

Fractions

(Latin for break)



A fraction is a part of something. The word fraction comes from a Latin word that means to break. If you fracture your leg, it means you broke your leg. Fractions are needed to measure these “fractured” parts of a whole. It would be nice if everything in the world could be measured and come out a whole number. Unfortunately for children who have to learn fractions, the world doesn’t work that way. When you weigh yourself, your weight is usually between two different weights, say 95 and 96 pounds. Because it is not important to have an exact weight, you could always round the measurement. But you cannot round all the time. What would happen to a cake if the recipe said $\frac{1}{8}$ cup of baking soda and you rounded to 0 cups, or even worse, 1 full cup. Your taste buds would be sorry you didn’t learn fractions.



A fraction has two parts $\longrightarrow$ $\frac{\text{numerator}}{\text{denominator}}$

The line in a fraction means two different things. The first meaning is “**divided by**”. Look at the fraction below:

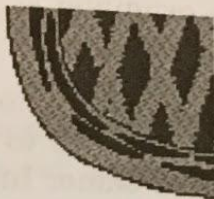
$\frac{2}{4} \longrightarrow$ Means 2 **divided by** 4.

The line in a fraction also means “**compare**”.

$\frac{2}{4} \longrightarrow$ Means 2 **compared to** 4.

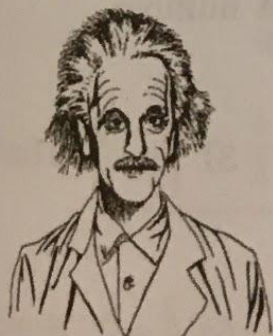
Adding Fractions

I have $\frac{1}{4}$ of a pie
and $\frac{1}{2}$ of a pie. How
much pie do I have?



It is much easier to add
fractions if you make
the denominators the
same. Change $\frac{1}{2}$ to $\frac{2}{4}$.
Now the problem is easy.
You have $\frac{3}{4}$ of a pie.

I can see how to add
 $\frac{1}{2}$ and $\frac{1}{4}$, but how
would I add $\frac{1}{3} + \frac{1}{4}$?



Adding fractions is easy if you remember two rules:

(1) Before you can add, you must change the denominators of the fractions so they are the same.

(2) To change the denominator of a fraction, you can multiply the denominator by any number you want — As long as you are fair and multiply the numerator by that same number.

Look at the following example: $\frac{1}{3} + \frac{1}{4}$

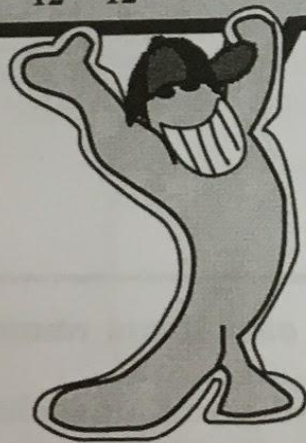
Since the denominators are not the same, we have to change them so they are the same. In this case, we will change both of the denominators to 12.

$$\begin{array}{r} \frac{1 \times 3}{4 \times 3} = \frac{3}{12} \\ + \frac{1 \times 4}{3 \times 4} = \frac{4}{12} \\ \hline \frac{7}{12} \end{array}$$

To get the first fraction's denominator to 12, we multiplied by 3. Then to be fair, we multiplied the numerator by 3.



To get the second fraction's denominator to 12, we multiplied by 4. Then to be fair to the numerator, we also multiplied it by 4. Now the fractions are easy to add:

$$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$


Try adding the fractions shown below. The hints after each problem give a number that the denominators should be changed to. Remember, you can multiply the denominator by any number, but you must be fair and multiply the numerator by the same number.

1) $\frac{2}{3} + \frac{1}{6}$ Hint: 6

2) $\frac{1}{2} + \frac{1}{3}$ Hint: 6

3) $\frac{1}{10} + \frac{2}{5}$ Hint: 10

4) $\frac{2}{3} + \frac{3}{4}$ Hint: 12

5) $\frac{3}{5} + \frac{2}{12}$ Hint: 60

6) $\frac{2}{5} + \frac{1}{4}$ Hint: 20

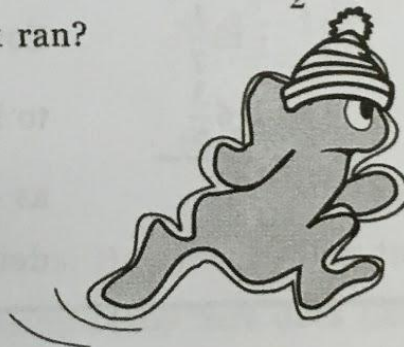
7) Kirsten saved half the money she needed for a new bike in the month of March. In April, she saved another $\frac{1}{8}$ of the money. What fraction of the money that she needs for the bike does she have saved?

8) Dan ate $\frac{1}{2}$ of an apple pie. He then ate $\frac{3}{8}$ of a pumpkin pie. How much pie did Dan eat altogether?



9) Mark had $2\frac{1}{3}$ pounds of gold and Michelle had $3\frac{2}{5}$ pounds of gold. How much gold did they have altogether?

10) Mark ran $\frac{1}{2}$ of a mile on Monday. On Tuesday he ran $\frac{3}{4}$ of a mile. On Wednesday and Thursday, he ran $\frac{1}{2}$ mile each day. What was the total distance Mark ran?



Subtracting Fractions

When you are subtracting fractions, just like in addition, you must make the denominators the same.

$$\frac{3}{4} = \frac{3}{4}$$

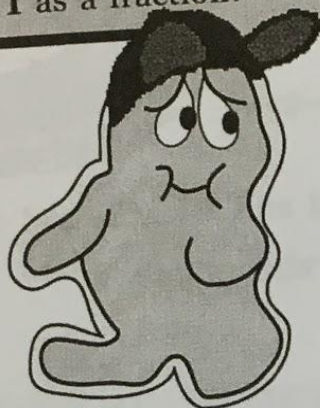
$$- \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$\frac{1}{4}$$

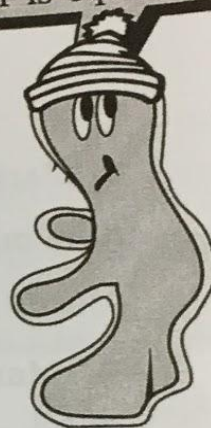
Make sure you are fair to the numerator and multiply it by the same number you multiplied the denominator by.



There is a very difficult kind of subtraction problem where you need to borrow. When you borrow, you borrow a 1, but how do you write 1 as a fraction?



