LOGARITHM

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1. Definition:

The logarithm of a number N to a base ' a ' is an exponent indicating the power to which the base ' a ' must be raised to obtain the number N. This number is designated as $log_a N$. (Read it "Log N on base a "). Here N is usually called argument of the Logarithm and ' a ' is called base of the Logarithm.

Hence: $log_a N = x \Leftrightarrow a^x = N, a > 0, a\neq 1$ and $N > 0$

By the definition of logarithm, $log_2 16$ is the exponent indicating the power to which 2 must be raised in order to obtain 16.

As $2^4 = 16$, hence $log_2 16 = 4$.

Similarly

Exponential Form **Logarithmic Form**

$$3^5 = 243 \Leftrightarrow log_3 243 = 5\; 5^4 = 625 \Leftrightarrow log_5 625 = 4\; 2^{-3} = \frac{1}{8} \Leftrightarrow log_2 \frac{1}{8} = -3\; 7^0 = 1 \Leftrightarrow log_7 1 = 0$$

Note that the expressions $log_3(-27)$, $log_1 16$, $log_0 5$ and $log_2 0$ has no sense in real numbers since the equations $3^x = -27, 1^x = 6, 0^x = 5, 2^x = 0$ are absurd for any real x, the reason being obvious that no such exponent x in real number could be found.

In general, the expression $log_a N$ is meaningful if and only if, $a > 0, a\neq 1$ and $N > 0$.

The existence and uniqueness of the number $log_a N$ follows from the properties of exponential functions.

Illustration 1:

If $log_4 m = 1.5$, then find the value of m.

Solution:

$log_4 m = 1.5 \Rightarrow m = 4^{3/2} \Rightarrow m = 8$

Illustration 2:

If $log_5 p = a$ and $log_2 q = a$, then prove that $\frac{p^4 q^4}{100} = 100^{2a-1}$

Solution:

$$log_5 p = a \Rightarrow p = 5^a \quad \Rightarrow log_2 q = a \Rightarrow q = 2^a \quad \Rightarrow \frac{p^4 q^4}{100} = \frac{5^{4a} \cdot 2^{4a}}{100} = \frac{(10)^{4a}}{100} = \frac{(100)^{2a}}{100} = 100^{2a-1}$$

Illustration 3:

The value of N, satisfying $log_a\left[1 + log_b\left\{1 + log_c\left(1 + log_p N\right)\right\}\right] = 0$ is -

(A) 4 (B) 3 (C) 2 (D) 1

Ans. (D)

Solution:

$1 + log_b\left\{1 + log_c\left(1 + log_p N\right)\right\} = a^0 = 1 \Rightarrow log_b\left\{1 + log_c\left(1 + log_p N\right)\right\} = 0 \Rightarrow 1 + log_c\left(1 + log_p N\right) = 1 \Rightarrow log_c\left(1 + log\right.$

2. Fundamental Logarithmic Identity:

From the definition of the logarithm of the number N to the base ' a ', we have an identity: $a^{\log_a N} = N, a > 0, a \neq 1$ and $N > 0$

This is known as the Fundamental Logarithmic Identity.

Note:

Using the basic definition of logarithm we have 3 important deductions:

(a) $\log_a 1 = 0$ i.e. logarithm of unity to any base is zero ($a > 0; a \neq 1$).

(b) $\log_N N = 1$ i.e. logarithm of a number to the same base is 1 . ($N > 0; N \neq 1$)

(c) $\log_{\frac{1}{N}} N =- 1 = \log_N \frac{1}{N}$ i.e. logarithm of a number to the base as its reciprocal is -1 .($N > 0; N \neq 1$)

3. The Principal Properties of Logarithms:

If m, n are arbitrary positive numbers where $a > 0, a \neq 1$ and x is any real number, then

(a) $\log_a mn = \log_a m + \log_a n$

(b) $\log_a \frac{m}{n} = \log_a m - \log_a n$

(c) $\log_a m^x = x\log_a m$

Illustration 4:

Find the value of $2\log\frac{2}{5} + 3\log\frac{25}{8} - \log\frac{625}{128}$

Solution:

$$2\log\frac{2}{5} + 3\log\frac{25}{8} + \log\frac{128}{625} = \log\frac{2^2}{5^2} + \log\left(\frac{5^2}{2^3}\right)^3 + \log\frac{2^7}{5^4} = \log\frac{2^2}{5^2}\cdot\frac{5^6}{2^9}\cdot\frac{2^7}{5^4} = \log 1 = 0$$

Illustration 5:

If $\log_e x - \log_e y = a, \log_e y - \log_e z = b \& \log_e z - \log_e x = c$, then find the value of $\left(\frac{x}{y}\right)^{b-c} \times \left(\frac{y}{z}\right)^{c-a} \times \left(\frac{z}{x}\right)^{a-b}$

Solution:

$$\log_e x - \log_e y = a \Rightarrow \log_e \frac{x}{y} = a \Rightarrow \frac{x}{y} = e^a \log_e y - \log_e z = b \Rightarrow \log_e \frac{y}{z} = b \Rightarrow \frac{y}{z} = e^b \log_e z - \log_e x = c \Rightarrow \log_e \frac{z}{x} = c$$

Illustration 6:

If $a^2 + b^2 = 23ab$, then prove that $\log\frac{(a+b)}{5} = \frac{1}{2}(\log a + \log b)$.

