$\begin{array}{lc} \rho_1 : & x + 2y = 6 \\ \rho_2 : & 2x + y = 8 \\ \hline \rho_1 - \frac{1}{2}\rho_2 : & 0 + \frac{3}{2}y = 2 \end{array}$$

$x - \frac{1}{2}(2x) = x - x = 0$ ✓

$2y - \frac{1}{2}(y) = \left(\frac{4}{2} - \frac{1}{2}\right)y = \frac{3}{2}y$ ✓

$6 - \frac{1}{2}(8) = 6 - 4 = 2$

another action: mult by $\frac{2}{3}$

$\frac{2}{3}\left(\frac{3}{2}y\right) = \frac{2}{3}(2)$

Fails: cannot just subtract ro...

Step 1 We want the first variable to disappear

$\rho_1: \quad x + 2y = 6$
$\rho_2: \quad 2x + y = 8$

$\rho_1 - \frac{1}{2}\rho_2: \quad 0 + \frac{3}{2}y = 2$

$x - \frac{1}{2}(2x) = x - x = 0$ ✓

$2y - \frac{1}{2}(y) = \left(\frac{4}{2} - \frac{1}{2}\right)y = \frac{3}{2}y$ ✓

$6 - \frac{1}{2}(8) = 6 - 4 = 2$

another action: mult by $\frac{2}{3}$ $\quad \frac{2}{3}\left(\frac{3}{2}y\right) = \frac{2}{3}(2)$

$\boxed{y = \frac{4}{3}}$

Step 2 Sub back to find x $\quad x + 2y = 6$
$x + 2\left(\frac{4}{3}\right) = 6$

Solve for $x = 6 - \frac{8}{3} = \frac{18}{3} - \frac{8}{3} = \frac{10}{3}$

$\boxed{x = \frac{10}{3}}$

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10/3 \\ 4/3 \end{pmatrix} \right\}$$

Check
$\left(\frac{10}{3}\right) + 2\left(\frac{4}{3}\right) = 6$?
$2\left(\frac{10}{3}\right) + \left(\frac{4}{3}\right) = 8$?

$\frac{10}{3} + 2\left(\frac{4}{3}\right) = \frac{10}{3} + \frac{8}{3} = \frac{18}{3} = 6$ ✓

$2\left(\frac{10}{3}\right) + \frac{4}{3} = \frac{20}{3} + \frac{4}{3} = \frac{24}{3} = 8$ ✓

HW2 Use this method to solve

$$\boxed{\begin{aligned} 2x + 5y &= 8 \\ 4x + 9y &= 10 \end{aligned}}$$

and then check your answer.

Example: Solve $\boxed{\begin{array}{l} 2x + 2y + 4z = 12 \\ x + y + z = 5 \\ x - y + z = 1 \end{array}}$

① Step 1: use row actions to simplify the system.

When we have many equations we keep track of the whole system as we proceed

$$\boxed{\begin{array}{l} 2x + 2y + 4z = 12 \\ x + y + z = 5 \\ x - y + z = 1 \end{array}} \xrightarrow[\text{copy the other rows}]{\rho_1 \to \frac{1}{2}\rho_1} \boxed{\begin{array}{l} x + y + 2z = 6 \\ x + y + z = 5 \\ x - y + z = 1 \end{array}}$$

If $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ solves this old system $\xRightarrow{\text{mult by } \frac{1}{2}}$ then $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ solves this new system

then $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ solve this old system $\xLeftarrow[\text{mult } \rho_1 \text{ by } 2]{}$ If $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ solves the new system

Row actions $\rho_i \to k\rho_i$ where $k \neq 0$

is reversable by mult by $\frac{1}{k}$

So we have the same solutions

Continuing with step 1 of our example:

more row actions

Next get rid of the first variable in rows 2+3

$$\boxed{\begin{array}{l} x+y+2z=6 \\ \cancel{x}+y+z=5 \\ x-y+z=1 \end{array}} \xrightarrow[\text{copy other rows}]{\rho_2 \to \rho_2-\rho_1} \boxed{\begin{array}{l} x+y+2z=6 \\ \quad\quad -z=-1 \\ x-y+z=1 \end{array}}$$

any solution of this system $\Longrightarrow$ is also a solution of this system

we can also go backwards

$$\boxed{\begin{array}{l} x+y+2z=6 \\ \cancel{x}+y+z=5 \\ x-y+z=1 \end{array}} \xleftarrow{\rho_2 \to \rho_2+\rho_1} \boxed{\begin{array}{l} x+y+2z=6 \\ \quad\quad -z=-1 \\ x-y+z=1 \end{array}}$$

