Vectors

Q1.

Relative to a fixed origin *O*

- the point A has position vector 5**i** + 3**j** + 2**k**
- the point B has position vector 2**i** + 4**j** + *a***k**

where *a* is a positive integer.

(a) Show that $|\overrightarrow{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2)

(Total for question = 3 marks)

Q2.

Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

(Total for question = 5 marks)

Q3.

Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

(3)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $|\overrightarrow{OD}|$, writing your answer as a fully simplified surd.

(3)

(Total for question = 6 marks)

Q4.

Given that the point *A* has position vector 4**i** – 5**j** and the point *B* has position vector –5**i** –2**j**,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

(Total for question = 4 marks)

Q5.

(i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$

The angle between vector **m** and vector **n** is 30°
Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

(Total for question = 5 marks)

Q6.

Relative to a fixed origin, points *P*, *Q* and *R* have position vectors **p**, **q** and **r** respectively.
Given that

- *P*, *Q* and *R* lie on a straight line
- *Q* lies one third of the way from *P* to *R*

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(Total for question = 3 marks)

Q7.

Given that the point *A* has position vector 3**i** – 7**j** and the point *B* has position vector 8**i** + 3**j**,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for question = 4 marks)

Q8.

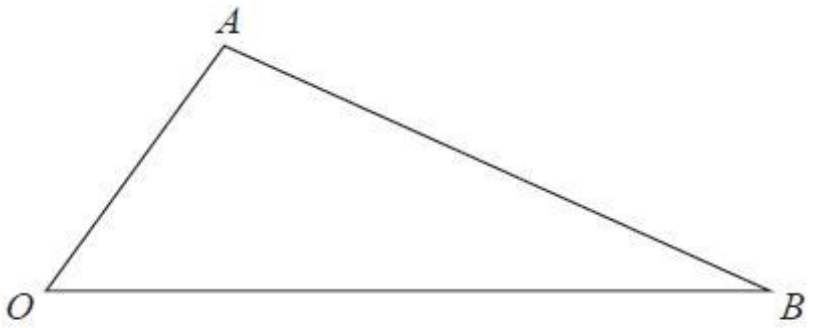

Figure 7

Figure 7 shows a sketch of triangle *OAB.*

The point *C* is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point *M* is the midpoint of *AB*.

The straight line through *C* and *M* cuts *OB* at the point *N*.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that *ON* : *NB* = 2 : 1

(2)

(Total for question = 6 marks)

Q9.

Relative to a fixed origin O,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC.

(2)

Points A, B, C and D are used to model the vertices of a running track in the shape of a quadrilateral.
Runners complete one lap by running along all four sides of the track.
The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

(Total for question = 6 marks)

Q10.

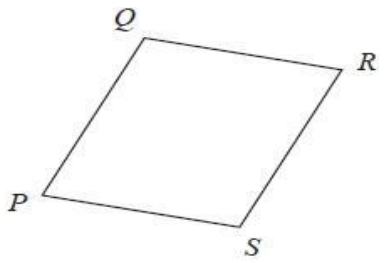

Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram *PQRS* is a rhombus.

(2)

(b) Find the exact area of the rhombus *PQRS*.

(4)

(Total for question = 6 marks)

Q12.

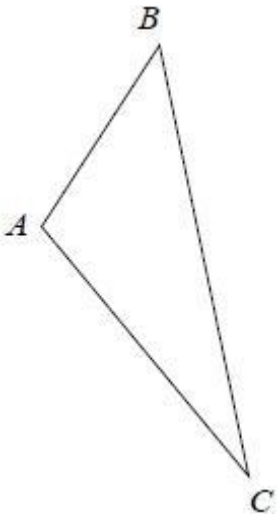

Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB}$ = 2**i** + 3**j** + **k** and $\overrightarrow{BC}$ = **i** – 9**j** + 3**k**,

show that ∠*BAC* = 105.9° to one decimal place.

(5)

(Total for question = 5 marks)

Q13.

Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + p\mathbf{j}$ where p is a constant

(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

(2)

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)

(Total for question = 7 marks)

Q14.

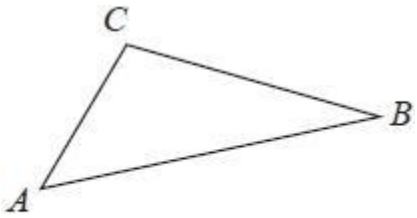

Figure 1

Figure 1 shows a sketch of triangle *ABC*.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that cos ABC = $\frac{9}{10}$

(3)

(Total for question = 5 marks)

Q15.

Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

(Total for question = 4 marks)