Solution:

$$a^2 + b^2 = (a + b)^2 - 2ab = 23ab \Rightarrow (a + b)^2 = 25ab \Rightarrow a + b = 5\sqrt{ab}$$

Using (i)

L.H.S. $= \log\frac{(a+b)}{5} = \log\frac{5\sqrt{ab}}{5} = \frac{1}{2}\log ab = \frac{1}{2}(\log a + \log b) =$ R.H.S.

Illustration 7:

If $\log_a x = p$ and $\log_b x^2 = q$, then $\log_x \sqrt{ab}$ is equal to (where $a, b, x \in R^+ - \{1\}$)-

(A) $\frac{1}{p} + \frac{1}{q}$ (B) $\frac{1}{2p} + \frac{1}{q}$ (C) $\frac{1}{p} + \frac{1}{2q}$ (D) $\frac{1}{2p} + \frac{1}{2q}$

Solution:

$log_a x = p \Rightarrow a^p = x \Rightarrow a = x^{\frac{1}{p}}$.

similarly $b^q = x^2 \Rightarrow b = x^{\frac{2}{q}}$

Now, $log_x \sqrt{ab} = log_x \sqrt{x^{1/p} x^{2/q}} = log_x x^{\left(\frac{1}{p}+\frac{2}{q}\right)\frac{1}{2}} = \frac{1}{2p} + \frac{1}{q}$

4. Base Changing Theorem:

Can be stated as "quotient of the logarithm of two numbers is independent of their common base."

Symbolically, $log_b m = \frac{log_a m}{log_a b}$, where $a > 0, a \neq 1, b > 0, b \neq 1$

Note:

(i) $log_b a \cdot log_a b = \frac{loga}{logb} \cdot \frac{logb}{loga} = 1$; hence $log_b a = \frac{1}{log_a b}$.

(ii) $a^{log_b c} = c^{log_b a}$

(iii) **Base power formula:** $log_{a^k} m = \frac{1}{k} log_a m$

(iv) The base of the logarithm can be any positive number other than 1, but in normal practice, only two bases are popular, these are 10 and $e(= 2.718$ approx). Logarithms of numbers to the base 10 are named as 'common logarithm' and the logarithms of numbers to the base e are named as Natural or Napierian logarithm**. We will consider $logx$ as $log_e x$ or lnx.**

(v) Conversion of base e to base 10 & viceversa:

$$log_e a = \frac{log_{10} a}{log_{10} e} = 2.303 \times log_{10} a; log_{10} a = \frac{log_e a}{log_e 10} = log_{10} e \times log_e a = 0.434 log_e a$$

(vi) Some important values: $log_{10} 2 \approx 0.3010; log_{10} 3 \approx 0.4771; ln2 \approx 0.693, ln10 \approx 2.303$

(vii) The positive real number ' n ' is called the antilogarithm of a number ' m ' to base ' a ' if $log_a n = m$ Thus, $log_a n = m \Leftrightarrow n = antilog_a m$

Illustration 8:

If a, b, c are distinct positive real numbers different from 1 such that

$\left(log_b a \cdot log_c a - log_a a\right) + \left(log_a b \cdot log_c b - log_b b\right) + \left(log_a c \cdot log_b c - log_c c\right) = 0$, then abc is equal to -

(A) 0 (B) e (C) 1 (D) none of these

Solution:

$\left(log_b a log_c a - 1\right) + \left(log_a b \cdot log_c b - 1\right) + \left(log_a c log_b c - 1\right) = 0 \Rightarrow \frac{loga}{logb} \cdot \frac{loga}{logc} + \frac{logb}{loga} \cdot \frac{logb}{logc} + \frac{logc}{loga} \cdot \frac{logc}{logb} = 3 \Rightarrow (loga)^3$

$\Rightarrow (loga + logb + logc) = 0$ [$\because$ If $a^3 + b^3 + c^3 - 3abc = 0$, then $a + b + c = 0$ if $a \neq b \neq c$]

$\Rightarrow logabc = log1 \Rightarrow abc = 1$

Illustration 9:

Evaluate: $81^{1/log_5 3} + 27^{log_3 36} + 3^{4/log_7 9}$

Solution:

$81^{log_3 5} + 3^{3log_9 36} + 3^{4log_9 7}$

$= 3^{4log_3 5} + 3^{log_3 (36)^{3/2}} + 3^{log_3 7^2} = 625 + 216 + 49 = 890.$

Illustration 10:

Show that $log_4 18$ is an irrational number.

Solution:

$log_4 18 = log_4 \left(3^2 \times 2\right) = 2log_4 3 + log_4 2 = 2\frac{log_2 3}{log_2 4} + \frac{1}{log_2 4} = log_2 3 + \frac{1}{2}$

assume the contrary, that this number $log_2 3$ is rational number.

$\Rightarrow log_2 3 = \frac{p}{q}$. Since $log_2 3 > 0$ both numbers p and q may be regarded as natural number

$\Rightarrow 3 = 2^{\frac{p}{q}} \Rightarrow 2^p = 3^q$

But this is not possible for any natural number p and q. The resulting contradiction completes the proof.

Illustration 11:

If in a right angled triangle, a and b are the lengths of sides and c is the length of hypotenuse and $c - b \neq 1$, $c + b \neq 1$, then show that $log_{c+b} a + log_{c-b} a = 2log_{c+b} a \cdot log_{c-b} a$.