The row action $\rho_i \longrightarrow \rho_i + k\rho_j$

is reversable $\rho_i \longrightarrow \rho_i - k\rho_j$

So we have some solutions before and after such a row action

$$\boxed{\begin{aligned} x+y+2z&=6\\ -z&=-1\\ x-y+z&=1\end{aligned}} \xrightarrow{\rho_3 \to \rho_3-\rho_1} \boxed{\begin{aligned} x+y+2z&=6\\ -z&=-1\\ 0x-2y-z&=-5\end{aligned}}$$

Switching Rows is Reversible

$$\rho_i \longleftrightarrow \rho_j$$

So we have the same solutions before + after this action

$$\boxed{\begin{aligned} x+y+2z&=6\\ -z&=-1\\ 0x-2y-z&=-5\end{aligned}} \xrightarrow[\text{copy other rows}]{\rho_2 \leftrightarrow \rho_3} \boxed{\begin{aligned} x+y+2z&=6\\ 0x-2y-z&=-5\\ -z&=-1\end{aligned}}$$

$$\boxed{\begin{aligned} x+y+2z&=6\\ 0x-2y-z&=-5\\ -z&=-1\end{aligned}} \xrightarrow[\rho_3 \to -\rho_3]{\rho_2 \to -\frac{1}{2}\rho_2} \underbrace{\boxed{\begin{aligned} x+y+2z&=6\\ y+\tfrac{1}{2}z&=\tfrac{5}{2}\\ z&=1\end{aligned}}}_{\text{Echelon Form}}$$

Neat each row begins with a new leading variable and the coefficient of that variable is 1.

Step 2 Sub up

pause + try

solution in next video.

$$\begin{array}{|l|}\hline x + y + 2z = 6 \\ 0x - 2y - z = -5 \\ \quad\quad -z = -1 \\ \hline \end{array} \xrightarrow[\rho_3 \to -\rho_3]{\rho_2 \to -\frac{1}{2}\rho_2} \underbrace{\begin{array}{|l|}\hline x + y + 2z = 6 \\ \quad y + \frac{1}{2}z = \frac{5}{2} \\ \quad\quad z = 1 \\ \hline \end{array}}_{\text{Echelon Form}}$$

Neat each row begins with a new leading variable and the coefficient of that variable is 1.

Step 2 Sub up

pause + try

solution in next video.

Observe that $z=1$ in the last row

go to one row above the last row: $y + \frac{1}{2}z = \frac{5}{2}$

and solve for its leader: $y = \frac{5}{2} - \frac{1}{2}z$

and sub in $z=1$: $y = \frac{5}{2} - \frac{1}{2}(1) = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

go the one row above that row: $x + y + 2z = 6$

and solve for its leader: $x = 6 - y - 2z$

and sub in $z=1$ and $y=2$: $x = 6 - (2) - 2(1) = 2$

So our solution set is $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \right\}$

The second part of the lesson may be watched after a break starting here.

****

Part II: How to Solve Any Linear System

Watch Playlist 313F21-2-8to12

****

"How to solve a Linear System" is in video 313F21-2-8 and the Example 1 rewritten using this technique is in video 313F21-2-8 and 313F21-2-9:

How to Solve a Linear System:

Step 1: Row Reductions to Echelon Form

Row Actions which are reversable

$$\left.\begin{array}{ll} \text{scale} & \bullet\ \rho_i \to k\rho_i \text{ where } k \neq 0 \\ \text{skew} & \bullet\ \rho_i \to \rho_i + k\rho_j \\ \text{switch} & \bullet\ \rho_i \leftrightarrow \rho_j \end{array}\right\} \text{later we will set up an algorithm for this step.}$$

Step 2: Sub up

Start with final row: solve for its leader

Next to last row: solve for its leader
sub in previous variables

Next row up: solve for its leader
sub in previous variable

When all rows are done we have a solution.

