CHAPTER 7

ARGUMENTS AND THE METHODS OF DEDUCTION

7.1 Arguments

An argument in logic is a sequence of statements, consisted of premises (one or more statements) and conclusion (one statement). An argument is valid only if the truth of all statements in the premises result in the true statement in conclusion. Therefore, in an argument there are some words which show inference: "Therefore", "So", "Hence", "This means", "Thus", "Since" and so on.

In an argument, premises contain statements which are put before the inference word, whereas the conclusion appears after the inference word

Example 7.1

If the concepts of sets or functions are required for students,

they will learn basic mathematics.

The concepts of sets and logic are required for the students.

Therefore, students will learn basic mathematics.

Here the first two statements are premises, and the third statement is conclusion.

Example 7.2

If death is a long sleep, death is a pleasant rest.

If death is a beautiful dream, death is happiness.

Death is a long sleep or a beautifl dream.

So, death is a pleasant rest or a happiness.

Example 7.3

If we take a taxi to go to the train station, we will be able to come to the station early, and we take a walk then we will be there lately. Hence, if we take a taxi and we take a walk to go to the train station, then we will be able to come to the station early and lately.

Supose we translate this English sentences into logical symbol as follows:

T : We take a taxi to go to the train station

C : We will be able to come to the station early

W: We take a walk to the train station

L : We will be in the train station lately.

Then the above argument can be represented as

$$(T \rightarrow C) \wedge (W \rightarrow L)$$
$$\therefore (T \wedge W) \rightarrow (C \wedge L).$$

7.2 Inductive and Deductive Inferences

Consider the following argument:

All ducks I observe in my village are in white colour.

I have seen so many ducks.

So, all ducks are in white colour.

At a glance, if we read this argument, it seems that this argument is a good one (valid). This can happen as all statements in the premis can result in the conclusion logically, although it just "a possible situation". An inference drawn from the premises to the conclusion can be true or false as the premises tell us possible things. The inference which is taken from the premieses based on the possible things is called **inductive inference**. In inductive inference we have a set of statements, containing particular facts, specific situation, and we reach the general situation in conlusion. In other words, we start with a number of specific facts and we end with a general conclusion.

Now consider an argument below:

All human are mortal.

Ratnasari is human.

This means, Ratnasari is mortal.

In this argument, if the statements in the premises are correct then the statement in the conclusion is also true, as there is no other possible situation other than "Ratnasari is mortal".

The correct inference with one and only one possibility is called **deductive inference**. Its argument is called **deductive argument**. When the inference is inductive, the argument is callled **inductive argument**.

Here is another example of deductive argument:

Example 7.4

If I pass the exam, I will you give you a reward.

I pass the exam.

So, I give you a reward.

Example 7.5

The flood has reached the city and the road has been blocked.

Hence, the flood has reached the city.

Example 7.6

Martini is in Bandung or Manila

She is not in Bandung.

This means, she is in Manila.

The three arguments above are examples of valid arguments, where their premises have definitely implied their conclusions. It is important to note that a deductive argument is not necessarily a collection of correct statements which are meaningful. This means that statements in an argument may consist of statements containing some possible things or even incorrect statements. The main important thing is that there is "connection" between the premises and the conclusion, and it does not really matter whether the statements are meaningful or not.

Example 7.7

In the argument below,

If Leila does "something", then the society will probably hate her.

In fact, she does “something”.

Thus, the society probably hate her.

The conclusion contains possibility. It cannot been denied, however, that this argument has a correct conclusion provided that its premises are correct. Obviously, this kind of argument is deductive argument.

7.3 Proving the Validity of Arguments

In this section we will learn how to distinguish valid deductive argument from invalid ones. Generally, valid deductive argument is considered as an argument with correct premises and conclusion. This assumption is not necessarily correct, as there are many arguments having correct premises and conclusion but its deductive is incorrect. On the other hand, there are also deductive arguments with incorrect premises and conclusion. Observe the following examples.

Example 7.8

Indonesia is more famous than Bali.

There are artists on stage.

Therefore, teachers are hero without any symbolic appreciation.

Example 7.9

All imaginary numbers are complex number.

2 is an imaginary number.

So, 2 is a complex number.

The first example does not show a valid argument. One may claim that all premises are correct, and so is the conlusion. The premises, however, do not imply the conclusion. So, this argument is not valid. On the other hand, the second argument describes correct statement as its first premis and its conclusion. However, we understand also, that 2 is not an imagimary number, and imaginary numbers are included in the set of complex numbers. So, it is easy to understand that the implication is “2 is a complex number”. Although, one of its permisses is incorrect, this argument is valid. This argument can be considered as a deductive argument.