Solution:

We know that in a right angled triangle

$c^2 = a^2 + b^2$

$c^2 - b^2 = a^2$ …(i)

$LHS = \frac{1}{log_a (c+b)} + \frac{1}{log_a (c-b)} = \frac{log_a (c-b) + log_a (c+b)}{log_a (c+b) \cdot log_a (c-b)}$

$= \frac{log_a \left(c^2 - b^2\right)}{log_a (c+b) \cdot log_a (c-b)} = \frac{log_a a^2}{log_a (c+b) \cdot log_a (c-b)}$ (using (i))

$= \frac{2}{log_a (c+b) \cdot log_a (c-b)} = 2log_{(c+b)} a \cdot log_{(c-b)} a$ = RHS

5. Logarithmic Equations:

Illustration 12:

$log_{\frac{1}{2}} \left(log_2 \sqrt{2}x\right) = 1$, then find x ?

Solution:

$log_{\frac{1}{2}} \left(log_2 \sqrt{2}x\right) = 1 \Rightarrow log_2 (\sqrt{2}x) = \frac{1}{2} \Rightarrow \sqrt{2}x = 2^{\frac{1}{2}} \Rightarrow x = 1$

Illustration 13:

Solve the equation $2log_2 \left(log_2 x\right) + log_{1/2} \left(\frac{3}{2} + log_2 x\right) = 1.$

Solution:

Let $log_2 x = t$

$\Rightarrow 2log_2(t) + log_{\frac{1}{2}}\left(\frac{3}{2} + t\right) = 1 \Rightarrow 2log_2 t - log_2\left(\frac{3}{2} + t\right) = 1 \Rightarrow log_2\left(\frac{t^2}{\frac{3}{2}+t}\right) = 1 \Rightarrow \frac{2t^2}{3+2t} = 2 \Rightarrow t^2 - 2t - 3 = 0 \Rightarrow (t + 1)$

Illustration 14:

Solve the equation $logx^2 - log(2x) = 3log3 - log6$.

Solution:

$logx^2 - log2x = 3log3 - log6, x > 0 \Rightarrow 2logx - log2 - logx = 3log3 - log2 - log3 \Rightarrow logx = 2log3 \Rightarrow logx = log9$

Illustration 15:

Solve the equation $\left(log_5 x\right)^2 + log_5 x + 1 = \frac{7}{log_5 x - 1}$

Solution:

Put $log_5 x = t$, we get $t^2 + t + 1 = \frac{7}{t-1}$

$(t - 1)\left(t^2 + t + 1\right) = 7 \Rightarrow t^3 + t^2 + t - t^2 - t - 1 = 7 \Rightarrow t^3 - 8 = 0 \Rightarrow (t - 2)\left(t^2 + 2t + 4\right) = 0 \Rightarrow t - 2 = 0; t^2 + 2t$

Now, $t = log_5 x$, so $log_5 x = 2$

$x = 5^2 \Rightarrow x = 25$

Illustration 16:

Solve the equation $|x - 1|^{log_2 x^2 - 2log_x 4} = (x - 1)^7$

Solution:

Obviously $x = 2$ is a solution. Since, left side is positive, $x - 1 > 0$.

The equation reduces $log_2 x^2 - 2log_x 4 = 7$

$\Rightarrow 2t - \frac{4}{t} = 7, t = log_2 x \Rightarrow 2t^2 - 7t - 4 = 0 \Rightarrow t = 4, -\frac{1}{2}$

But $t > 0$ since $x > 1$. $\therefore t = 4$

$\Rightarrow x = 2^4 = 16 \therefore x = 2, 16$

Illustration 17:

Solve the equation $x^{log_3 x^2 + \left(log_3 x\right)^2 - 10} = \frac{1}{x^2}$

Solution:

Taking log_3 on both sides, we get

$\left(2t + t^2 - 10\right)t = -2t,\ t = log_3 x \Rightarrow t\left(t^2 + 2t - 8\right) = 0 \Rightarrow t = 0, 2, -4 \Rightarrow x = 1, 9, \frac{1}{81}$

Illustration 18:

Solve the equation $4^{log_2 lnx} = lnx - (lnx)^2 + 1$

Solution:

$4^{log_2 lxx} = 2^{2log_2(lnx)} = 2^{log_2(lnx)^2} = (lnx)^2 \Rightarrow (lnx)^2 = lnx - (lnx)^2 + 1 \Rightarrow 2(lnx)^2 - lnx - 1 = 0 \Rightarrow lnx = 1, -\frac{1}{2}$

But $lnx > 0$
$\therefore lnx = 1 \Rightarrow x = e$

Illustration 19:

Solve the equation x: $log_{x+1}\left(x^2 + x - 6\right)^2 = 4$

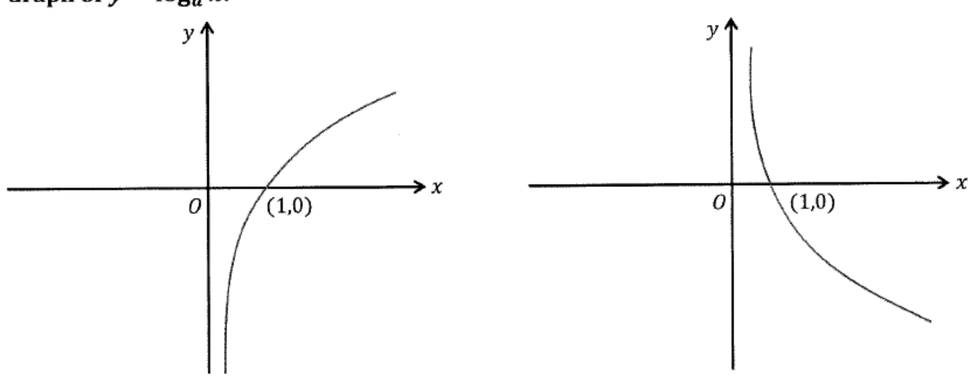
Solution:
We have,

$log_{x+1}\left(x^2 + x - 6\right)^2 = 4 \Rightarrow \left(x^2 + x - 6\right)^2 = (x + 1)4 = \left(x^2 + 2x + 1\right)^2 \Rightarrow \left(x^2 + x - 6 - x^2 - 2x - 1\right)\left(x^2 + x - 6\right.$

The values $x =- 7$ and $x =- 5/2$ are rejected because they make the base $x + 1$ negative Hence, $x = 1$ is the only solution of the given equation.