Example above rewritten (method to use for HW + Exams)

$$\boxed{\begin{aligned} 2x + 2y + 4z &= 12 \\ x + y + z &= 5 \\ x - y + z &= 1 \end{aligned}} \longrightarrow$$

Step 1 Row actions moving towards Echelon Form

- make the 1^{st} coefficient (upper left) a $\underline{1}x$

$$\boxed{\begin{aligned} 2x+2y+4z&=12\\ x+y+z&=5\\ x-y+z&=1\end{aligned}} \xrightarrow[\text{copy other lines}]{\rho_1 \to \frac{1}{2}\rho_1} \boxed{\begin{aligned} \boxed{1x}+1y+2z&=6\\ x+y+z&=5\\ x-y+z&=1\end{aligned}}$$

- get rid of the first variable in rows 2 to n using row action $\rho_i \to \rho_i - k\rho_1$

$$\rho_2 \to \rho_2-\rho_1$$
$$\rho_3 \to \rho_3-\rho_1$$

$$\longrightarrow \boxed{\begin{aligned} \boxed{1x}+1y+2z&=6\\ 0x+0y-z&=-1\\ 0x-2y-z&=-5\end{aligned}}$$

x is the first leader

notice x only appears in first row.

- put our next leading variable, y, in the second row

$$\xrightarrow{\rho_2 \leftrightarrow \rho_3} \boxed{\begin{aligned} \boxed{1x}+1y+2z&=6\\ 0x\ \boxed{-2y}-z&=-5\\ 0x+0x-z&=-1\end{aligned}}$$

1^{st} leader is x
2^{nd} leader is 1

- make this leading variable have coefficient $=1$

$$\xrightarrow{\rho_2 \to -\frac{1}{2}\rho_2} \boxed{\begin{aligned} \boxed{1x}+1y+2z&=6\\ 0x+\boxed{1y}+\tfrac{1}{2}z&=\tfrac{5}{2}\\ 0x+0y\ \boxed{-z}&=-1\end{aligned}}$$

1^{st} leader is x
2^{nd} leader is y

- our third leader is z, needs coeff $=1$

$$\xrightarrow{\rho_3 \to -\rho_3} \boxed{\begin{aligned} \boxed{1x}+1y+2z&=6\\ 0x+\boxed{1y}+\tfrac{1}{2}z&=\tfrac{5}{2}\\ 0x+0y+\boxed{1z}&=1\end{aligned}}$$

leaders for each row has coeff $=1$
and has 0's to the left + below

Echelon Form.

Step 2 Sub up
pause + try
solution in next
video.

Echelon Form

Neat each row
begins with a new
leading variable
and the coefficient of
that variable is 1.

Observe that $z=1$ in the last row

go to one row above the last row: $y+\frac{1}{2}z=\frac{5}{2}$

and solve for its leader: $y=\frac{5}{2}-\frac{1}{2}z$

and sub in $z=1$: $y=\frac{5}{2}-\frac{1}{2}(1)=\frac{5}{2}-\frac{1}{2}=\frac{4}{2}=2$

go the one row above that row: $x+y+2z=6$

and solve for its leader: $x=6-y-2z$

and sub in $z=1$ and $y=2$: $x=6-(2)-2(1)=2$

So our solution set is $\left\{\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}\right\}$

Example 2 is in video 313F21-2-10

Example 2

Solve
$$\begin{cases} 2x+2y+4z=12 \\ x+y+z=5 \\ 2x+2y+2z=8 \end{cases}$$

Classwork!
Use our method

Step 1: Row reduction changing the system to Echelon Form

- Upper Left Coefficient to 1

$$\xrightarrow{\rho_1 \to \frac{1}{2}\rho_1} \begin{cases} \boxed{x}+y+2z=6 \\ x+y+z=5 \\ 2x+2y+2z=8 \end{cases}$$

upper left

our 1st leader is x

- Get rid of 1st leader in 2nd – nth rows (change 1st coeff to 0) using skew actions $\rho_i \to \rho_i - k\rho_1$

$$\xrightarrow[\rho_3 \to \rho_3 - 2\rho_1]{\rho_2 \to \rho_2 - \rho_1} \begin{cases} \boxed{x}+y+2z=6 \\ 0x+0y-z=-1 \\ 0x+0y-2z=-4 \end{cases}$$

y is never a leader!
z is our next leader

- Make the coefficient of next leader $= 1$

$$\xrightarrow{\rho_2 \to -\rho_2} \begin{cases} \boxed{x}+y+2z=6 \\ 0x+0y+\boxed{z}=1 \\ 0x+0y-2z=-4 \end{cases}$$

box our leaders!
x and z
(y is not a leader)