You can write the number 1 as a fraction in many different ways: $\frac{4}{4}$ or $\frac{2}{2}$ or $\frac{9}{9}$. As a matter of fact, any fraction with an equal numerator and denominator is equal to 1.



Look at this subtraction problem:

$$\begin{array}{r} 12\frac{1}{7} \\ - 6\frac{3}{7} \\ \hline \end{array}$$

You cannot subtract $\frac{3}{7}$ from $\frac{1}{7}$ so you need to borrow 1 from the 12. We will write 1 as $\frac{7}{7}$ because our fractions have a 7 as a denominator.

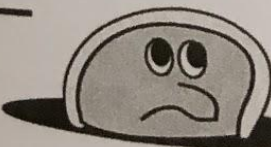
$$\begin{array}{r} 11\frac{1}{7} + \frac{7}{7} \\ - 6\frac{3}{7} \\ \hline \end{array}$$

Notice that the 12 is reduced to 11

because we borrowed 1 from the 12. Now

we can add $\frac{1}{7} + \frac{7}{7}$ and get $\frac{8}{7}$

$$\begin{array}{r} 11\frac{8}{7} \\ - 6\frac{3}{7} \\ \hline 5\frac{5}{7} \end{array}$$



The answer is $5\frac{5}{7}$. That was a little confusing, so I think I'll practice a few subtraction problems. Who took my hat?

Try the following subtraction problems. The first two are done for you.

1)
$$\begin{array}{r} 9 \\ - 6\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 8\frac{3}{3} \\ - 6\frac{1}{3} \\ \hline 2\frac{2}{3} \end{array}$$
 ← Borrowing one

2)
$$\begin{array}{r} 13\frac{1}{4} \\ - 5\frac{1}{2} \\ \hline \end{array}$$

Making the denominators the same $\rightarrow$

$$\begin{array}{r} 13\frac{1}{4} \\ - 5\frac{2}{4} \\ \hline \end{array}$$

Borrowing one $\rightarrow$

$$\begin{array}{r} 12\frac{1}{4} + \frac{4}{4} \\ - 5\frac{2}{4} \\ \hline 7\frac{3}{4} \end{array}$$

3) $\frac{3}{5} - \frac{1}{4}$

4) $2\frac{1}{2} - 1\frac{1}{16}$

5) $7 - 2\frac{1}{3}$

6) $8\frac{1}{4} - 3\frac{1}{3}$

7) Larry bought 10 candy bars. He gave $2\frac{2}{3}$ bars to his sister and $3\frac{2}{5}$ bars to his brother. How many candy bars does Larry have left?



8) If Anna's hair is $12\frac{1}{2}$ inches long and she gets $5\frac{1}{8}$ inches cut off during a haircut, how long will Anna's hair be after her haircut?

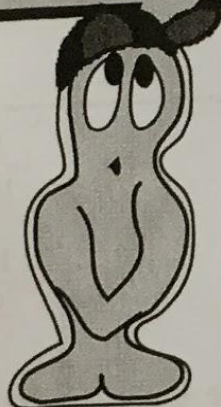
9) A carpenter bought a 10 foot board. He used three $2\frac{1}{5}$ foot long pieces for shelves. How much wood does he have left?

10) When Rick walks along the road, the distance to school is $3\frac{2}{3}$ miles. If he takes a path through the woods, the distance is $1\frac{7}{8}$ miles. How much shorter is the path through the woods?

Improper Fractions

When a fraction's numerator is larger than its denominator, we call it an improper fraction because it needs changing. If you had nine halves of something-----say pies-----you wouldn't say you had nine half pieces of pie, you would say you had $4\frac{1}{2}$ pies. If you had 22 quarter pieces of pie, you wouldn't say you had $\frac{22}{4}$ pies, you would say you had $5\frac{1}{2}$ pies.

To change improper fractions to mixed numbers, you simply divide the numerator by the denominator.



A good thing to remember about fractions is that the line in a fraction means divide.



Change the following improper fractions to whole numbers and proper fractions. (These are called mixed numbers.) The first one is done for you.

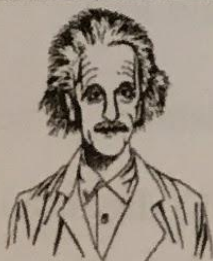
1) $\frac{7}{2} = 3\frac{1}{2}$

2) $\frac{8}{3}$

3) $\frac{10}{2}$

4) $\frac{15}{1}$

5) $\frac{24}{6}$



One of the most important things to remember about fractions is that the line in a fraction means divide. If you wanted to turn a fraction such as $\frac{1}{8}$ into a decimal, all you would need to do is the division problem: $1 \div 8 = .125$.

Mixed Numbers

When you divide or multiply fractions, sometimes you have to temporarily change mixed numbers into improper fractions. To change a mixed number into an improper fraction, say $5\frac{2}{3}$, you will need to multiply the whole number by the denominator of the fraction.

$$5\frac{2}{3}$$

$$5 \times 3 = 15$$

Now add the numerator $\longrightarrow 15 + 2 = 17$

17 is your improper fraction numerator.

The denominator stays the same. $\longrightarrow \frac{17}{3}$

Change the following mixed numbers into improper fractions. The first one is done for you.

1) $5\frac{2}{7} = \frac{37}{7}$

2) $8\frac{1}{3}$

3) $6\frac{3}{4}$

4) $21\frac{1}{2}$

5) $12\frac{2}{9}$

Reducing Fractions

If you had $\frac{2}{4}$ of a pie, you would probably say you had $\frac{1}{2}$ of a pie. You have just reduced a fraction. When we reduce fractions, we don't change the amount of the fraction, we just say the fraction in a different way. When you reduce fractions, you need to see if there is any number by which you can evenly divide both the numerator and the denominator. For example: $\frac{9}{12}$

Both the top and the bottom

of the fraction can be evenly

divided by 3.

$\longrightarrow \frac{9}{12}$ turns into $\frac{3}{4}$

Reduce the following fractions to their lowest terms. The first two are done for you.