6. Graphs of Logarithmic Function and its Inequalities:

Graph of $y = \log_a x$:



When $a > 1$ When $0 < a < 1$

(i) $log_a x < log_a y \Leftrightarrow \{x < y \ if \ a > 1 \ x > y \ if \ 0 < a < 1$

(ii) If $a > 1$, then $log_a x < p \Rightarrow 0 < x < a^p$ and $log_a x > p \Rightarrow x > a^p$

(iii) If $0 < a < 1$, then $log_a x < p \Rightarrow x > a^p$ and $log_a x > p \Rightarrow 0 < x < a^p$

Note:
(i) If base of logarithm is greater than 1 then logarithm of greater number is greater.
i.e. $log_2 8 = 3, log_2 4 = 2$ etc. and if base of logarithm is between 0 and 1 then on that base logarithm of greater number is smaller. i.e. $log_{1/2} 8 =- 3, log_{1/2} 4 =- 2$ etc.
(ii) It must be noted that whenever the number and the base are on the same side of unity then logarithm of that number to that base is positive, however if the number and the base are located on different side of unity then logarithm of that number to that base is negative.
e.g. $log_{10}\sqrt[3]{10} = \frac{1}{3}$; $log_{\sqrt{7}} 49 = 4$; $log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$; $log_2\left(\frac{1}{32}\right) =- 5$; $log_{10}(0.001) =- 3$

Illustration 20:

Solve for $: x^{log_5 x} > 5$

Solution:

as $x > 0$ (for existance)

now solving inequality

$x^{log_5 x} > 5$. Taking ' log ' with base ' 5 ' we have $log_5 x \cdot log_5 x > 1$

$\Rightarrow \left(log_5 x - 1\right)\left(log_5 x + 1\right) > 0 \Rightarrow log_5 x > 1$ or $log_5 x < - 1$

$\Rightarrow x > 5$ or $x < 1/5$. Also we must have $x > 0$

Thus, $x \in (0, 1/5)$ or $x \in (5, \infty)$

Illustration 21:

Solve for $: log_3(2x + 1) < log_3 5$.

Solution:

Checking existance

$2x + 1 > 0 \Rightarrow x > - \frac{1}{2}$

Now solving inequality we have $2x + 1 < 5$

$\Rightarrow 2x < 4 \Rightarrow x < 2 \Rightarrow x \in (- 1/2, 2)$

Illustration 22:

Solve for x: $\left(log_{10} 100x\right)^2 + \left(log_{10} 10x\right)^2 + log_{10} x \le 14$.

Solution:

Checking existance

$x > 0$

Now solving inequality,

Let $u = log_{10} x$

$(2 + u)^2 + (1 + u)^2 + u \le 14 \Rightarrow u^2 + 4u + 4 + u^2 + 2u + 1 + u \le 14 \Rightarrow 2u^2 + 7u - 9 \le 0 \Rightarrow 2u^2 + 9u - 2u - 9 \le 0 \Rightarrow u($

Illustration 23:

Solve for $: log_3((x + 2)(x + 4)) + log_{1/3}(x + 2) < \frac{1}{2} log_{\sqrt{3}} 7$.

Solution:

Checking existence,

$(x + 2)(x + 4) > 0$ and $(x + 2) > 0$

$x < - 4$ or $x > - 2$ and $x > - 2$

Now solving inequality.

$log_3((x + 2)(x + 4)) + log_{1/3}(x + 2) < \frac{1}{2} log_{\sqrt{3}} 7$.

$\Rightarrow log_3(x + 2)(x + 4) - log_3(x + 2) < log_3 7 \Rightarrow log_3(x + 4) < log_3 7 \Rightarrow x + 4 < 7 \Rightarrow x < 3 \Rightarrow - 2 < x < 3 \Rightarrow x \in (- 2,$

Illustration 24:

Solve for x: $log_{1/3}log_4\left(x^2 - 5\right) > 0$

Solution:

Checking existence,

(i) $log_4\left(x^2 - 5\right) > 0 \Rightarrow x^2 - 5 > 1$

$\Rightarrow (x - \sqrt{6})(x + \sqrt{6}) > 0 \ \Rightarrow x \in (-\infty, -\sqrt{6}) \cup (\sqrt{6}, \infty)$

(ii) $x^2 - 5 > 0 \Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

solving inequality,

$log_{\frac{1}{3}}log_4\left(x^2 - 5\right) > 0 \ \Rightarrow log_4\left(x^2 - 5\right) < 1 \ \Rightarrow x^2 - 5 < 4 \Rightarrow x^2 - 9 < 0 \ \Rightarrow (x - 3)(x + 3) < 0$ ...(iii)

∴ Answer: (i) ∩ (ii) ∩ (iii)

$\Rightarrow x \in (-3, -\sqrt{6}) \cup (\sqrt{6}, 3)$

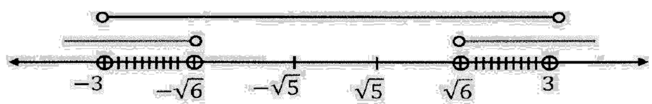


Illustration 25:

Solve for x: $\frac{log_2\left(4x^2 - x - 1\right)}{log_2\left(x^2 + 1\right)} > 1$

Solution:

Checking existence

(i) $x^2 + 1 \neq 1 \Rightarrow x \neq 0$

(ii) $4x^2 - x - 1 > 0$

Now solving inequality,

$\frac{log_2\left(4x^2 - x - 1\right)}{log_2\left(x^2 + 1\right)} > 1 \ \Rightarrow log_2\left(4x^2 - x - 1\right) > log_2\left(x^2 + 1\right) \Rightarrow log_2\left(4x^2 - x - 1\right) - log_2\left(x^2 + 1\right) > 0 \ \Rightarrow log_2\frac{4x^2 - x - 1}{x^2 + 1} > 0$

Note: $x < -\frac{2}{3}$ and $x > 1$; $4x^2 - x - 1 > 0$

Illustration 26:

Solve for : $log_4\left(3^x - 1\right)log_{1/4}\left(\frac{3^x - 1}{16}\right) \leq \frac{3}{4}$

Solution:

Checking existence $3^x - 1 > 0 \Rightarrow x > 0$,

Now solving inequality

$log_4\left(3^x - 1\right)log_{1/4}\left(\frac{3^x - 1}{16}\right) \leq \frac{3}{4} \Rightarrow log_4\left(3^x - 1\right) \cdot \left[-log_4\left(3^x - 1\right) + log_4 16\right] \leq \frac{3}{4} \Rightarrow log_4\left(3^x - 1\right)\left[-log_4\left(3^x - 1\right) + 2\right] \leq$

Put $log_4\left(3^x - 1\right) = t \Rightarrow -t^2 + 2t \leq \frac{3}{4}$

$\Rightarrow -4t^2 + 8t - 3 \leq 0 \Rightarrow 4t^2 - 8t + 3 \geq 0 \Rightarrow 4t^2 - 6t - 2t + 3 \geq 0 \Rightarrow 2t(2t - 3) - 1(2t - 3) \geq 0 \Rightarrow (2t - 3)(2t - 1) \geq 0$

$\Rightarrow log_4\left(3^x - 1\right) \leq \frac{1}{2}$ or $log_4\left(3^x - 1\right) \geq \frac{3}{2}$

$\Rightarrow 0 < 3^x - 1 \leq 4^{1/2}$ or $3^x - 1 \geq 4^{3/2}$

$\Rightarrow 1 < 3^x \leq 3$ or $3^x \geq 9$
$\Rightarrow 0 < x \leq 1$ or $x \geq 2$
$\Rightarrow x \in (0, 1] \cup [2, \infty)$

Illustration 27:
Solve for : $log_{1/3}\left(x^2 - 6x + 18\right) - 2log_{1/3}(x - 4) < 0$
Solution:
Checking existence
(1) $x^2 - 6x + 18 > 0 \Rightarrow x \in R$
(2) $x - 4 > 0 \Rightarrow x > 4$
Now solving inequality
$\Rightarrow log_{1/3}\left(x^2 - 6x + 18\right) - log_{1/3}(x - 4)^2 < 0$

$\Rightarrow log_{1/3}\frac{\left(x^2-6x+18\right)}{(x-4)^2} < 0$ and $2log_{1/3}(x - 4) = log_{1/3}(x - 4)^2$
only when $x - 4 > 0$, so we get
$\Rightarrow log_{1/3}\left(\frac{x^2-6x+18}{(x-4)^2}\right) < 0 \ and \ x - 4 > 0 \Rightarrow x > 4$ …(i)
$\Rightarrow x^2 - 6x + 18 > (x - 4)^2 \Rightarrow x^2 - 6x + 18 > x^2 - 8x + 16$
$2x + 2 > 0 \Rightarrow x > -1 \Rightarrow x \in (-1, \infty)$ ..(ii)
from equation (i) and (ii), we get $x \in (4, \infty)$

Illustration 28:
Solve for : $log_e\left(x^2 - 2x - 2\right) \leq 0$
Solution:
The values of x satisfying the inequality $log_e\left(x^2 - 2x - 2\right) \leq 0$ must be such that $0 < x^2 - 2x - 2 \leq 1$
we have, $x^2 - 2x - 2 > 0 \Rightarrow (x - 1)^2 > 3$
$\Rightarrow |x - 1|^2 > 3 \Rightarrow |x - 1| > \sqrt{3} \Rightarrow x - 1 > \sqrt{3}$ or $x - 1 < -\sqrt{3}$
$\Rightarrow x > 1 + \sqrt{3}$ or $x < 1 - \sqrt{3}$ …(1)
Again $x^2 - 2x - 2 \leq 1 \Rightarrow x^2 - 2x \leq 3 \Rightarrow (x - 1)^2 \leq 4 \Rightarrow |x - 1|^2 \leq 4$
$\Rightarrow |x - 1| \leq 2 \Rightarrow -2 \leq x - 1 \leq 2 \Rightarrow -1 \leq x \leq 3$ …(2)
The value of x satisfying both the inequalities equation (1) and (2) are given by; Hence,
$x \in [-1, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, 3]$

Illustration 29:
Solve for : $log_x\left(2x - \frac{3}{4}\right) > 2$
Solution:
For existence of logarithm
$2x - \frac{3}{4} > 0$ and $x > 0$ and $x \neq 1$
so, $x \in \left(\frac{3}{8}, \infty\right) - \{1\}$
To find the value of x satisfying the inequality $log_x[2x - (3/4)] > 2$

Case I. Let $0 < x < 1$

Then, $log_x[2x - (3/4)] > 2 \Rightarrow [2x - (3/4)] < x^2$

$\Rightarrow x^2 - 2x + (3/4) > 0 \Rightarrow 4x^2 - 8x + 3 > 0 \Rightarrow (2x - 1)(2x - 3) > 0$

$\Rightarrow \left(x - \frac{1}{2}\right)[x - (3/2)] > 0 \Rightarrow x > 3/2$ or $x < 1/2$

$\Rightarrow x < 1/2$ because we have $0 < x < 1$.