- Get rid of 2nd leader in lower rows (3rd row)

$\rho_3 \to \rho_3 + k\rho_2 \longleftarrow$ use ρ_2 to do this

$$\xrightarrow{\rho_3 \to \rho_3 + 2\rho_2} \begin{cases} \boxed{x}+y+2z=6 \\ 0x+0y+\boxed{z}=1 \\ 0x+0y+0z=-2 \end{cases}$$

scratch
$-4+2(1) = -4+2 = -2$

This is in Echelon Form!
zeroes under + left of leaders ✓
coeffs of leaders are 1 ✓

Step2 sub up: Start with bottom line.

$0x + 0y + 0z = -2$ which is $0 = -2$

Impossible!

So our Echelon Form has no solutions!

but all our actions were reversable

So our original system has no solution.

Solution Set $= \emptyset$ Empty set!

Example 3 is in the last two videos 313F21-2-11 and 313F21-2-12 which you should watch pausing and trying as you work:

Example 3: Solve
$$\boxed{\begin{aligned} 1x+1y+1z+1w&=0\\ 1x+1y-1z+1w&=0\\ 2x+3y+2z+1w&=0 \end{aligned}}$$

Classwork

Step 1: Reduce to Echelon Form Using Row Actions

- Upper Left coeff is a 1 ✓ so our first leader is x

$$\boxed{\begin{aligned} \boxed{1x}+1y+1z+1w&=0\\ 1x+1y-1z+1w&=0\\ 2x+3y+2z+1w&=0 \end{aligned}}$$

- Get rid of first leader in rows below ρ_1 using $\rho_i \to \rho_i + k\rho_1$

pause + try to find row actions for ρ_2 + ρ_3

$$\xrightarrow{\substack{\rho_2 \to \rho_2 - \rho_1 \\ \rho_3 \to \rho_3 - 2\rho_1}} \boxed{\begin{aligned} \boxed{1x}+1y+1z+1w&=0\\ 0x+0y-2z+0w&=0\\ 0x+\boxed{1y}+0z-1w&=0 \end{aligned}}$$

2nd leader is y

move up to row 2

move 2nd leader into 2nd row

$$\xrightarrow[\text{switch!}]{\rho_3 \leftrightarrow \rho_2} \boxed{\begin{aligned} \boxed{1x}+1y+1z+1w&=0\\ 0x+\boxed{1y}+0z-1w&=0\\ 0x+0y\boxed{-2z}+0w&=0 \end{aligned}}$$

pause + try

do we have a coeff = 1 on 2nd leader? Yes

do we have zeroes below it? Yes

Next look for third leader in next column
the next leader is z.
It is already in 3^{rd} row.
No switch needed.
Must make 3^{rd} leader's coeff = 1

$$\xrightarrow{\rho_3 \to \frac{-1}{2}\rho_3} \boxed{\begin{array}{l} \boxed{1x} + 1y + 1z + 1w = 0 \\ 0x + \boxed{1y} + 0z - 1w = 0 \\ 0x + 0y + \boxed{1z} + 0w = 0 \end{array}}$$

Echelon Form!

Step 2:

Sub Up

last row $0x + 0y + \boxed{1z} + 0w = 0$

Solve for the leader $z = 0 - 0w - 0x - 0y = 0$

so $\boxed{z = 0}$

second to last row

$$0x + \boxed{1y} + 0z - 1w = 0$$

Solve for the leader $\boxed{y = 0 + 1w}$

Sub in leaders (but w is not a leader)

"w is free" not a leader

So w can have any value in $\mathbb{R}$

next to last row

$$\boxed{1x} + 1y + 1z + 1w = 0$$

Solve for the leader

$$x = 0 - y - z - w$$

Sub in previous leaders

$$x = 0 - (0 + 1w) - (0) - w$$

formula for y

formula for z

free so keep.

simplify:

$$\boxed{x = -w - w = -2w}$$

Already at top so stop

$$\left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -2w \\ 0 + 1w \\ 0 \\ w \end{pmatrix} : w \in \mathbb{R} \text{ free} \right\}$$

free = free leaders = formulas

Many Solutions "A line"

Homework:

Be sure to watch both playlists of videos before doing this homework!