1) $\frac{12}{15}$ The top and bottom can be evenly divided by 3 $\longrightarrow \frac{4}{5}$

2) $\frac{25}{100}$ Divided by 5 $\longrightarrow \frac{5}{20}$ Can still be divided by 5 $\longrightarrow \frac{1}{4}$

3) $\frac{12}{14}$

4) $\frac{6}{9}$

5) $\frac{15}{20}$

6) $\frac{10}{45}$

7) $\frac{21}{24}$

8) $\frac{24}{48}$

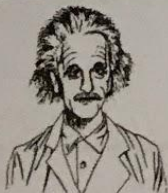
Multiplying Fractions

Multiplying fractions is much easier than addition or subtraction of fractions. When you multiply fractions, you simply multiply the numerators and then multiply the denominators:

$$\frac{1}{7} \times \frac{2}{5} = \frac{2}{35}$$

When you are multiplying mixed numbers such as $2\frac{1}{5} \times 3\frac{1}{4}$, you need to change them into improper fractions and then multiply. Let's see how this works in the mixed numbers given above:

$$2\frac{1}{5} \times 3\frac{1}{4} \longrightarrow \frac{11}{5} \times \frac{13}{4} = \frac{143}{20} \longrightarrow 7\frac{3}{20}$$



Before you multiply fractions, you should always reduce them if you can.

Try the following multiplication problems. The first one is done for you.

$$1) 35 \times 2\frac{1}{8} \longrightarrow \frac{35}{1} \times \frac{17}{8} \longrightarrow \frac{595}{8} \longrightarrow 74\frac{3}{8}$$

$$2) \frac{1}{2} \times \frac{1}{8}$$

$$3) \frac{1}{3} \times \frac{2}{3}$$

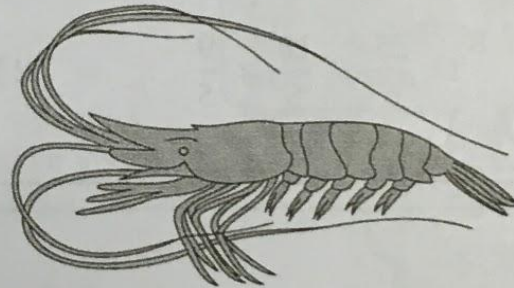
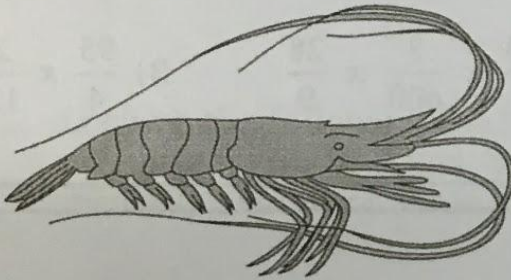
$$4) 2\frac{2}{3} \times 4\frac{1}{2}$$

$$5) 3\frac{2}{5} \times 2\frac{1}{3}$$

$$6) 9\frac{1}{2} \times \frac{1}{2}$$

$$7) 5\frac{1}{3} \times 3$$

8) Bonnie bought $8\frac{1}{5}$ pounds of shrimp at $9\frac{1}{2}$ dollars per pound. How much did Bonnie spend on shrimp?



9) If 18 people ate $\frac{2}{3}$ of a pie each, how many pies did they eat?

10) Ashley walks $\frac{1}{3}$ of a mile to school each day and $\frac{1}{3}$ of a mile home. How far does she walk in a school year that has 180 days?



Cross Reducing

Sometimes you can avoid multiplying large fractions by cross reducing before you multiply. When you reduce a normal fraction, you are using a number that can be evenly divided into the numerator and the denominator. Look at the fraction multiplication problem below:

$$\frac{10}{21} \times \frac{7}{24} \times \frac{12}{25}$$

If we went ahead and multiplied, not only would we have a lot of multiplying to do, but we would end up with a very large fraction. In cross reducing, we match up top and bottom numbers and reduce them. Look at how the matching numbers reduce.

$$\frac{\boxed{10}}{\textcircled{21}} \times \frac{\textcircled{7}}{\textcircled{24}} \times \frac{\textcircled{12}}{\boxed{25}} \longrightarrow \frac{2}{3} \times \frac{1}{2} \times \frac{1}{5} \longrightarrow \frac{2}{30} \longrightarrow \frac{1}{15}$$

Try cross reducing the following fractions before you multiply.

1) $\frac{25}{6} \times \frac{18}{3} \times \frac{9}{5}$

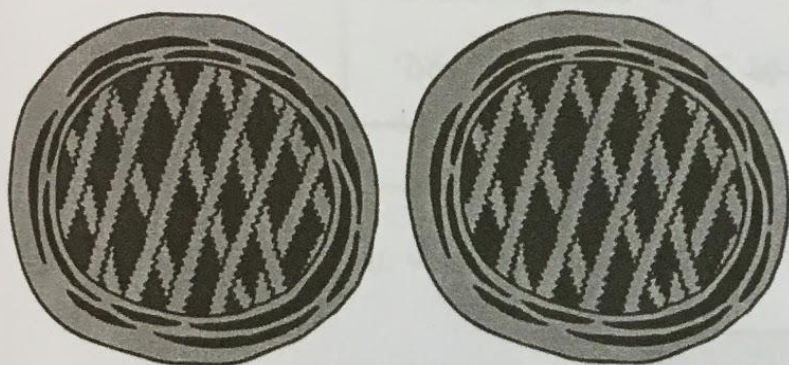
2) $\frac{100}{7} \times \frac{3}{600} \times \frac{28}{9}$

3) $\frac{95}{4} \times \frac{24}{125}$

Dividing Fractions

$$2 \div \frac{1}{4}$$

The division problem $2 \div \frac{1}{4}$ can be said another way: How many times will $\frac{1}{4}$ of a pie fit into 2 pies?



When you ask it like that, the problem is very easy. A quarter of a pie will fit into two pies 8 times.

Look at the following fraction division problems and see if you can solve them mentally. The first three are done for you.

1) $4 \div \frac{1}{2} \rightarrow \frac{1}{2}$ will fit into four 8 times.

2) $2\frac{1}{2} \div \frac{1}{4} \rightarrow \frac{1}{4}$ will fit into two and a half 10 times.

3) $3\frac{1}{4} \div \frac{1}{8} \rightarrow \frac{1}{8}$ will fit into three and a quarter 26 times.

