# Fuzzy Interpretation of the quantum Heisenberg uncertainly principle at high energy

H. Torres-Silva

With quantum mechanics came the Heisenberg principle, also known as the uncertainty principle, which tells us that it is impossible to know the position and speed of a particle simultaneously. Furthermore, it turns out that this Heisenberg principle is an intrinsic quality of nature. Its smallest building blocks of matter simply *are* fuzzy and diffuse objects. “The uncertainty principle means that you can’t know everything about a certain system at any given time”. Heisenberg [1], spoke of *Unschärfe*, the literal translation of which is "fuzziness" rather than "uncertainty". Since the relation $\Delta x \Delta p \geq \hbar / 2$ is a consequence of the fact that $\psi(x)$ and $\varphi(p)$ of a particle are related to each other via a Fourier transformation. The fuzziness relation for position and momentum follows via $p = \hbar k$ where $\hbar = h / 2\pi$ is the reduced Planck constant and $k$ is the wave number. It says that the fuzziness of a position (as measured by $\Delta x$) and the fuzziness of the corresponding momentum (as measured by $\Delta p = \hbar \Delta k$) must be such that their product equals at least $\hbar / 2$. The Heisenberg principle is given by $\Delta x \Delta p \geq \hbar / 2$, or ($\Delta x \Delta k \geq 1/2$), for the electron at non-relativistic speeds. The derivation of the Fourier transform inequality relation between the uncertainty in position and the uncertainty in wavenumber was derived by Schrodinger using the Schwartz inequality [2,3].

In this work we review the Heisenberg uncertainty principle of subatomic particles of the standard model at high energies when the ratio between the speed of the subatomic particle is close to the speed of light $v / c \approx 0.5 - 1$. On this energy scale, you cannot measure the location in space and time because fuzziness quantum effects increase when the energy increases, and the particle size decreases.

In this article, a new approach to this inequality is presented using the Sugeno fuzzy integral, giving $\Delta x \cdot \Delta k = 0.6810$ as most probable value to the minimum of Heisenberg's formula for particles with spin 1/2. Thus, we obtain a value of $v/c = 0.6180....$ at high energies which satisfy $\Delta x \Delta p \geq \hbar/2$. Here c is the light velocity and we find that where the four forces of nature appear unified. We study the problem of quantum particles in one-dimensional because we want to compare it with string theory, which at very high energies shows fuzziness in space-time that is greater than the result indicated by Heisenberg [1].

## 1 Introduction

The fuzzy theory is an appropriate tool for describing those phenomena that occur in the interaction of particles and the forces of nature because this theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely studied. Therefore, fuzzy theory can be merged with quantum theory for describing its concepts which are difficult to explain by conventional logic.
The goal of this paper is to help to close the gap between quantum theory concepts and conventional language. The main matter of this discussion is the uncertainty principle. In this paper, a new approach to this principle, based on fuzzy theory, is presented. The uncertainty principle formulated by Heisenberg [1,2] in 1927, can be stated as: Any measurement of the position x of a particle with the error $\Delta x$ causes the disturbance $\Delta p$ on its momentum p satisfying

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad (1)$$

It should be emphasized that Heisenberg not only derived this relation from the famous γ-ray microscope thought experiment, but he also gave a mathematical justification, in which he indeed proved Eq. (1) for Gaussian wave functions. Subsequently Schrodinger [3,4] proved it for general wave functions in terms of the duality particle- wave function [5−8]. Recently, there are reviews of this principle, but it is recognized that it does not modify what was developed by Heisenberg. [9−15]. As a related topic associated with this uncertainty inequality, it is important to make the connection of the