Use this method from class:

How to Solve a Linear System

[redacted]

[redacted] Do Row Actions to Echelon Form:

- Make upper left leader into a 1 using scaling (or switch if 0) Put a box on the leader.
- Put zeroes below this leader using skew by leader's row
- Move down to the next row and make sure the next column has a leader which is 1 by repeating the red step then repeat the blue step
- Move down again.... and again Until in Echelon Form
  Each leader should be a 1 in a box
  Below and left of leaders are 0s.

Check the answers to each problem before doing the next.

Lesson 2 Homework (answers below)

(HW1) $\boxed{\begin{aligned} 3x+3y-9z&=6 \\ x-y+z&=0 \\ x+4y+2z&=5 \end{aligned}}$ What is the first row action you should do? Do the row action

(HW2) $\boxed{\begin{aligned} \boxed{x}+3y+4z&=0 \\ 2x+7y+9z&=0 \\ 4x+12y+12z&=4 \end{aligned}}$ What row actions are needed here to get zeroes under the boxed leader? Do them!

(HW3) $\boxed{\begin{aligned} \boxed{x}+y+z+4w&=0 \\ 0x+0y+5z+5w&=10 \\ 0x+2y+4z+6w&=12 \\ 0x+0y+5z+5w&=15 \end{aligned}}$ What is the 2nd leader? Move it to the second row using a switch then make its coeff = 1.

(HW4) $\boxed{\begin{aligned} x+y+z&=5 \\ 0x+0y+1z&=6 \\ 0x+0y+0z&=0 \end{aligned}}$ Box the leaders. Is this in Echelon form? If not, complete the row actions to Echelon form

(HW5) $\boxed{\begin{aligned} x+y+z&=5 \\ 0x+0y+1z&=6 \\ 0x+y+0z&=0 \end{aligned}}$ Box the leaders. Is this in Echelon form? If not, complete the row actions to Echelon form

(HW6) Solve the system in (HW5) by subing up

(HW7) Solve the system in (HW4)

(HW8) Solve the system in (HW1)

(HW9) Solve the system in (HW2)

(HW10) Solve the system in (HW3)

HW Solutions (only check after trying each)

Note your solution is only correct if you do the same row actions in the same order exactly as solved below. If you do something different, and do not know why it is wrong, send a question.

Homework Solutions

HW1

$$\begin{array}{l} 3x + 3y - 9z = 6 \\ x - y + z = 0 \\ x + 4y + 2z = 5 \end{array}$$

What is the first row action you should do? Do the row action

Solution: Must make the first coeff. in the upper left a 1

$$\xrightarrow{\rho_1 \to \frac{1}{3}\rho_1} \boxed{\begin{array}{l} 1x + 1y - 3z = 2 \\ x - y + z = 0 \\ x + 4y + 2z = 5 \end{array}}$$

because

$\frac{1}{3}(3x+3y-9z) = \frac{1}{3}(6)$

$x + y - 3z = 2$

scratch

HW2

$$\begin{array}{l} \boxed{x} + 3y + 4z = 0 \\ 2x + 7y + 9z = 0 \\ 4x + 12y + 12z = 4 \end{array}$$

What row actions are needed here to get zeroes under the boxed leader? Do them!

Always skew $\rho_i \to \rho_i + k\rho_1$

$$\xrightarrow{\begin{array}{l}\rho_2 \to \rho_2 - 2\rho_1 \\ \rho_3 \to \rho_3 - 4\rho_1\end{array}} \boxed{\begin{array}{l} x + 3y + 4z = 0 \\ 0x + 1y + 1z = 0 \\ 0x + 0y - 4z = 4 \end{array}}$$

scratch

$12 - 4 \cdot 3 = 0$

$12 - 4 \cdot 4 = 12 - 16 = -4$

$4 - 4 \cdot 0 = 4$

6:13 PM Fri Aug 13

Linear Algebra 0

HW3

$$\begin{aligned} \boxed{x} + y + z + 4w &= 0 \\ 0x + 0y + 5z + 5w &= 10 \\ 0x + \boxed{2y} + 4z + 6w &= 12 \\ 0x + 0y + 5z + 5w &= 15 \end{aligned}$$