4) $\frac{1}{2} \div \frac{1}{8}$

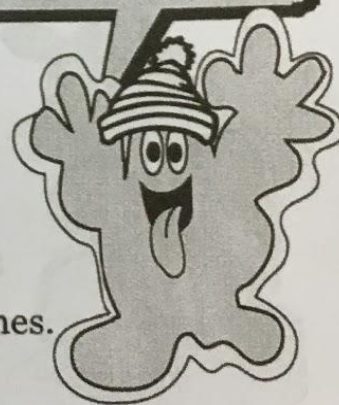
5) $2\frac{1}{4} \div \frac{1}{4}$

6) $\frac{3}{4} \div \frac{1}{8}$

7) $5 \div \frac{1}{5}$

8) $20\frac{1}{2} \div \frac{1}{4}$

9) $\frac{1}{2} \div \frac{1}{16}$



Sometimes it is impossible to mentally solve fraction division problems. When you are faced with this type of problem, you can solve it by using the method described below:

Problem: $\frac{6}{9} \div \frac{1}{18}$

Step 1 $\longrightarrow$ Flip the **second** fraction over and change the division sign to multiplication. $\longrightarrow \frac{6}{9} \times \frac{18}{1}$

Step 2 $\longrightarrow$ Cross reduce and then multiply. $\longrightarrow \frac{6}{1} \times \frac{2}{1} = \frac{12}{1} = 12$

That seems easy enough, but how do you do word problems?



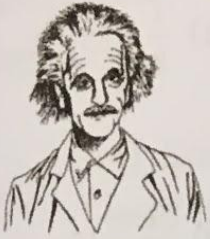
Dan had $6\frac{1}{2}$ candy bars that he wanted to give to 13 friends. How much candy would each friend get?

Step 1 $\longrightarrow$ Make $6\frac{1}{2}$ an improper fraction $\longrightarrow \frac{13}{2} \div \frac{13}{1}$

Step 2 $\longrightarrow$ Flip the second fraction and change division sign to multiplication $\longrightarrow \frac{13}{2} \times \frac{1}{13}$

Step 3 $\longrightarrow$ Cross reduce and then multiply $\longrightarrow \frac{1}{2}$

Each friend would receive $\frac{1}{2}$ of a candy bar.



When you are doing fraction division problems, it is very important to make sure that the fractions are in the proper order. The fraction that you are splitting up (dividend) always goes first:

This fraction goes second) This fraction goes first

Let's try another example:

A board that is $15\frac{3}{4}$ feet long is going to be cut into shelves that will be $2\frac{1}{4}$ feet long. How many shelves will there be?

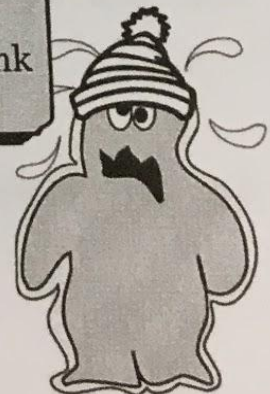
Since $15\frac{3}{4}$ is what is being split up, it will be the first fraction.

- | | | |
|--------|-------------------------------------|-------------------------------------|
| Step 1 | (Write problem) | $15\frac{3}{4} \div 2\frac{1}{4}$ |
| Step 2 | (Improper fractions) | $\frac{63}{4} \div \frac{9}{4}$ |
| Step 3 | (Flip and change to multiplication) | $\frac{63}{4} \times \frac{4}{9}$ |
| Step 4 | (Cross reduce and multiply) | $\frac{7 \times 1}{1 \times 1} = 7$ |

Cross reducing is fun and saves a lot of time.



I agree it saves time and energy, but I think you need to find some new hobbies if you think cross reducing is fun.



1) $2\frac{1}{2} \div \frac{1}{5}$

2) $9\frac{3}{5} \div \frac{5}{8}$

3) $\frac{1}{8} \div 25$

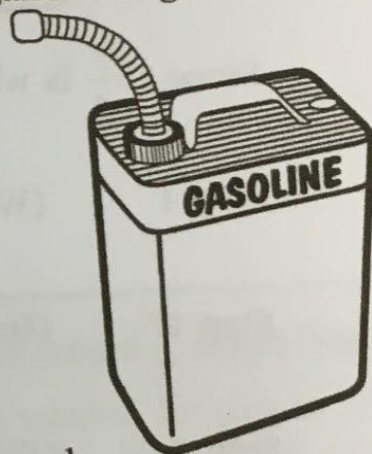
4) $20\frac{2}{5} \div 1\frac{1}{5}$

5) $12 \div \frac{1}{12}$

6) $\frac{1}{12} \div 12$

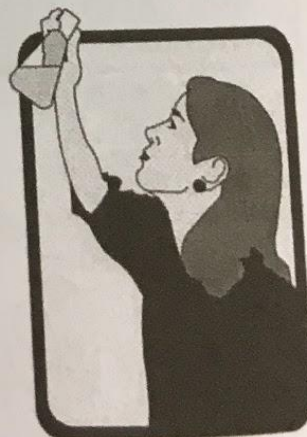
7) There were 48 people attending a picnic. If $3\frac{3}{7}$ pies were to be split evenly among the people at the picnic, how much pie would each person get?

8) Grace drove 159 miles and used $5\frac{3}{10}$ gallons of gas. How far could she go with one gallon of gas?



9) Claire had to cut a board that was $9\frac{1}{5}$ feet long into 6 equal pieces. How long would each piece be?

10) Heather worked for $15\frac{3}{4}$ hours. She was paid a total of $102\frac{3}{8}$ dollars. What is Heather's hourly wage?



Fractions

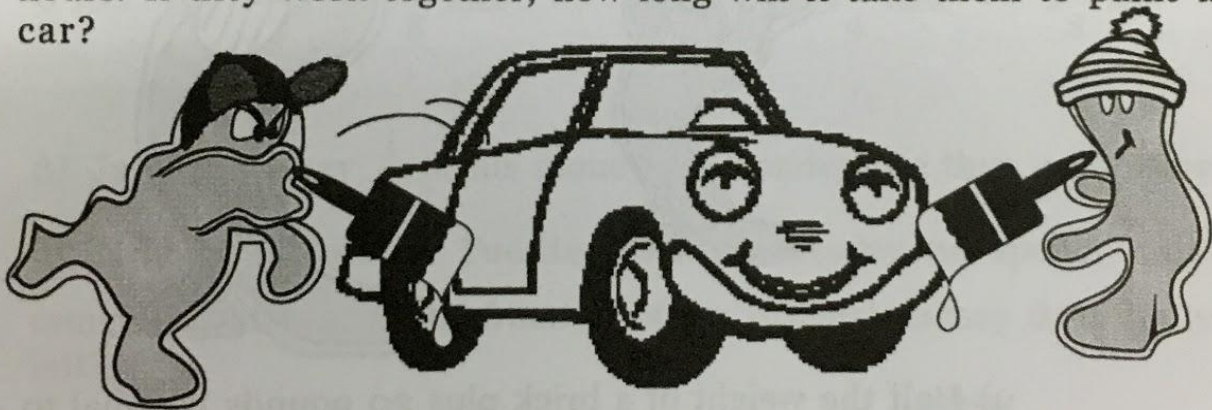
Level 1

1) Emily needs to read 85 pages from her history book. If she can read $1\frac{1}{16}$ pages per minute, how long will it take her to finish her assignment?