Based on the above explanation, we can see that a good deduction is not necessarily resulted from correct constructive statements. Any deduction in deductive argument is called valid deductive. This means valid is a good or a correct deduction without refering to the truth value of its statements. A deductive argument is valid if its conclusion is the logical implication of its premises.

Consider the following argument:

All human are creature.

Ratnasari is human.

So, Ratnasari is creature.

All angels are Greece person.

Ratnasari is an angel.

Therefore, Ratnasari is Greece person.

The first argument can be classified as a valid one. We are not in doubt that it is a valid argument, as intuitively, all statements in the premises and conclusion are correct, and the conclusion is the logical impact from its premises. What about the second argument? Is it correct? This argument is the same as the first one in terms of its structure.

All human are creature.	All angels are Greece person.
Ratnasari is human.	Ratnasari is an angel.
So, Ratnasari is creature.	So, Ratnasari is Greece person.

The second argument is also valid, although it is not easy to see its validity directly, as its premises are incorreect. The second argument, however, is identical to the argument in the left hand side, except the word "human" and "creature" which are replaced by "angel" and "Greece person". This means that these two arguments has the same structure. Schematicaly, both arguments have the following structure:

All *A* are *B*.

Ratnasari is *A*.

Therefore*,* Ratnasari is *B*.

This argument, consisting the word "Ratnasari", will remain the same, although it is replaced by other names. Its representation can be expressed as

All *A* are *B*.

C is *A*.

Therefore, *C* is *B*.

The structure of the above arguments consists of symbols containing statement variables, in such a way that when other statements are substituted into these statement variables, the result is an argument. The resulted argument from substitution process is called substitution instance of the argument. Statement variables in specific arguments are usually symbolized by small letters (lowercase) such as *p*, *q*, *r*, *s*, and so on.

An invalid argument has at least one substitution instance with correct premis and incorrect conclusion. On the other hand, a valid argument has no substitution instance in which its premises are correct and its conclusion is incorrect. Therefore, to find out whether a particular argument is valid or not, we can recognise it from its structure, if it is the same as the given valid arguments. In other words, to find out whether a given argument is valid, it is sufficient to show that the given argument has the same structure as the particular valid argument.

Now, what about the following arguments? Are they valid?

Nurmala is in Bandung or Tasikmalaya.

She is not in Bandung,

So, she is in Tasikmalaya.

We can see that this argument can be classified as a valid deductive argument. Its symbol can be represented as in the following:

A or *B*.

Not *A*.

Therefore, *B*.

Let *A* : Nurmala is in Bandung.

B : Nurmala is in Tasikmalaya.

Then the above argument can be converted into:

$A \vee B$

$\neg A$

$\therefore B$.

The specific form of this argumnent can be represented as

$p \vee q$

$\neg p$

$\therefore q.$

There is another method to find out whether or not a given argument is valid. This can be conducted by using truth table. However, before we use this table, first we should find a conditonal statement structure corresponds to the given statement. Remember, that each conditional statement always correponds to an argument. The premises of the argument correspond to the antecedent of the given conditional statement, whereas the conclusion corresponds to its consequent.

Example 7.10

Suppose we have an argument:

p

q

$\therefore p.$

This argument corresponds to conditional statement $(p \wedge q) \to p$. The next proving step is to show whether the truth table of the statement $(p \wedge q) \to p$ is a tautology. If it is a tatutology, the argument which we are proving is valid.

Example 7.11

Write a corresponding conditional statement which corresponds to the following argument:

$q \to r$

$p \to q$

$\therefore p \to r.$

Solution

The corresponding statement to the above argument is

$$((q \to r) \wedge (p \to q)) \to (p \to r).$$

7.4 Rules of Inference

Proving the validity of an argument having one or more simple states by using truth table is usually time consuming and, of course, boring. There is a short , simple, and straight strategy which can be applied to solve this problem. This can be done by the deriving conclusion from the argument. This means, that we produce a conclusion from the premises by applying a sequence of elementary valid arguments. Here, we mean by an elementary valid argument as a form of valid simple argument. Each substituion instance of an elementary argument is also a valid argument.

Example 7.12

Suppose we have an argument symbolized by the following:

$\neg A \vee \neg B$

$\neg\neg A$

$\therefore \neg$ I.

This argument is valid, as it is a substution instance of an elementary valid argument:

$p \vee q$

$\neg p$

$\therefore q$.

by substituting $\neg A$ for p and $\neg B$ for q.