Fourier transform inequality identity to quantum mechanics. This can be achieved with a short review of the de Broglie wave concept of quantum particles, and the resultant Schrödinger wave equation. Specifically, de Broglie proposed that a free quantum particle, which has a precise momentum, $p$ , could be modeled as an infinite extent quantum wave function, where: $\psi(x) = \psi_0 e^{ikx}$ which has a precise wavenumber, $k$ , that is associated with the momentum, as $p = \hbar k$ , where $\hbar$ is the normalized Planck's constant. With the additional Einstein concept of the frequency $\omega$ of the free particle wave function being associated with the precise energy $E$ , as $E = \hbar\omega$ the time dependent wave function, $\psi(x,t) = \psi_0 e^{i(kx-\omega t)}$ also satisfies the Schrödinger wave function equation for a free particle,

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x}\psi(x,t) \qquad (2),$$

which it is essentially an energy balance equation, where the energy of a free particle is $E = \hbar\omega = p^2/2m = (\hbar k)^2/2m$ . In order to extend the concept of the infinite extent wave function, equation (2), to the situation that the quantum particle might have a finite spatial extent $\Delta x$ , it was necessary to consider an infinite linear superposition of precise momentum, or wavenumber eigenstate, wave functions, with $p = \hbar k$ , using a Fourier integral representation approach, where

$$\psi(x+\Delta x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\varphi(k+\Delta k)e^{ik(x+\Delta x)} \quad , \quad \varphi(k+\Delta k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\psi(x+\Delta x)e^{-ik(x+\Delta x)} \qquad (3)$$

This imply that there must be an infinite spectrum of momentum components of such a wave function, since each wavenumber component has the property that $p = \hbar k$ , and thus, if the particle has a finite spatial extent, $\Delta x$ , then it will also have a finite wavenumber extent, $\Delta k$ , which indicates that it also has a

finite momentum extent, $\Delta p = \hbar \Delta k$ . Consequently, due to the inverse variance, or uncertainty width relation, between $\Delta x$ and $\Delta k$ , found from the Fourier transform and the variational analysis given above, the quantum uncertainty principle is given by $\Delta x \cdot \Delta k \geq 1/2$ or $\Delta x \cdot \Delta p \geq \hbar / 2$ .According Heisenberg if we apply quantum mechanics to unique objects (e.g, quantum black hole, leptons like electrons, muons or inter galactic photons etc..), then the probabilities resulting from equations of quantum mechanics at high energies can be calculated using a fuzzy interpretation of quantum mechanics $[16,17]$. The probabilities are interpreted as degrees of belief from fuzzy logic operations well defined as shown in section 2.

## 2. About Fuzzy Sets Theory

A fuzzy sunset A of a classical set U is characterized by its membership function

$$\varphi_A : U \to [0,1]$$

If $\varphi_A$ Is the set $\{0,1\}$ , then we have a crisp subset. $[16,17,18]$

**Definition of $\alpha$ -level**

Let A be a subset of $U$ and $\alpha \in [0,1]$ , then the $\alpha$ level of the fuzzy subset A is the classical subset of $U$ that is defined as

$[A]_\alpha = \{x \in U : \varphi_A(x) \geq \alpha\}$, for $\alpha \in [0.1]$

We are interested in on the fuzzy subsets $A$ of $\Re$ so their $\alpha$ levels are given by

$[A]_\alpha = [a^-{}_\alpha, a^+{}_\alpha]$, $a^-{}_\alpha \leq a^+{}_\alpha$, for each $\alpha \in (0,1]$ . We use the symbol $\Re_\alpha$ to denote the class of these fuzzy subsets

**Definition of Fuzzy measure,**

Let $\Lambda$ be a sigma $\sigma$ - algebra of a classical set $\Omega$ omega. A map $\mu : \Lambda \to [0,\infty)$ is called a fuzzy measure such satisfies:

a) $\mu(\varnothing) = 0$ and b) if $A, B \in \Lambda$ and $A \subseteq B$ then $\mu(A) \leq \mu(A)$

The definition of measure of Sugeno (1974) [18] considered the boundary conditions $\mu(\Omega) = 1$ in a). that is a normalization of the fuzzy measure.