What is the 2^{nd} leader?
Move it to the second row using a switch
then make its coeff = 1.

x is the first leader, all zeroes below x ✓
now check the 2nd column and we see 2y
so our second leader is y, box it

$$\xrightarrow[\text{switch}]{\rho_3 \leftrightarrow \rho_2} \begin{aligned} \boxed{x} + y + z + 4w &= 0 \\ 0x + \boxed{2y} + 4z + 6w &= 12 \\ 0x + 0y + 5z + 5w &= 10 \\ 0x + 0y + 5z + 5w &= 15 \end{aligned} \xrightarrow{\rho_2 \to \frac{1}{2}\rho_2} \begin{aligned} \boxed{x} + y + z + 4w &= 0 \\ 0x + 1y + 2z + 3w &= 6 \\ 0x + 0y + 5z + 5w &= 10 \\ 0x + 0y + 5z + 5w &= 15 \end{aligned}$$

HW4

$$\begin{aligned} \boxed{1x} + y + z &= 5 \\ 0x + 0y + \boxed{1z} &= 6 \\ 0x + 0y + 0z &= 0 \end{aligned}$$

Box the leaders.
Is this in Echelon form?
If not, complete the row actions to Echelon form

no leader in third row

we have zeroes below and to the left of our leaders ✓

do the leaders have coeff = 1? $1x = x$ ✓ $1z = z$ ✓

So yes in Echelon form

HW5

$$\begin{aligned} \boxed{x} + y + z &= 5 \\ 0x + 0y + \boxed{1z} &= 6 \\ 0x + \boxed{y} + 0z &= 0 \end{aligned}$$

Box the leaders.
Is this in Echelon form?
If not, complete the row actions to Echelon form

First entry in each row without a zero coefficient is a temporary leader.
Not yet Echelon form because we have 1y to the left + below 1z
So we need to switch $\rho_2 \leftrightarrow \rho_3$.

$$\longrightarrow \begin{aligned} \boxed{x} + y + z &= 5 \\ 0x + \boxed{y} + 0z &= 0 \\ 0x + 0y + \boxed{1z} &= 6 \end{aligned}$$

(HW5)
$\boxed{x} + y + z = 5$
$0x + 0y + \boxed{1z} = 6$
$0x + \boxed{y} + 0z = 0$

Box the leaders.
Is this in Echelon form?
If not, complete the row actions to Echelon form

First entry in each row without a zero coefficient is a temporary leader.
Not yet Echelon form because we have 1y to the left + below 1z
So we need to switch $\rho_2 \leftrightarrow \rho_3$.

$\boxed{x} + y + z = 5$
$0x + \boxed{y} + 0z = 0$
$0x + 0y + \boxed{1z} = 6$

If your answer checks in Echelon form but not original system then your error is in the row actions

(HW6) Solve the system in (HW5) by subing up

$\boxed{1x} + 1y + 1z = 5$
$0x + 1y + 0z = 0$
$0x + 0y + \boxed{1z} = 6$

$\boxed{z} = 6$
$0x + \boxed{y} + 0z = 0$
Solve for y $\boxed{y = 0}$
$x + y + z = 5$
Solve for x $x = 5 - y - z$
Sub in y + z $x = 5 - 0 - 6 = -1$ $\boxed{x = -1}$

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 6 \end{pmatrix} \right\}$$

Check
$1(-1) + 1 \cdot 0 + 1 \cdot 6 = 5$ ✓
$0(-1) + 1 \cdot 0 + 0 \cdot 6 = 0$ ✓
$0(-1) + 0 \cdot 0 + 1 \cdot 6 = 6$ ✓

Check in original system too!

$-1 + 0 + 6 = 5$ ✓
$0 + 0 + 6 = 6$ ✓
$0 + 0 + 0 = 0$ ✓

(HW4)

$$\begin{aligned} \boxed{1x} + y + z &= 5 \\ 0x + 0y + \boxed{1z} &= 6 \\ 0x + 0y + 0z &= 0 \end{aligned}$$