In the rules of Inference we carry out deduction. This deduction is not only drawing a conclusion from its premises, but also constrcuting arguments under a sequence of a relativey simple proving strategy. The process of inference is again continued. Each produced conclusion (consisting of some parts) is considered as a new conclusion which can be infered to obtain next conclusion, and this process is repeated until the final conclusion is obtained. This procedure will be elaborately discussed in detail as the Rule of Inference below.

(a) Modus Ponen

We start with an example:

(1) If I pass the exam, we will go for watching movies in a cinema.
(2) If we go for watching movies in a cinema, we will miss our friend's birthday party.
(3) I pass the exam.
Therefore, we miss our friend's birthday party.

We know that the statement

$P \rightarrow Q$

P

$\therefore Q$

is a valid argument, which can be proved intuitively or by applying a truth table. The argument of this structure is called Modus Ponen (MP).

It is obvious that the first premis (1) and the third premis (3) in the above argument have the structure of modus ponen. As the result, we can draw a conclusion by making use of modus ponen. We obtain for (1) and (3) a new statement (4):

(4) We will go for watching movies in a cinema.

We repeat this step once again. Since (2) and (4) have also premis in modus ponen structure, we have

(5) We will miss our friend's birthday party.

From these steps we obtain (5), whereas (5) is the conclusion we look for. So, the given argument is a valid one.

In a simple form, we can write the whole process of the above proving steps as follows:

(1) $P \rightarrow Q$
(2) $Q \rightarrow R$
(3) P.

We focus our attention on the premises (1) and (3) which are obtained from the statement Q by making use of modus ponen. Hence, we can add

(4) Q

(obtained from (1) and (3) by using modus ponen). Then form (2) and (4) by using the similar process we have R, so we add the following:

(5) R

(from (2) and (4) by applying modus ponen).

The complete process can be written as follows.

1.	$P \supset Q$	Pr.
2.	$Q \supset R$	Pr.
3.	P	Pr/ $\therefore R$.
4.	Q	1, 3, MP
5.	R	2, 4, MP.

The symbols in the left hand side are statements of arguments. On the right hand side, all symbols represent the reason for the corresponding statements in the left hand side. Pr stands for Premis, while MP stands for modus ponen. The symbol Pr is sometimes omitted. All notation in the left hand side are explanation. The whole sequence and structure is called "direct proof" (formal proof).

Example 7.13

If corruption cannot be eradicated or there is no new investment coming, then if the revenue of the country cannot be lifted, the country will have financial problems.

The revenue of the country cannot be lifted.

If there is no new investment coming, the corruption will not be able to be eradicated or there will be no new investment coming.

There is no new investment coming.

So, the country will be bankrupt.

It is not to easy to see whether or not this argument is valid. Let's first convert the above argument into symbols as follows.

P: Corruption cannot be eradicated

Q: There is no new investment coming

R: The revenue of the country cannot be lifted

S: The country will have financial problems.

T: The country will be bankrupt.

We obtain a series of argument:

1.	$(P \vee Q) \rightarrow (\neg S \rightarrow \mathrm{R})$	Pr.
2.	$\neg S$	Pr.
3.	$Q \rightarrow T$	Pr.
4.	$T \rightarrow (P \vee Q)$	Pr.
5.	Q	Pr./ $\therefore R$.

From (3) and (5) we obtain T (using modus ponen), whereas between T and (4) results in $P \vee Q$. If $P \vee Q$ is combined with (1), we will have $\neg S \rightarrow \mathrm{R}$. The statement $\neg S \rightarrow \mathrm{R}$ together with (2) will give R, where R is the conclusion we are looking for. The whole proof can be rewritten as follows.

1.	$(P \vee Q) \supset (\neg S \rightarrow R)$	Pr.
2.	$\neg S$	Pr.
3.	$Q \rightarrow T$	Pr.

4. $T \rightarrow (P \vee Q)$ Pr.
5. Q Pr./ ∴ R.
6. T 3, 5, MP.
7. $P \vee Q$ 4, 6, MP.
8. $\neg S \rightarrow R$ 1, 7, MP.
9. R. 2, 8, MP.

We will require more valid arguments than just modus ponen. The following argument is also valid, but it cannot be proved by using modus ponen only.

1. $P \rightarrow Q$ Pr.
2. $Q \rightarrow R$ Pr.
3. $\neg P \rightarrow S$ Pr.
4. $\neg R$. Pr./ ∴ S.

(b) Modus Tollen

Form the same context, we now try to reveal teh last information: a correct conditional statement with incorrect consequence must have an incorrect antecedent. This conditional statement corresponds to the following argument:

$P \rightarrow Q$

$\neg Q$

∴ $\neg P$.