**Definition of Lebesgue measure,**

Let $A \in \Re_\alpha$ and $\alpha \in [0,1]$. The usual Lebesgue $\mu(A)$ of the $\alpha$ -level of $A$ is given by

$$\mu([A]_\alpha) = a_\alpha^+ - a_\alpha^-$$

The usual Lebesgue measure is a fuzzy measure (Roman, Flores) [19 – 21].

**Definition of Sugeno integral**

Let $\mu$ be a fuzzy measure on $(\Re, \Sigma)$. If $f \in F^\mu(\Re)$ and $A \in \Sigma$, then the Sugeno integral of $f$ on $A$ with respect to the fuzzy normalized measure $\mu$. The Sugeno integral is given by

$$\int_A f d\mu = \bigvee_{\alpha \geq 0} \left[ \alpha \wedge \mu(A \cap \{f \geq \alpha\}) \right], \quad A \in \Sigma,$$

Where $\vee$, $\wedge$ denote the operations sup and inf on $[0,\infty]$, respectively. If $A = \Re$ then

$$\int_{\Re} f d\mu = \int f d\mu = \bigvee_{\alpha \geq 0} \left[ \alpha \wedge \mu\{f \geq \alpha\} \right] \quad (4)$$

**Remark**. Consider the distribution function $F$ associated to $f$ : $F(\alpha)=\mu(A\cap\{f\geq\alpha\})$ , then, due to the prepositions $\mu(A\cap\{f\geq\alpha\})\geq\alpha$ and $\mu(A\cap\{f\geq\alpha\})\leq\alpha$ we have

$$F(\alpha)=\int_A fd\mu=\alpha \qquad (5)$$

Thus, from computational point of view the fuzzy integral can be calculated solving the equation $F(\alpha)=\alpha$ .

In connection with our problem, is important to observe that, physically a point particle in space closely resembles a fuzzy point in Zadeh fuzzy set theory $[16]$ . In the usual set theory, a point either belongs or does not belong to a subset. But according to Zadeh' fuzzy set theory a point can be a member of a subset with probabilistic or uncertain feature. a fuzzy point $p_f$ in the set $\mathbf{Z}$ is a fuzzy subset with a membership function: $\mu_{p_f}(x)=y$ for $x=x_0$ or $x=0$ otherwise, where $0<x<1$ , where a fuzzy point $p_f$ is said to have support $x_0$ and $x$ values. The point in the set $\mathbf{Z}$ at which $\mu_{p_f}(x)>0$ constitute the support of the fuzzy point.

Here, we need a function $f(x)$ with $f(0)=0$ , $f(1)=0$ and $f_{\max}(x)=1$ so we choose $f(x)=4x(1-x)$ ; this function fully satisfies the postulates of the fuzzy theory indicated on this section 2.

In this context we define the function $f(x)=\varphi_A:\Omega\to[0,1]$ given by $f(x)=4x(1-x)$ ; $f(x)$ which must be a membership of a fuzzy subset $F$ of $\mathbb{R}$ whose $\alpha-$ levels are given by

$$[F]_\alpha=\{x\in\mathbb{R}:4x(1-x)\geq\alpha\}=\left[\frac{1-\sqrt{1-\alpha}}{2},\frac{1+\sqrt{1-\alpha}}{2}\right]$$

If $\mu$ is the usual Lebesgue measure on $\Omega = \mathbb{R}$ , then the level function $F(\alpha)$ is

$$F(\alpha) = \mu([F]_\alpha) = \frac{1+\sqrt{1-\alpha}}{2} - \frac{1-\sqrt{1-\alpha}}{2} = \sqrt{1-\alpha}$$

Thus, the Sugeno integral is

$$\int_\Omega f d\mu = \sup_{\alpha \in [0,1]} \left[ \alpha \wedge \sqrt{1-\alpha} \right]$$

Since $F(\alpha)$ has a decreasing function part, we have

$$\int_\Omega f d\mu = \alpha = F(\alpha) = \frac{\sqrt{5}-1}{2} = 0.61803 \qquad (6)$$

Which gives the fixed point of $F(\alpha)$ . This numerical result between $[0.5,1]$ is the most likely for Heisenberg's principle under the fuzzy approach at high energies at the beginning of the universe. We note that this result corresponds to a transcendental number $F(\alpha) = 1/\varphi$ , where $\varphi = 1.6180.....$ is the irrational golden number.