2) Gordan worked for $7\frac{5}{8}$ hours and was paid $13\frac{3}{4}$ dollars per hour. How much did Gordan earn?

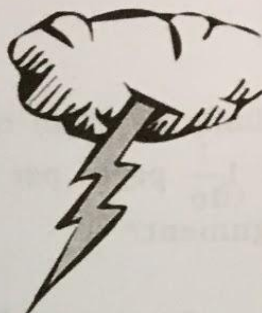
3) Jay can paint a car in 6 hours. Mike can paint the same car in 3 hours. If they work together, how long will it take them to paint the car?



4) Zach bought $8\frac{1}{8}$ pies and ate $5\frac{2}{5}$ of them over a week's time. How many pies does he have left?

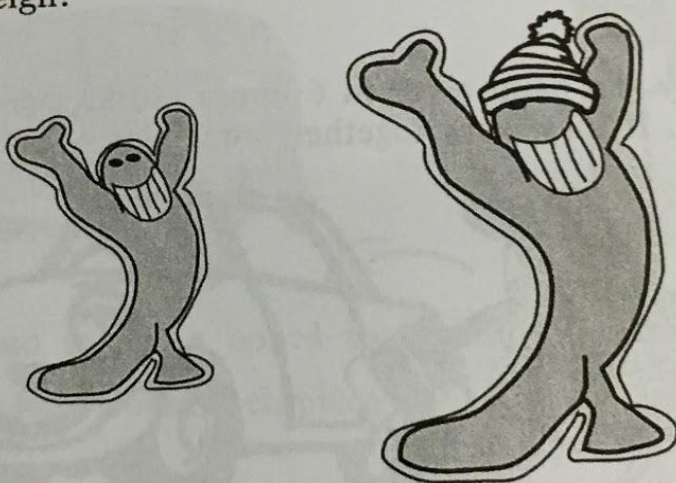
5) Jayme had 7 pizzas for a birthday party of 42 people. How much pizza should each person get?

- 6) If thunder travels at $\frac{1}{5}$ of a mile per second, how far will thunder travel in $12\frac{1}{5}$ seconds?

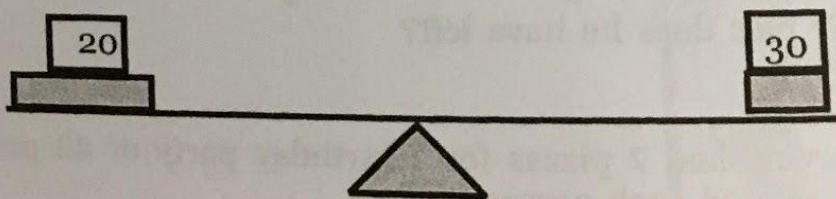


- 7) A store spends $\frac{1}{5}$ of its income on salaries and $\frac{1}{6}$ of its income on advertising. If the store pays \$180,000 for salaries, how much does it spend on advertising?

- 8) Nik is $\frac{7}{12}$ of his dad's weight. If Nik weighs 140 pounds, what does his dad weigh?



- 9) Half the weight of a brick plus 20 pounds is equal to $\frac{1}{3}$ the weight of the brick plus 30 pounds. How much does the brick weigh?



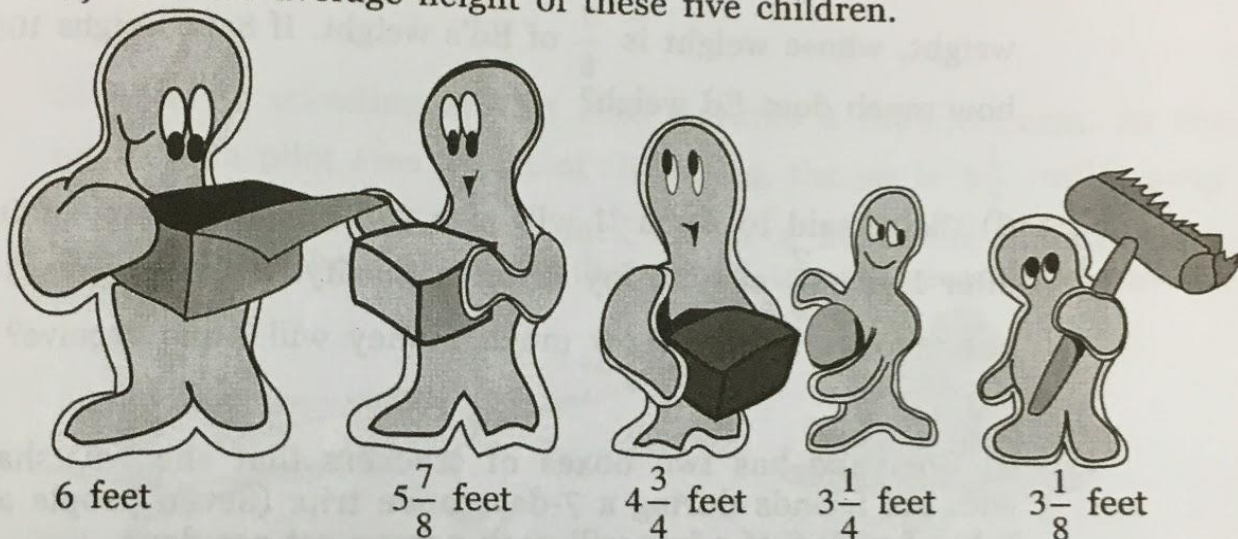
- 10) If it takes 5 hours to paint $\frac{5}{6}$ of a truck, how long will it take to paint the whole truck?

Fractions

Level 2

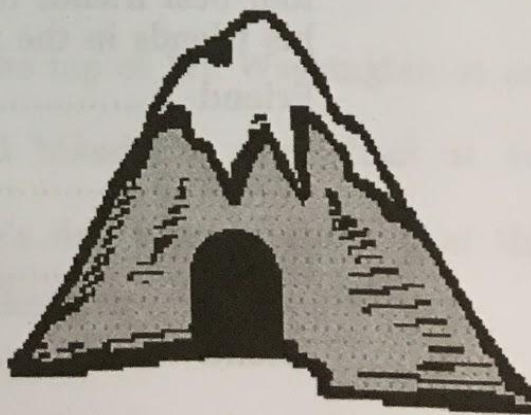
1) A meter is defined as the distance light travels in $\frac{1}{299,792,458}$ of a second. How many meters does light travel in $\frac{1}{4}$ of a second?