This construction is called modus tollen, abbreviated as MT. Tollen is derived from Greece word "tollere", which means "denying". We deny a particular part of the conditional statement.

Now consider the above argument:

1. $P \rightarrow Q$ Pr.
2. $Q \rightarrow R$ Pr.
3. $\neg P \rightarrow S$ Pr.
4. $\neg R$. Pr./ ∴ S.

We can see that the statement (2) and (4) correspond to the premises of MT, so we have

1. $P \rightarrow Q$ Pr.
2. $Q \rightarrow R$ Pr.
3. $\neg P \rightarrow S$ Pr.
4. $\neg R$ Pr./ ∴ S.
5. $\neg Q$ 2, 4, MT.
6. $\neg P$ 1, 5, MT.
7. S. 3, 6, MP.

The construction of MP and MT are usually mixed up with other invalid construction. This fallacy is so famous, debatable, since it has similar construction as MP and MT, but the form is incorrect. First, the form of the argument:

$P \to Q$

Q

$\therefore\ P.$

Is called fallacies of affirming concequent.

The other invalid form is:

$P \to Q$

$\neg P$

$\therefore\ \neg Q.$

which is called fallacies of denying antecedent.

(a) **Simplification**

In this section we add a number of other simple valid construction for checking formal validity of an argument.

Now consider the following argument:

(1) If Shinta reads the textbook, Reni reads it too.
(2) Shinta and Shanty read the textbook.

So, Shanty reads the textbook.

This argument is valid and its construction is as follows:

(1) $P \to Q$ Pr.
(2) $P \wedge R$ Pr./ $\therefore\ Q.$

Usual strategy for proving it in ordered deduction is by drawing conclusion that from $P \wedge R$ can be drawn conclusion P. This principle is called Simplification, abbreviated as Simp.

We now complete the proof of the above argument:

1.	$P \to Q$	Pr.
2.	$P \wedge R$	Pr./ $\therefore\ Q.$
3.	P	2, Simp.
4.	Q	1, 3, MP.

The other valid simple arguments in the Rules of Inference can be listed below.

(d) Conjunction (Conj)

P

Q

$\therefore\ P \wedge Q$

(e) **Hypothetical Syllogism (HS)**

$P \rightarrow Q$

$Q \rightarrow R$

$\therefore\ P \rightarrow S.$

(f) **Disjunctive Syllogism (DS)**

$A \vee B$

$\neg A$

$\therefore\ B.$

(g) Constructive Dilemma (CD)

$A \supset B$

$C \supset D$

$A \vee C$

$\therefore\ B \vee D.$

(h) Destructive Dilemma (DD)

$A \supset B$

$C \supset D$

$\neg B \vee \neg D$

$\therefore \neg A \vee \neg C.$

(i) Addition (Add)

A

$\therefore\ A \vee B.$

7.5 Rules of Replacement

In this section we will discuss a new rule which support the Rules of Inference: **Rules of Replacement**. When we discussed equivalency, we understood that two statements are called logically equivalent if they have the same truth value. So, if the whole parts or some parts of a compound statement are changed by a statement which is logically equivalent to the replaced statement, then the truth value of the new compound statement is the same as that of the original statement. This rules are also called **Principle of Extensionality**.

To make it easy to use the Rules of Replacement, the list of the equivalency are mentioned below. Note that to see the equivalency of these statements, we can use the truth table.

1. De Morgan's Theorem (de M)

$\neg\neg (p \wedge q) \equiv (\neg p \vee \neg q)$

$\neg (p \vee q) \equiv (\neg p \wedge \neg q)$.

2. Commutation (Comm)

$(p \vee q) \equiv (q \vee p)$

$(p \wedge q) \equiv (q \wedge p)$.

3. Association (Ass)

$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$

$[p \wedge (q \wedge r)] \equiv [(p \wedge q) \wedge r]$.

4. Distribution (Distr)

$[p \wedge (q \vee r)] \equiv [(p \wedge q) \vee (p \wedge r)]$

$[p \vee (q \wedge r)] \equiv [(p \vee q) \wedge (p \vee r)]$.

5. Double Negation (DN)

$p \equiv \neg\neg p$.

6. Transposition (Trans)

$(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$.

7. Material Implication (Impl)

$(p \rightarrow q) \equiv (\neg p \vee q)$.

8. Material Equivalence (Equiv)

$(p \Leftrightarrow q) \equiv [(p \rightarrow q) \wedge (q \rightarrow p)]$

$(p \Leftrightarrow q) \equiv [(p \wedge q) \vee (\neg p \wedge \neg q)]$.