The Sugeno integral was defined only for functions whose range is contained in $[0,1]$ and for normalized fuzzy measures $[22]$ . Also, the quantum wave functions $\psi(x)$ of subatomic particles (as leptons; electrons muons, baryons like the quark's family with spin 1/2, intergalactic massive photons or primordial black hole $[23,24]$ ), are normalized like $\int \psi(x)\psi^*(x)dx = 1$ .

The chosen function $f(x) = g(x)h(x)$ satisfies the fuzzy Chebyshev´s inequality, so in comparation with the classical Chebyshev´s result, the fuzzy Chebyshev´s reverse inequality does not hold $[20]$ so

$$\int_0^1 f d\mu \geq \int_0^1 g d\mu \int_0^1 h d\mu \rightarrow 0.6180 \geq 0.2 \times 0.5 = 0.1$$ .

## 3. Interpretation of Heisenberg uncertainly principle under the fuzzy theory approach

The particle-waveform duality stars from the comparison of the magnitudes of the behavior of a particle understood as a wave and as a particle, a value is obtained for the wavelength that the wave motion associated with subatomic particle would have, which is given by $\lambda = \hbar / p$ , where v is the velocity of the particle and m is its mass. This magnitude, called the de Broglie wavelength, increases with decreasing speed, and vice versa $[25]$, $\lambda = \hbar / p$ or $p = \hbar k = mv$, with Einstein relation $E = \hbar\omega = mc^2$.

From section 2. the chosen membership $f(x) = ax(1-x)$ is a continuous function derived from the first-order nonlinear difference equation $z_{n+1} = rz_n(1-z_n)$ where time is implicit $[26,27]$ so making $x_{n+1} = rx_n(1-x_n)$ with $x_{n+1} = r/s(x_n)$, $r = (1+a)\Delta t$, $s = a\Delta t$ so $x_{n+1} = rx_n - sx_n^2)$, then we have, $x_{n+1} = x_n(1+a)\Delta t - a\Delta t x_n^2)$. Thus, we obtain $\Delta x/\Delta t = (x_{n+1} - x_n)/\Delta t \to dx/dt = ax(1-x)$. In this form we have a differential equation $f(x) = dx/dt = ax(1-x)$; with $a = 4$ which is a normalized equation in $[0.1]$.

The normalized function f(x) in our case corresponds to the ratio of the speed of the massive particle and the speed of light such that $v/c = 0.6182$ or $mv/mc = 0.6182$ and by definition $p = mv$. On the other hand, the quantum size of a subatomic particle characterized by the Compton wavelength $[28]$ gives by $\lambda = \lambda/2\pi = \hbar/mc$ so we have $mc = \hbar/\lambda \approx \hbar/\Delta x$. Here we find a point of contact with string theory if this string is quantized canonically, such that we get $(\Delta x^2) \approx \lambda^2$. We consider the linear part of the bisector of $\alpha \wedge H(\alpha)$ close to $0.6182$, then we obtain $\Delta p \cdot \Delta x = 0.6182\hbar$. In this form, for particles at high

velocity we obtain the new Heisenberg uncertainly principle $\Delta p \cdot \Delta x = 0.6182\hbar$ or $\Delta k \cdot \Delta x = 0.6182$, being $k$ the wave number. This result is valid for particles like the electron, muon and tau and quarks with spin ½ derived from the standard model.

Thus, the Heisenberg uncertainly principle at high velocity under the fuzzy theory is obtained giving a value fixed greater than the value determined by Heisenberg at low velocity for the case of electrons with spin ½.
$\Delta p \cdot \Delta x\big|_{Fuzzy} = 0.6182\hbar > \Delta p \cdot \Delta x\big|_{Heisenberg} = 0.5\hbar$.