2) Find the average height of these five children.



3) Jay gave away $\frac{1}{2}$ of his money on Monday. He then gave away $\frac{1}{3}$ of what was left on Tuesday. On Wednesday he spent $\frac{1}{4}$ of his remaining money. What fraction of his original money does he have left?

4) Eight workers took $2\frac{1}{4}$ days to dig half of a tunnel. If only two workers are going to finish the tunnel, how long will it take them?



5) What is $\frac{1}{8}$ of $\frac{3}{11}$ of 66?

6) Use a fraction to compare a white blood cell's life span to a nerve cell's life span.

White blood cell.....	14 days
Red blood cell.....	120 days
Liver cell.....	18 months
Nerve cell.....	100 years

7) Sara's weight is $\frac{1}{2}$ of Luke's weight, whose weight is $\frac{3}{4}$ of Dan's weight, whose weight is $\frac{7}{8}$ of Ed's weight. If Sara weighs 105 pounds, how much does Ed weigh?

8) Claire said to Anna "I will give you half of my remaining money after I give $\frac{7}{8}$ of it to my favorite charity. If Claire gave \$43,750 to her favorite charity, how much money will Anna receive?

9) Christine has two boxes of crackers that she will share evenly with six friends during a 7-day canoe trip. (Seven people altogether) What fraction of a box will each person eat per day?



10) Ben, who owns a music company, decided that he wanted his four best friends to be part owners in the business. He gave shares to his friends in the following way:

Friend 1..... $\frac{1}{3}$ of the company

Friend 2..... $\frac{1}{4}$ of the company

Friend 3..... $\frac{1}{5}$ of the company

Friend 4..... $\frac{1}{6}$ of the company

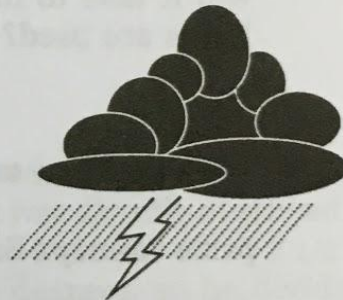
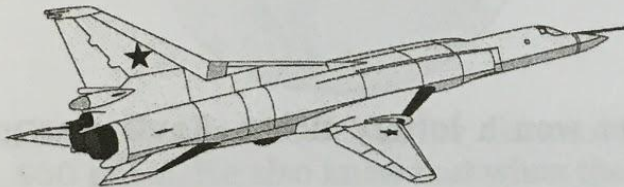
What fraction of the company does Ben still own?

Einstein Level



1) Shannon bought a piece of plywood that was $4\frac{1}{2}$ feet wide and $8\frac{1}{4}$ feet long. She cut out a three foot diameter circle for a table she was making. Shannon then sold the leftover plywood to a friend for \$1.75 per square foot. How much did Shannon's friend pay for the leftover plywood?

2) A jet is traveling at 720 mph towards a thunderstorm. At the instant the pilot sees a bolt of lightning, the jet is $8\frac{1}{2}$ miles away from the storm. How long until the pilot hears the thunder from that lightning bolt? (Assume that sound takes 5 seconds to travel a mile.)



3) A father left all his money to his three children. Child A received $\frac{1}{6}$ of his father's money while child B received $\frac{1}{7}$. Child C, who received the remainder of the money was given \$23,490. How much money did child A receive?

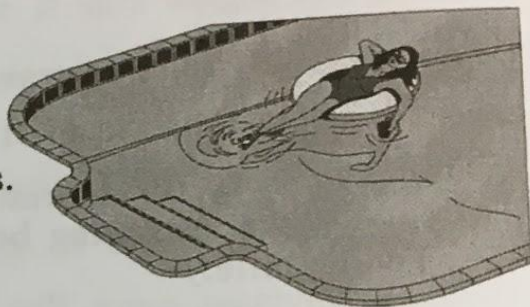
4) Luke hiked up a $4\frac{1}{2}$ mile trail to the top of Mt. Washington at an average speed of $2\frac{1}{7}$ mph. His dad hiked the same trail at an average speed of $2\frac{1}{4}$ mph. If Luke's dad reaches the top of the mountain at noon, what time will Luke arrive?

5) When his dad reaches the top, what fraction of a mile does Luke have remaining?

6) Wendy spent $\frac{3}{8}$ of her money at the first store she visited. She then spent $\frac{3}{4}$ of her remaining money at the second store. If Wendy had \$20 left, how much money did she start with?

7) Kyle can paint a fence in 4 hours while Gabe takes 6 hours to paint the same fence. If they work together, how long will it take them to paint the fence?

8) Three hoses are filling a swimming pool. If only hose A was used, it would take six hours to fill the pool, while using only hose B would take two hours. Hose C is identical to hose A. How long will it take to fill the pool if all three hoses are used?



9) When her mother won a lottery ticket, Kayla was given $\frac{1}{3}$ of the prize money. Each of her four brothers received $\frac{1}{4}$ of the remaining money. When Kayla and one of her brothers put their money together, they had \$75,000. What was the total amount of the lottery winnings?

10) A business is planning to provide sandwiches at their company picnic. Adults eat $1\frac{3}{4}$ times the amount that school-aged children eat, and babies eat half of what school-age children eat. 56 adults, 14 school-age children and 28 babies are attending the picnic. If adults eat $\frac{5}{8}$ of a sandwich, how many sandwiches should be ordered?



Solutions

Chapter 6: Fractions

(Page 86)

- 1) $5/6$
- 2) $5/6$
- 3) $5/10 = 1/2$
- 4) $17/12 = 1\frac{5}{12}$
- 5) $23/30$
- 6) $13/20$
- 7) $5/8$
- 8) $7/8$ of the pie
- 9) $5\frac{11}{15}$ pounds of gold
- 10) $2\frac{1}{4}$ miles

(Page 89)

- 1) $2\frac{2}{3}$
- 2) $7\frac{3}{4}$
- 3) $7/20$
- 4) $1\frac{7}{16}$
- 5) $4\frac{2}{3}$
- 6) $4\frac{11}{12}$
- 7) $3\frac{14}{15}$
- 8) $7\frac{3}{8}$
- 9) $3\frac{2}{5}$
- 10) $1\frac{19}{24}$

(Page 90)

- 1) $3\frac{1}{2}$
- 2) $2\frac{2}{3}$
- 3) 5
- 4) 15
- 5) 4

(Page 91)

- 1) $37/7$
- 2) $25/3$
- 3) $27/4$
- 4) $43/2$
- 5) $110/9$

(Page 92)