$\therefore$ But for $log[2x - (3/4)]$ to be meaningful, we must have

$2x - (3/4) > 0 \Rightarrow x > 3/8$

Therefore, if $0 < x < 1$, the values of x satisfying the given inequality are given by:

$3/8 < x < 1/2$

Case II. Let $x > 1$

Then, $log_x[2x - (3/4)] > 2 \Rightarrow [2x - (3/4)] > x^2$

$\Rightarrow x^2 - 2x + (3/4) < 0 \Rightarrow 4x^2 - 8x + 3 < 0 \Rightarrow (2x - 1)(2x - 3) < 0 \Rightarrow \left(x - \frac{1}{2}\right)[x - (3/2)] < 0 \Rightarrow 1/2 < x < 3/2$

But we have $x > 1$

$\therefore$ We must have $1 < x < 3/2$ and obviously these values of x make $2x - (3/4) > 0$

Therefore, if $x > 1$, the values of x satisfying the given inequality are given by, $1 < x < 3/2$

$x \in \left(\frac{3}{8}, \frac{1}{2}\right) \cup \left(1, \frac{3}{2}\right)$

Illustration 30:

Solve for : $log_{0.5}\left(x^2 - 5x + 6\right) \geq -1$

Solution:

Checking existence $x^2 - 5x + 6 > 0$,

$\Rightarrow x \in (-\infty, 2) \cup (3, \infty)$

Now solving inequality

$log_{0.5}\left(x^2 - 5x + 6\right) \geq -1 \Rightarrow 0 < x^2 - 5x + 6 \leq (0.5)^{-1} \Rightarrow x^2 - 5x + 6 \leq 2 \{x^2 - 5x + 6 > 0 \; x^2 - 5x + 6 \leq 2 \Rightarrow x \in [1, 2$

Hence, solution set of original inequation: $x \in [1, 2) \cup (3, 4]$

Illustration 31:

Solve for : $log_2 x \leq \frac{2}{log_2 x - 1}$.

Solution:

Let $log_2 x = t$

$t \leq \frac{2}{t-1} \Rightarrow t - \frac{2}{t-1} \leq 0 \Rightarrow \frac{t^2 - t - 2}{t-1} \leq 0 \Rightarrow \frac{(t-2)(t+1)}{(t-1)} \leq 0 \Rightarrow t \in (-\infty, -1] \cup (1, 2]$

or $log_2 x \in (-\infty, -1] \cup (1, 2]$

or $x \in (0, \frac{1}{2}] \cup [2, 4]$

Illustration 32:
Find all x such that $log_{1/2} x > log_{1/3} x$.

Solution:
We have $log_{1/2} x > log_{1/3} x$.

$\Rightarrow -log_2 x > -log_3 x \Rightarrow log_2 x < log_3 x \Rightarrow log_2 x < \frac{log_2 x}{log_2 3}$

$\Rightarrow log_2 3 log_2 x < log_2 x$ (as $log_2 3 > 0$)

$\Rightarrow log_2 x \left(log_2 3 - 1\right) < 0$

Since $log_2 3 - 1 > 0$, from the latter inequality we obtain $log_2 x < 0$, hence $x < 1$. But the original inequality is meaningful only when $x > 0$. Therefore all x that satisfy the original inequality lie in the interval $0 < x < 1$.
Answer: $x \in (0, 1)$

Illustration 33:
Solve the inequation: $log_{2x+3} x^2 < log_{2x+3}(2x + 3)$

Solution:
For existence of logarithm

$$(i)\ x^2 > 0\ (ii)\ 2x + 3 > 0\ (iii)\ 2x + 3 \neq 1 \Rightarrow x \in \left(-\frac{3}{2}, \infty\right) - \{-1, 0\} \#(i)$$

Now solving inequality

Case I: $0 < 2x + 3 < 1$

$$\Rightarrow -\frac{3}{2} < x < -1\ \ \because log_{2x+3} x^2 < log_{2x+3} 2x + 3\ \ \Rightarrow x^2 > 2x + 3\ \ \Rightarrow (x - 3)(x + 1) > 0\ \ \Rightarrow x \in (-\infty, -1) \cup (3, \infty$$

Case II: $2x + 3 > 1 \Rightarrow x > -1$

$\because log_{2x+3} x^2 < log_{2x+3} 2x + 3 \Rightarrow x^2 < 2x + 3 \Rightarrow (x - 3)(x + 1) < 0$

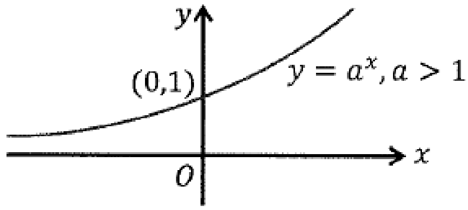
$\Rightarrow x \in (-1, 3)$; but $x > -1$