There is another form of Heisenberg's uncertainty principle for simultaneous measurements of energy and time, $\Delta E \Delta t \geq \hbar / 2$, where $\Delta E$ is the uncertainty in energy and $\Delta t$ is the uncertainty in time. The uncertainty principle for energy and time can be of great significance if the lifetime of particle is very short is unstable and interacting with bosons. Then $\Delta t$ is very small, and $\Delta E$ is very large. Some nuclei and exotic particles have extremely short lifetimes (as small as $10^{-25} s$), causing uncertainties in energy as great as many GeV ($10^{9} eV$). At high energies $v/c \leq 1$ $\Delta pc\Delta x \geq \Delta mc^2 \Delta x \geq \Delta E \Delta x \geq \hbar c / 2$ so $\Delta E \Delta x / c = \Delta E \Delta x / \Delta x (\Delta t)^{-1} = \Delta E \Delta t \geq \hbar / 2$ and fuzzy result is $\Delta E \cdot \Delta t = 0.6182\hbar$. It is so well established experimentally that the uncertainty in decay energy is used to calculate the lifetime of short-lived states. Some nuclei and particles are so short-lived that it is difficult to measure their lifetime. But if their decay energy can be measured, its spread is $\Delta E$, and this is used in the uncertainty principle. In our case $\Delta E \cdot \Delta t = 0.6182\hbar$ is used to calculate the lifetime $\Delta t$.

## 4. Summary

To our knowledge, this article determines for the first time the Heisenberg uncertainly/indeterminacy principle under the fuzzy theory giving a minimum value fixed greater than the value determined by Heisenberg, for the case of leptons (electrons, muons) and baryons (quarks) with spin ½ at relativistic velocities, $v/c \geq 0.5$, $\Delta p \cdot \Delta x\big|_{Fuzzy} = 0.6182\hbar > \Delta p \cdot \Delta x\big|_{Quantum} = 0.5\hbar$. To Obtain this

result, the function chosen here is based on " Tópicos de lógica fuzzy e biomatemática" of L C Barros and R C Bassanezi, pg 193-195, (2006).

In the standard model all elementary subatomic particles are considered as dimensionless points, while in the fuzzy model considered in this work, there is a minimum distance for each subatomic particle. Our fuzzy approach can be a clue and a bridge to study the string theory where particles have one dimension like strings that vibrate at a characteristic frequency and unifies all fundamental forces including quantum gravity.

The smallest of the subatomic particle sizes is determined by the Planck conditions, that is the string theory at very high energy ( $E_p \approx 10^{19} GeV$ ) has a minimum length given by the Planck length. $l_p = \Delta x_{\min} \approx 10^{-33} cm$ , where also $l_p$ is the Schwarzschild radius of a black hole of mass $m_p$ and $t_p \approx 10^{-42} s$ is the evaporation via Hawking radiation $[28,29]$ . At this energy level, string theory imparts a "fuzziness" to all our familiar notions of space and time. On string theory particles, Heisenberg's uncertainty principle increases like $\Delta x \Delta p \geq \hbar / 2 + \alpha' \Delta p^2 / \hbar)$ where $\alpha' \approx const$ . Here, we can compare this result with our fuzzy result $\Delta x \Delta p \geq \hbar / 2 + 0.16182\hbar$ . Also, as string theory theoretically give rise to natural fuzzy dark matter candidate in the form of ultralight axions $[30]$ or like a axial Higgs boson, our approach also take into account the dark matter.

The standard model still presents some problems derived from uncertainty in the measurements of the muon, the appearance of tetraquarks, and pentaquarks and more elementary particles than those theoretically indicated by this model. One of the main objectives of the Large Hadron Collider, LHC-CERN in its new stage with 13.6 TeV, possibly will allow find new exotic particles and the Higgs field will be studied in greater detail where fuzziness will be more and more important at high energies. Here, fuzzy approach will have a decisive role.