- 1) $4/5$
- 2) $1/4$
- 3) $6/7$
- 4) $2/3$
- 5) $3/4$
- 6) $2/9$
- 7) $7/8$
- 8) $1/2$

(Page 93)

- 1) $74\frac{3}{8}$
- 2) $1/16$
- 3) $2/9$
- 4) 12
- 5) $7\frac{14}{15}$
- 6) $4\frac{3}{4}$
- 7) 16
- 8) \$77.90
- 9) 12 pies

(Page 94)

- 1) 45
- 2) $2/9$
- 3) $4\frac{14}{25}$

(Page 95)

- 1) 8
- 2) 10
- 3) 26
- 4) 4
- 5) 9
- 6) 6
- 7) 25
- 8) 82
- 9) 8

(Page 98)

- 1) $12\frac{1}{2}$
- 2) $15\frac{9}{25}$
- 3) $1/200$
- 4) 17
- 5) 144
- 6) $1/144$
- 7) $1/14$ pie
- 8) 30 miles
- 9) $1\frac{8}{15}$ feet

Level 1 (Page 99-100)

Note: It is often very difficult for students to figure out what numbers to multiply or divide when problems contain fractions. If students substitute the number "2" for the smaller number in the problem and a "10" for the larger number, it makes the steps to solving the problem much easier to see.

1) 80 minutes

If you substitute 2 and 10, the problem becomes: Emily needs to read 10 pages and reads 2 pages per minute, how long will it take her? Now it is obvious that division is the correct operation.

$$\frac{85}{1} \div \frac{17}{16} \text{ is the same as } \frac{85}{1} \times \frac{16}{17} = 80$$

2) \$104.84

$$7\frac{5}{8} \times 13\frac{3}{4} \text{ is the same as } \frac{61}{8} \times \frac{55}{4} = \frac{3355}{32} = 104.84$$

3) 2 hours

In one hour, Jay can paint $\frac{1}{6}$ of the car while Mike can paint $\frac{1}{3}$ of the car. Together they can paint $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ of the car in one hour. It will take them 2 hours to paint the whole car.

4) $2\frac{29}{40}$ pies

$$8\frac{1}{8} - 5\frac{2}{5} = 2\frac{29}{40}$$

5) $\frac{1}{6}$ pizza

$$7 \text{ pizzas} \div 42 \text{ people} = 7/42 = 1/6$$

6) 2.44 miles

If thunder travels at 2 miles per second, how far will it travel in 10 seconds? Now it is obvious that you need to multiply to get the answer: $2 \times 10 = 20$ miles

$$\frac{1}{5} \times 12\frac{1}{5} \text{ is the same as } \frac{1}{5} \times \frac{61}{5} = \frac{61}{25} = 2.44 \text{ miles}$$

7) \$150,000

If a store spends $\frac{1}{5}$ of its income on salaries (\$180,000), then its total income must be $5 \times$

$$\$180,000 = \$900,000$$

$$\frac{1}{6} \text{ of } \$900,000 \text{ is equal to } \$150,000$$

8) 240 pounds

If $\frac{7}{12}$ of his dad's weight is 140 pounds, then $\frac{1}{12}$ is $140 \div 7 = 20$ pounds
 If $\frac{1}{12}$ is equal to 20 pounds, then $\frac{12}{12}$ must equal $12 \times 20 = 240$ pounds.

9) 60 pounds

n = weight of the brick

$$\frac{1}{2}n + 20 = \frac{1}{3}n + 30$$

Subtract $\frac{1}{3}n$ from both sides: $\frac{1}{6}n + 20 = 30$

Subtract 20 from both sides: $\frac{1}{6}n = 10$

Multiply both sides by 6: $n = 60$

10) 6 hours

If it takes 5 hours to paint $\frac{5}{6}$ of a truck, it must take one hour to paint only $\frac{1}{6}$ of the truck.
 $\frac{6}{6}$ of the truck would therefore take 6 hours.

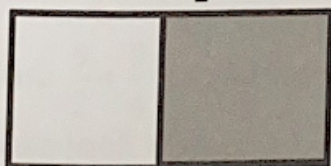
Level 2 (Page 101-102)
1) 74,948,114.5 meters

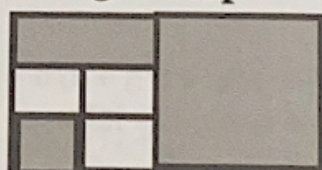
Change numbers to 2 and 10 to help the brain understand the problem: If light travels a meter in 2 seconds, how far does it travel in 10 seconds? Here it is easy to see that you would divide 10 by 2 = 5 meters.

Changing the process to the real problem: $\frac{1}{4} \div \frac{1}{299,792,458} = 74,948,114.5$

2) $4\frac{3}{5}$ feet

Add the fractions and divided by 5: $6 + 5\frac{7}{8} + 4\frac{3}{4} + 3\frac{1}{4} + 3\frac{1}{8} = 23$ $23 \div 5 = 4\frac{3}{5}$

3) $\frac{1}{4}$
1st Step

2nd Step

3rd Step


The small squares in the third figure are each $\frac{1}{12}$ of the large rectangle: $\frac{3}{12} = \frac{1}{4}$

4) 9 days

If one worker finished the tunnel: $8 \times 2\frac{1}{4} = 18$ days 2 workers: $18 \div 2 = 9$ days

5) $2\frac{1}{4}$

$$\frac{1}{8} \times \frac{3}{11} \times \frac{66}{1} \text{ (Cross reduce)} = \frac{1}{8} \times \frac{3}{1} \times \frac{6}{1} = \frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$$

6) $\frac{7}{18,250}$

$$\frac{\text{White blood cell: 14 days}}{\text{Nerve cell: } 365 \times 100 \text{ years} = 36,500 \text{ days}} = \frac{14}{36,500} \text{ (Reduce)} = \frac{7}{18,250}$$

7) 320 pounds

Sara's weight: 105 pounds

Luke's weight is twice as much as Sara's weight: 210 pounds

Luke's weight of 210 pounds is $\frac{3}{4}$ of Dan's weight. If $\frac{3}{4}$ is 210, then $\frac{1}{4}$ must be $210 \div 3 = 70$ pounds. If $\frac{1}{4} = 70$ then $\frac{4}{4}$ (or all) must equal 280 pounds

Dan's weight: 280 pounds

Dan's weight of 280 pounds is $\frac{7}{8}$ of Ed's weight. $\frac{1}{8}$ of Ed's weight must be $280 \div 7 = 40$ pounds. If $\frac{1}{8} = 40$ pounds, then $\frac{8}{8} = 8 \times 40 = 320$ pounds

