AB Calculus Practice 5.1

Finite limits

1. If a limit as x approaches a finite number yields an un-workable result (like $\frac{0}{0}$), there's a good chance you need to _____________ the top and bottom and cancel something out.

2. For $f(x)$ to be continuous at the point at which $x = a$, which of the following does not necessarily need to be true?

a. $\lim_{x \to a^+} f(x)$ must exist

b. $\lim_{x \to a^-} f(x)$ must exist

c. $\lim_{x \to a} f(x)$ must exist

d. $\lim_{x \to a^+} f(x)$ must be equal to $\lim_{x \to a^-} f(x)$

e. $\lim_{x \to a^+} f'(x)$ must be equal to $\lim_{x \to a^-} f'(x)$

f. $\lim_{x \to a} f(x)$ must be equal to $f(a)$

3. Any x-value which causes you to divide _____ _____________ will result in a _________________.

4. A _____________ discontinuity, also called a "hole," occurs when a factor on top and bottom of a fraction cancels out.

5. For $f(x)$ to be differentiable at the point at which $x = a$, the left- and right-hand ____________ must match and the left- and right-hand ________________ must also match.

6. To do L'Hôspital's Rule, you find a _____________ by taking the _____________________ of the top and bottom of a fraction.

7. L'Hôspital's Rule only applies when evaluating a limit that gives you ___ or ___.

bold *numbers - calculator* ***permitted***

8. $\lim_{x \to -3} \frac{x^2 + 6x + 8}{x^2 - x - 6}$ is

9. $\lim_{x \to 5} \frac{x^2 - 2x - 15}{x^2 - 25}$ is

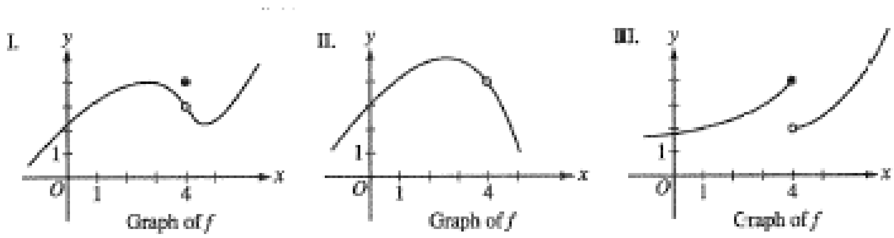
10. $\lim_{x \to 3^-} \frac{|x-4|}{x-4}$ is

11. If

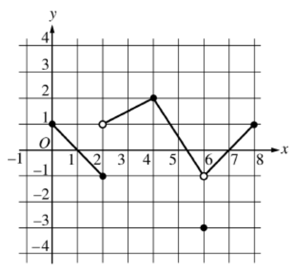
$$f(x) = \ln x \qquad \text{for} \qquad 0 < x \leq 3$$

$$f(x) = x \ln x \qquad \text{for} \qquad 3 < x \leq 5$$

then $\lim_{x \to 3} f(x)$ is



12. Of the graphs shown, for which does $\lim_{x \to 4} f(x)$ not exist?



13. The figure above shows the graph of the function *f*. Which of the following statements are false?

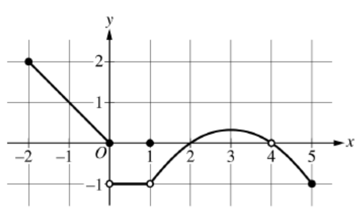
Circle all that apply

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 2^+} f(x) = f(2)$

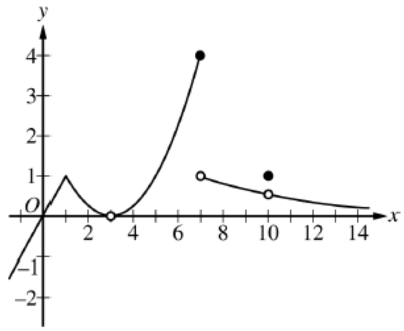
III. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

IV. $\lim_{x \to 6} f(x) = f(6)$



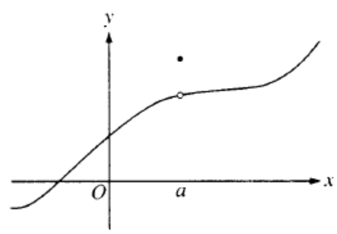
Graph of f

14. The graph of the function *f* is shown above. For what values of *a* does $\lim_{x \to a} f(x) = -1$?



Graph of f

15. The graph of the function f is shown above. At what value(s) of x does f have a removable discontinuity?



16. The graph of a function f is shown above. Which of the following statements about f is true?

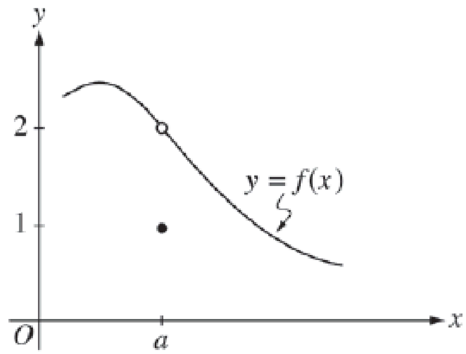
(A) $\lim\limits_{x \to a} f(x)$ is equal to $f(a)$

(B) f is continuous at $x = a$

(C) $\lim\limits_{x \to a} f(x) = 0$

(D) $\lim\limits_{x \to a^+} f(x)$ is equal to $\lim\limits_{x \to a^-} f(x)$.

(E) $\lim\limits_{x \to a} f(x)$ does not exist



17. The graph of a function f is shown in the figure above. Which of the following statements is false?

(A) $x = a$ is in the domain of f.

(B) f is continuous at $x = a$

(C) $\lim_{x \to a} f(x) = 2$

(D) $f(a) = 1$

(E) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

18. Let f be the function given by $f(x) = \frac{(x-3)^2(x+2)}{(x-3)(x+4)}$. For which value(s) of x is f not continuous?

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

19. The function f is defined above. For what value of k, if any, is f continuous at $x = 2$?

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \ge 2 \end{cases}$$

20. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

$$f(x) = \begin{cases} 6 + cx & \text{for } x < 1 \\ 9 + 2\ln x & \text{for } x \ge 1 \end{cases}$$

21. Let f be the function defined above, where c is a constant. If f is continuous at $x = 1$, what is the value of c ?

$$f(x) = \begin{cases} \dfrac{x^2 - 7x + 10}{b(x-2)} & \text{for } x \ne 2 \\ b & \text{for } x = 2 \end{cases}$$

22. Let f be the function defined above. For what value of b, if any, is f continuous at $x = 2$?

x	0	1	2
$f(x)$	1	k	2

23. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{5}{2}$ must have at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 **(E) 3**

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

24. The table above shows selected values of a continuous function f. For $0 \le x \le 13$, what is the fewest possible number of times $f(x) = 3.5$?

$$f(x)=\begin{cases}3x+5 & \text{when } x<-1\\ -x^2+3 & \text{when } x\geq -1\end{cases}$$

25. If f is the function defined above, then $f'(-1)$ is

26. If a $\neq$ 0, then $\lim_{x\to a}\frac{x^3-a^3}{x^4-a^4}$ is

27. $\lim_{x\to -4}\frac{tan(x+4)}{4e^{x+4}+x}$ is

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
2	0	0	5	7

28. The third derivative of the function f is continuous on the interval (0, 4). Values for f and its first three derivatives at $x = 2$ are given in the table above. What is $\lim_{x\to 2}\frac{(x-2)^2}{f(x)}$?