Roll No…………………….. **Total No. of Printed Pages: 1**

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B.Sc. (Hons.) Math (Semester – 4th)

LINEAR ALGEBRA

Subject Code: BMATS1402

Paper ID: [19131217]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Define Subspace of a vector space.
b) Define Quotient Space.
c) What is Basis of vector space?
d) Define Linear Transformation.
e) Define Null space of a linear transformation.
f) Define Rank of a linear transformation.
g) Find the characteristic polynomial of the matrix $A = [2\ 3\ 1\ 7\ ]$
h) Define Non- Singular Transformation.
i) Define Invertible operator.
j) Define Invariant Subspace.

Section – B **(5 marks each)**

Q2. Prove that the intersection of two subspaceW_1 and W_2 of a vector space V(F) is also a subspace.

Q3. Find a basis and dimension of the subspace W of R^4 generated by the following vectors: $(1, 2, 3, 5), (2, 3, 5, 8), (3, 4, 7, 11), (1, 1, 2, 3)$.
Also extend these to a basis of R^4.

Q4. Find the eigen values and the corresponding eigen vector of the following matrix $A = [1\ 4\ 2\ 3\ ]$

Q5. Show that the Linear Transformation, $T: R^3 \to R^3$ defined by $T(x, y, z) = (x, \lambda y, z)$ where λ is a fixed real non-zero is isomorphism.

Q6. Let T be a linear operator on R^2 defined by

$$T(x, y) = (4x - 2y, 2x + y)$$

Find the matrix of T relative to the basis B ={(1,1);(-1,0)}

Section – C **(10 marks each)**

Q7. The linear transformation $T: R^3 \to R^3$ be defined by

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$$

Find a basis and dimension of

i) Its range space
ii) Its null space

Also verify Rank-Nullity Theorem.

Q8. Prove that, two finite dimensional vector spaces $V(F)$ and $W(F)$ over the same field F are isomorphic iff they have the same dimensions.

Q9. Given that $B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ is basis of R^3. Find the dual basis of B.