Ed's weight: 320 pounds

8) \$3,125If $\frac{7}{8}$ of Claire's money is \$43,750, then $\frac{1}{8}$ of her money must be $\$43,750 \div 7 = \$6,250$ If $\frac{1}{8}$ is equal to \$6,250, then $\frac{8}{8}$ (or all) of Claire's money must be equal to

$$\$6,250 \times 8 = \$50,000$$

$$\$50,000 - \$43,750 = \$6,250 \text{ remaining}$$

$$\text{Half of } \$6,250 = \$3,125$$

9) 2/49 of a box2 boxes $\div$ 7 people = $\frac{2}{7}$ of a box for each person for the whole trip.Because the trip is a 7 day trip, the $\frac{2}{7}$ of a box must be divided by 7 to find the amount for

each day. $\frac{2}{7} \div \frac{7}{1} = \frac{2}{7} \times \frac{1}{7} = \frac{2}{49}$

10) 1/20Ben gave away $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{57}{60}$ of his company.Ben had $\frac{3}{60}$ of his company left. $\frac{3}{60}$ reduces to $\frac{1}{20}$

Einstein Level (Page 103-104)**1) \$52.61**

Area of plywood: $4.5 \text{ feet} \times 8.25 \text{ feet} = 37.125 \text{ square feet}$

Area of round table top: $3.14 \times 1.5 \times 1.5 = 7.065 \text{ square feet}$

(Area of circle is $\pi \times r^2$)

Amount of leftover wood is found by subtracting: $37.125 - 7.065 = 30.06 \text{ square feet}$

Cost: $30.06 \text{ square feet} \times \$1.75 = \$52.605$

2) 21.25 seconds

720 miles in one hour (3600 seconds) is the same as $3600 \div 720 = 5 \text{ seconds per mile}$

A jet traveling at 720 miles per hour takes 5 seconds to travel one mile.

Sound takes 5 seconds to travel one mile.

Because the jet and the sound are traveling towards each other, they get a mile closer every 2.5 seconds.

Two miles would take 5 seconds; three miles would take 7.5 seconds; four miles would take 10 seconds. They need to cover 8.5 miles, so it will take them $8.5 \times 2.5 = 21.25 \text{ seconds}$

3) \$5670

After Child A and Child B are given their money, there is $1 - \frac{1}{6} - \frac{1}{7} = \frac{29}{42}$ of the money left. (The number one in the subtraction problem stands for the whole amount.)

If $29/42$ are equal to \$23,490, then $1/42$ is equal to $\$23,490 \div 29 = \810

If $1/42$ is equal to \$810, then $42/42$ (all the money) is equal to $42 \times \$810 = \$34,020$

$1/6$ of \$34,020 = \$5670

4) 6 minutes later (12:06)

If Luke hiked a 10 mile trail at a speed of 2 miles per hour, it is obvious that it would take him $10 \div 2 = 5 \text{ hours}$ This tells us that we must divide.

Luke: $\frac{9}{2} \div \frac{15}{7}$ is the same as $\frac{9}{2} \times \frac{7}{15} = \frac{21}{10}$ or $2\frac{1}{10} \text{ hours}$

Dad: $\frac{9}{2} \div \frac{9}{4}$ is the same as $\frac{9}{2} \times \frac{4}{9} = \frac{4}{2}$ or 2 hours

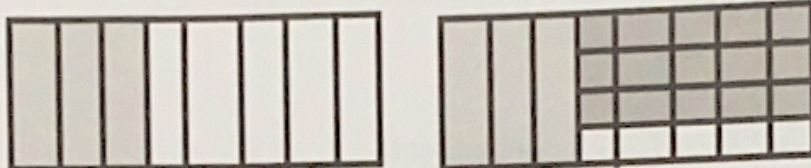
Luke took $1/10$ of an hour longer to climb the mountain. $1/10$ of an hour is 6 minutes

5) $3/14$ mile

How far will Luke travel in $1/10$ hour?

Because he travels $2\frac{1}{7}$ miles in an hour, he will travel $2\frac{1}{7} \times \frac{1}{10} = \frac{3}{14}$ miles in $1/10$ of an hour.

6) \$128



The remaining 5 white squares are $1/32$ each. If $5/32$ are equal to \$20, then $1/32$ is equal to $\$20 \div 5 = \4 . If $1/32$ is equal to \$4, then $32/32$ are equal to $32 \times 4 = \$128$.

7) 2 hours and 24 minutes

Kyle will paint $1/4$ of the fence in one hour.

Gabe will paint $1/6$ of the fence in one hour.

Together they will paint $1/4 + 1/6 = 5/12$ of the fence in one hour. If $5/12$ are equal to 60 minutes, then $1/12$ is equal to $60 \text{ minutes} \div 5 = 12 \text{ minutes}$.

If $1/12$ is equal to 12 minutes, then $12/12$ are equal to $12 \times 12 = 144 \text{ minutes}$.

8) 1 hour and 12 minutes

Hose A fills $1/6$ of the pool in one hour.

Hose B fills $1/2$ of the pool in one hour.

Hose C fills $1/6$ of the pool in one hour.

Together they fill $1/6 + 1/2 + 1/6 = 5/6$ of the pool in one hour.

If $5/6$ are equal to 60 minutes, then $1/6$ is equal to $60 \text{ minutes} \div 5 = 12 \text{ minutes}$.

If $1/6$ is equal to 12 minutes, then $6/6$ are equal to $6 \times 12 = 72 \text{ minutes}$.

9) \$150,000

Sister	Brother	Brother
	Brother	Brother

From the drawing, it is easy to see that each brother had $1/6$ of the total.

Sister's $1/3 + \text{brother's } 1/6 = 1/2$ of the total = \$75,000

If $1/2$ the total is equal to \$75,000, then the total is equal to \$150,000

10) 45 sandwiches

Adults: $5/8$ of a sandwich

$$56 \times 5/8 = 35 \text{ sandwiches}$$

School children: The problem states that $1\frac{3}{4}$ of the children's amount = $5/8$ $1\frac{3}{4}n = 5/8$

Divide both sides by $7/4$ $n = 5/14$ of a sandwich $5/14 \times 14 \text{ children} = 5 \text{ sandwiches}$

Babies: Babies eat half as much as school children, but there are twice as many babies. Therefore, the babies also need 5 sandwiches.

$$35 + 5 + 5 = 45 \text{ sandwiches}$$