9. Exportation (Exp)

$[(p \wedge q) \rightarrow r] \equiv [p \rightarrow (q \rightarrow r)]$.

10. Tautology (Taut)

$p \equiv (p \vee p)$

$p \equiv (p \wedge p)$.

Example 7.14

Below is an example of the proof of argument validity. The expressions in the right hand side explain the reason of the argument.

$(A \vee B) \rightarrow (C \wedge D)$. Pr.

1. $\neg C$	Pr. / $\therefore \neg B$.
2. $\neg C \vee \neg D$	2, Add.
3. $\neg (C \wedge D)$	3, de M.
4. $\neg (A \vee B)$	1, 4, MT.
5. $\neg A \wedge \neg B$	5, de M.
6. $\neg B \wedge \neg A$	6, Comm.
7. $\neg B$.	7, Simpl.

7.6 The Rules of Conditional Proof

We have already understood that a conditional statement corresponds to an argument. For example, $((P \vee Q) \wedge \neg P] \rightarrow Q$ corresponds to the argument

$P \vee Q$

$\neg P$

$\therefore Q$

From the above correspondence, it can be inferred that the antecedent of the statement corresponds to the premis of the argument, whereas the consequence of the statement corresponds to the conclusion of the argument.

If we have a statement in the form of $A \rightarrow (B \rightarrow C)$, then this statement is logically equivalent to a statement in the form of $(A \wedge B) \rightarrow C$, based on the Principle of Exportation. If the statement of the form $A \rightarrow (B \rightarrow C)$ is a Tautology, then the statement of the form $(A \wedge B) \rightarrow C$ is also a Tautology, since both statements are logically equivalent, one to another. The argument corresponds to the statement $A \rightarrow (B \rightarrow C)$ is

A

$\therefore B \rightarrow C$.

Whereas the argument corresponds to the statement $(A \wedge B) \rightarrow C$ is:

A

B

$\therefore\ C.$

Both arguments above are valid, if all corresponding statements are tautology. This means, that if we would like to show that following argument

A

$\therefore\ B \rightarrow C$

then we can draw a conclusion of the validity of the argument by converting it into an argument in the following construction:

A

B

$\therefore\ C$

This principle is called Rule of Conditional Proof.

Here we get an additional premis obtained from the antecedent of the conclusion. The next steps are deriving a consequence wich has to be proved. By making use of Rules of Conditional Proof, we can convert the following

$A \rightarrow B$

$C \rightarrow D$

$\neg B \vee \neg D$

$\neg A \vee \neg B$

$\therefore\ A \rightarrow \neg C.$

into

$A \rightarrow B$

$C \rightarrow D$

$\neg B \vee \neg D$

$\neg A \vee \neg B$

A

$\therefore\ \neg$C.

The complete proof is as follows:

1. $A \rightarrow B$ Pr.

2. $C \rightarrow D$ Pr.
3. $\neg B \vee \neg D$ Pr.
4. $\neg A \vee \neg B$ Pr./ $\therefore A \rightarrow \neg C$.
5. A / $\therefore \neg C$ (CP).
6. B 1, 5, MP.
7. $\neg\neg B$ 6, DN
8. $\neg D$ 3, 7, DS
9. $\neg C$ 2, 8, MT.

We can apply Rules of Conditional Proof as many as we need in proving a valid argument.

Example 7.15

$A \rightarrow (B \rightarrow C)$

$C \rightarrow (D \wedge E)$

$\therefore A \rightarrow (B \rightarrow D)$.

The proof of the validity of this argument can be constructed by converting this rargument into the following structure:

$A \rightarrow (B \rightarrow C)$

$C \rightarrow (D \wedge E)$

A

$\therefore B \rightarrow D$.

The resulted argument after underwent one process of Rules of Conditional Proof implementation is still in conditional statement form. So, we can reapply this rule to have:

$A \rightarrow (B \rightarrow \mathrm{C})$

$C \rightarrow (D \wedge E)$

A

B

$\therefore D$.

The complete proof can be presented as in the following sequence:

1. $A \rightarrow (B \rightarrow C)$ Pr.
2. $C \rightarrow (D \wedge E)$ Pr./ $\therefore A \rightarrow (B \rightarrow D)$.
3. A Pr./ $\therefore B \rightarrow D$. (CP).
4. B Pr./ $\therefore D$. (CP).
5. $B \rightarrow C$ 1, 3, MP.
6. C 5, 4, MP.
7. $D \wedge E$ 2, 6, MP.

8. *D*. 7, Simp.