Box the leaders.
Is this in Echelon form?
If not, complete the row actions to Echelon form

no leader in third row

we have zeroes below and to the left of our leaders ✓

do the leaders have coeff $=1$? $1x = x$ ✓ $1z = z$ ✓

So yes in Echelon form

check

$1x + 1y + 1z = 5$

$1(-1-y) + 1y + 1\cdot 6 = -1-y+y+6 = -1+6 = 5$ ✓

$0x + 0y + 1z = 6$

$1(6) = 6$ ✓

$0x + 0y + 0z = 0$

$0 = 0$ ✓

(HW7) Solve the system in (HW4)

Already in Echelon Form
So we sub up.

last row

$0x + 0y + 0z = 0$

$0 = 0$ ok

second to last row:

$0x + 0y + \boxed{1z} = 6$

$z = 6$

top row:

$\boxed{1x} + 1y + 1z = 5$

Solve for x $\quad x = 5 - y - z$

sub in $z = 6$ $\quad x = 5 - y - 6 = -1 - y$

y is not a leader so y is free!

$y = y$ ← free to be himself.

Must show this work

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1-y \\ y \\ 6 \end{pmatrix} : \underset{\text{free}}{y} \in \mathbb{R} \right\}$$

← Final Answer

HW8 Solve the system in HW1

HW1 $\boxed{\begin{array}{l} 3x+3y-9z=6 \\ x-y+z=0 \\ x+4y+2z=5 \end{array}}$ What is the first row action you should do? Do the row action

Solution: Must make the first coeff. in the upper left a 1

$\xrightarrow{A\to\frac{1}{3}\rho_1}$ $\boxed{\begin{array}{l} \boxed{1x}+1y-3z=2 \\ x-y+z=0 \\ x+4y+2z=5 \end{array}}$ ⟵ Not yet Echelon Form

Need zeroes below the first leader x

$\rho_i \to \rho_i + k\rho_1$

$\xrightarrow[\rho_3\to\rho_3-\rho_1]{\rho_2\to\rho_2-\rho_1}$ $\boxed{\begin{array}{l} \boxed{1x}+1y-3z=2 \\ 0x\ \boxed{-2y}+4z=-2 \\ 0x+3y+5z=3 \end{array}}$ Next leader is y need coeff to be 1 (pause + try)

$\xrightarrow{\rho_2\to\frac{-1}{2}\rho_2}$ $\boxed{\begin{array}{l} \boxed{1x}+1y-3z=2 \\ 0x+\boxed{1y}-2z=1 \\ 0x+3y+5z=3 \end{array}}$ Need zeroes under 2^{nd} leader

$\rho_i \to \rho_i + k\rho_2$

$\xrightarrow{\rho_3\to\rho_3-3\rho_2}$ $\boxed{\begin{array}{l} \boxed{1x}+1y-3z=2 \\ 0x+\boxed{1y}-2z=1 \\ 0x+0y+\boxed{11z}=0 \end{array}}$

$5-3(-2)=5+6=11$

$3-3(1)=0$

Almost Echelon form

Need Coeff=1 on third leader!

$\xrightarrow{\rho_3\to\frac{1}{11}\rho_3}$ $\boxed{\begin{array}{l} \boxed{1x}+1y-3z=2 \\ 0x+\boxed{1y}-2z=1 \\ 0x+0y+\boxed{1z}=0 \end{array}}$ Echelon Form.

0 to left + below leaders

Sub up

bottom row $1z=0$ so $\boxed{z=0}$

next row $0x+1y-2z=1$

solve for leader $y=1+2z$

sub in $z=0$ $y=1+2\cdot 0=1$ $\boxed{y=1}$

next row (top) $1x+1y-3z=2$

solve for leader $x=2-y+3z$

sub in $z=0$ and $y=1$ $x=2-1+3(0)=1$

$\boxed{x=1}$

$$\text{Solution Set} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

Check in original system

$3(1)+3(1)-9(0)=6$ ✓

$1-1+0=0$ ✓

$1+4(1)+2(0)=5$ ✓

HW9 Solve the system in HW2

Solution (submit showing all work)

Your final answer should be a set

with $x=1$ $y=1$ $z=-1$

If you did not get this email me

QUESTION to look at your work

HW10 Solve the system in HW3

Solution (submit showing all work)

$\emptyset$

If you did not get this

email me QUESTION

to look at your work

It is very important to email me if you do not understand why any of your problems are incorrect. See how to email questions at the top of this document.

You can use your Lehman id and hand instead of your face in your selfie. This can be helpful if you are not dressed well or are shy or have difficulty taking a selfie.