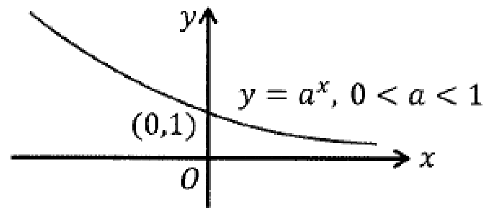
$\Rightarrow x \in (-1, 3)$ intersection with (i) $\Rightarrow x \in (-1, 3) - \{0\}$

$\therefore x \in$ case I $\cup$ case II

$\Rightarrow x \in \left(-\frac{3}{2}, -1\right) \cup (-1, 0) \cup (0, 3)$

7. Graph of Exponential Function, its Equation and Inequalities





If $a^{f(x)} > b \Rightarrow [f(x) > log_a b \; when \; a > 1 \; f(x) < log_a b \; when \; 0 < a < 1$

Note: Domain of a^x is R and Range is $(0, \infty)$.

Illustration 34:

Solve the equation $3^x \cdot 8^{\frac{x}{x+2}} = 6$.

Solution:

Some students solved this equation thus: rewriting it as

$$3^x \cdot 2^{\frac{3x}{x+2}} = 3^1 \cdot 2^1$$

They chose a root x so that the exponents of the respective bases were the same:

$x = 1, \frac{3x}{x+2} = 1$

hence the "answer" $x = 1$.

But this "answer" is incorrect in the sense that only one root of the equation is found and nothing has been said about any other roots. Actually, if the exponents on the appropriate bases are equal, then the products of these powers are equal, however the converse is not in any way implied and is simply incorrect. For instance, the equation

$3^1 . 2^1 = 3^2 . 2^{log_2(2/3)}$

is valid, but $1 \neq 2$ and $1 \neq log_2(2/3)$. Therefore, the foregoing reasoning may lead to a loss of roots, and this is exactly what occurred in the equation at hand.

Taking logarithms of both members of the original to the base 10 , we get

$x log_{10} 3 + \frac{3x}{x+2} log_{10} 2 = log_{10} 6$

or $x^2 log_{10} 3 + x\left(3log_{10} 2 + 2log_{10} 3 - log_{10} 6\right) - 2log_{10} 6 = 0$

We now have to solve this quadratic equation. This can be done using a familiar formula, but we will try to simplify the solution by an ingenious device, since we have alreadyseen, by trial and error, that $x_1 = 1$ is a root of the original equation and, consequently, satisfies the equivalent quadratic equation. For this reason, by Viete's theorem the second root of the quadratic equation is $x_2 = \left(-2log_{10} 6\right)/log_{10} 3 = -2log_3 6$ and so the original equation has two roots; $x_1 = 1, x_2 = -2log_3 6$.

Thus, it is useful to be able to guess a root, but never consider the guessing as the whole solution.

Illustration 35:

$\{2^{y-x}(x + y) = 1, \; (x + y)^{x-y} = 2.$

Solution:
The domain of definition is $x + y > 0$.
$\{x + y = \frac{1}{2^{y-x}} = 2^{x-y},\ (x + y)^{x-y} = 2.$

From the first equation we find $x + y = 2^{x-y}$ and substitute it into the second equation.
Then $\left(2^{x-y}\right)^{x-y} = 2 \Rightarrow 2^{(x-y)^2} = 2 \Rightarrow (x - y)^2 = 1.$
The solution of the given system is the solution of the collection of systems
$\{x - y = 1,\ x + y = 2,$ or $\{x - y = -1,\ x + y = \frac{1}{2}.$
Answer: $\left(\frac{3}{2}, \frac{1}{2}\right), \left(-\frac{1}{4}, \frac{3}{4}\right).$

Illustration 36:
Solve for : $2^{x+2} > \left(\frac{1}{4}\right)^{\frac{1}{x}}$
Solution:
We have $2^{x+2} > 2^{-\frac{2}{x}}$. Since the base $2 > 1$, we have $x + 2 > -\frac{2}{x}$
(the sign of the inequality is retained).
Now $x + 2 + \frac{2}{x} > 0 \Rightarrow \frac{x^2+2x+2}{x} > 0$
$\Rightarrow \frac{(x+1)^2+1}{x} > 0 \Rightarrow x \in (0, \infty)$

Illustration 37:
Solve for : $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$
Solution:
We have $\left(\frac{5}{4}\right)^{1-x} < \left(\frac{16}{25}\right)^{2(1+\sqrt{x})}$ or $\left(\frac{4}{5}\right)^{x-1} < \left(\frac{4}{5}\right)^{4(1+\sqrt{x})}$
Since the base $0 < \frac{4}{5} < 1$, the inequality is equivalent to the inequality $x - 1 > 4(1 + \sqrt{x})$
$\Rightarrow \frac{x-5}{4} > \sqrt{x}$

Now, R.H.S. is positive
$\Rightarrow \frac{x-5}{4} > 0 \Rightarrow x > 5$ ...(i)
we have $\frac{x-5}{4} > \sqrt{x}$
both sides are positive, so squaring both sides
$\Rightarrow \frac{(x-5)^2}{16} > x$ or $\frac{(x-5)^2}{16} - x > 0$
Or $x^2 - 26x + 25 > 0$ or $(x - 25)(x - 1) > 0$
$\Rightarrow x \in (-\infty, 1) \cup (25, \infty)$ ...(ii)
intersection (i) & (ii) gives $x \in (25, \infty)$

8. Characteristic and Mantissa:
For any given number N, logarithm can be expressed as $log_a N$ = Integer + Fraction. The integer part is called characteristic and the fractional part (always taken non negative) is called mantissa. When the value of $log_{10} N$ is

given, then to find digits of ' N ' we use only the mantissa part. The characteristic is used only in determining the number of digits in the integral part (if ≥ 1) or the number of zeros after decimal & before first non-zero digit in the number (if $0 < N < 1$).