**References**

1. Heisenberg, W. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Z. Phys. 43, 172–198 (1927).
2. Heisenberg, W. The Physical Principles of the Quantum Theory, (University of Chicago Press, Chicago, 1930)
3. Schrodinger E, Uber die kraftefreie bewegung in der relativistischen quantenmechanik. Sitzungsberichte der Preuischen Akademie der Wissenschaften. Physikalisch-mathematische Klasse, 1930.
4. Schrodinger E, Die gegenwartige Natur wissennschuften 23, 807-812, 1935
5. Kennard, E. H. Zur Quantenmechanik einfacher Bewegungstypen. Z. Phys. 44, 326–352 (1927).
6. von Neumann, J. Mathematische Grundlagen der Quantenmechanik. Springer, Berlin, (1932).
7. Bohm, D. Quantum Theory. Prentice-Hall, New York, (1951).
8. Messiah, A. Mécanique Quantique volume I, Dunod, Paris, (1959). [Quantum Mechanics Vol. I, (North-Holland, Amsterdam, 1959)].
9. Schiff, L. I. Quantum Mechanics. MacGraw-Hill, New York, (1968).
10. Wiseman, H. M. Extending Heisenberg's measurement-disturbance relation to the twin-slit case. Found. Phys. 28, 1619–1631 (1998).
11. Ozawa, M. Physical content of Heisenberg's uncertainty relation: limitation and reformulation. Phys. Lett. A 318, 21–29 (2003).
12. Ozawa, M. Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement. Phys. Rev. A 67, 042105/1–042105/6 (2003).
13. Watanabe, Y., Sagawa, T. & Ueda, M. Uncertainty relation revisited from quantum estimation theory. Phys. Rev. A, 84, 042121 (2011).
14. Fujikawa, K. Universally valid Heisenberg uncertainty relation. Phys. Rev. A, 85, 062117 (2012).
15. Rozema, L. A. et al. Violation of Heisenberg's measurement-disturbance relationship by weak measurements. Phys. Rev. Lett. 109, 100404 (2012).
16. Zadeh, L. A. (1965). Fuzzy sets, Information and Control, 8, 338.
17. Pykacz, J, fuzzy quantum logics, Int. J. Theor. Physics, vol 83, N 7, 1403-1416, 1994.
18. Sugeno M, Theory of fuzzy integrals and Applications, PHD thesis, Tokyo Institute of Technology, 1974.
19. Roman H, Flores A, Chalco Y. the fuzzy integral for monotone functions. Applied Mathematics and Computation, 185, 492-498, (2007).
20. Flores A, Roman H, A Chebyshev type inequality for fuzzy integrals, Applied Mathematics and Computation, 190, 2, 1178-1184, (2007).

21. Roman H, Flores A, Sugeno integral and geometric inequalities , International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15, 01, 1-11, (2007).
22. Murofushi H. Sugeno M. Fuzzy measures and fuzzy integrals. Theory and applications 3-41. (2000).
23. Torres H. and Souza A, Do Intergalactic Photons Attract or Repel Each Other? Científica, vol. 21, núm. 1, pp. 43-48, enero-junio (2017).
24. Torres H et al, Number-phase Quantization Approach for Bohr's Black Hole, Prespacetime Journal, Volume 7, Issue 1, pp. 154-162, (2016).
25. Weinberger P, Revisiting Louis de Broglie paper in tge Philosophical Magazine, Phil. Mag. Letts, 86, 7, 405-410, (2006).
26. May R, simple mathematical model with very complicated dynamics, Nature, vol 261, 459-467, (1976).
27. Barros L C, Bassanezi R C. Tópicos de Lógica Fuzzy e Biomatemática, cap 7, pag 195, (2006).
28. Sidharth B.G, Strings and Planck oscillators, Chaos Solitons and Fractals, 30, 300-311. (2006).
29. Witten E, What every physicist should know about String Theory, 38-43, (2015).
30. Cicoli M et al, Fuzzy Dark Matter candidates from string theory, Journal of High Energy Physics volume 107, 1-44, (2022).
31. Google: chiral quiral H. Torres-Silva