Note:

(i) The mantissa part of logarithm of a number is always non-negative ($0 \leq m < 1$)

(ii) If the characteristic of $log_{10}N$ be ' C ' and $C \geq 0$, then the number of digits in N is $(C + 1)$

(iii) If the characteristic of $log_{10}N$ be ' $-C$ ' and $C > 0$, then there exist $(C - 1)$ zeros after decimal in N.

In summary, if characteristic of $log_{10}N$ is ' C ' then number of digits ($N \geq 1, N \in N$) or number of zeros after decimal in $N(0 < N < 1) = |C + 1|$.

EXERCISE - 0
SINGLE CORRECT TYPE QUESTIONS

1. If $a^4 \cdot b^5 = 1$ then the value of $log_a\left(a^5 b^4\right)$ equals

   (A) 9/5 (B) 4 (C) 5 (D) 8/5

2. The expression $log_p log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{.... pp}}}}_{n\ radical\ sign}$, where $p \geq 2, p \in N; n \in N$ when simplified is

   (A) independent of p (B) independent of p and of n

   (C) dependent on both p and n (D) positive

3. If a, b, c are positive real numbers such that $a^{log_3 7} = 27; b^{log_7 11} = 49$ and $c^{log_{11} 25} = \sqrt{11}$. The value of $\left(a^{\left(log_3 7\right)^2} + b^{\left(log_7 11\right)^2} + c^{\left(log_{11} 25\right)^2}\right)$ equals

   (A) 489 (B) 469 (C) 464 (D) 400

4. If $log_2\left(x^2 + 1\right) + log_{13}\left(x^2 + 1\right) = log_2\left(x^2 + 1\right) log_{13}\left(x^2 + 1\right)$, $(x \neq 0)$, then $log_7\left(x^2 + 24\right)$ is equal to

   (A) 1 (B) 2 (C) 3 (D) 4

5. Given $log_3 a = p = log_b c$ and $log_b 9 = \frac{2}{p^2}$. If $log_9\left(\frac{a^4 b^3}{c}\right) = \alpha p^3 + \beta p^2 + \gamma p + \delta (\forall p \in R - \{0\})$, then $(\alpha + \beta + \gamma + \delta)$ equals

   (A) 1 (B) 2 (C) 3 (D) 4

6. Sum of all the solution(s) of the equation $log_{10}(x) + log_{10}(x + 2) - log_{10}(5x + 4) = 0$ is -

   (A) -1 (B) 3 (C) 4 (D) 5

7. If x_1 and x_2 are the roots of equation $e^{3/2} \cdot x^{2 ln x} = x^4$, then the product of the roots of the equation is -

   (A) e^2 (B) e (C) $e^{3/2}$ (D) e^{-2}

8. If x satisfies the inequality $log_{25} x^2 + \left(log_5 x\right)^2 < 2$, then $x \in$

   (A) $\left(\frac{1}{25}, 5\right)$

   (B) $(1, 2)$

   (C) $(4, 5)$

   (D) $(0, 1)$

9. If $x = \log_2\left(\sqrt{56+\sqrt{56+\sqrt{56+\sqrt{56+..\infty}}}}\right)$, then which of the following statement holds good ?

(A) $x < 0$ (B) $0 < x < 2$ (C) $2 < x < 4$ (D) $3 < x < 4$

MULTIPLE CORRECT TYPE QUESTIONS

10. Which of the following statements is(are) correct?

(A) $7^{1/7} > (42)^{1/14} > 1$ (B) $log_3(5)log_7(9)log_{11}(13) >- 2$

(C) $\sqrt{99+70\sqrt{2}} + \sqrt{99-70\sqrt{2}}$ is rational (D) $\frac{1}{log_4 3} + \frac{1}{log_7 3} > 3$

11. If $\left(log_\beta \alpha\right)^2 + \left(log_\alpha \beta\right)^2 = 79, (\alpha > 0, \beta > 0, \alpha\neq 1, \beta\neq 1)$ then value of $\left(log_\beta \alpha\right) + \left(log_\alpha \beta\right)$ can be -

(A) 7 (B) -9 (C) 9 (D) -7

12. The solution of the equation $5^{log_a x} + 5x^{log_a 5} = 3, (a > 0, a\neq 1)$ is

(A) $a^{-log_5 2}$ (B) $a^{log_5 2}$ (C) $2^{-log_5 a}$ (D) $2^{log_5 a}$

13. The equation $log_{x^2}16 + log_{2x}64 = 3$ has :

(A) one irrational solution (B) no prime solution

(C) two real solutions (D) one integral solution

14. The equation $x^{[log_3 x)^2 - \frac{9}{2}log_3 x + 5]} = 3\sqrt{3}$ has

(A) exactly three real solution (B) at least one real solution

(C) exactly one irrational solution (D) At least one imaginary root

15. Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1 , such that $2\left(log_a c + log_b c\right) = 9log_{ab}c$, then the possible value of $log_a b$.

(A) 1/2 (B) 3 (C) 1/3 (D) 2

16. If $2^{x+y} = 6^y$ and $3^{x-1} = 2^{y+1}$, then the value of $(log3 - log2)/(x - y)$ is

(A) 1 (B) $log_2 3 - log_3 2$ (C) $log(3/2)$ (D) $log3 - log2$

17. Which of the following statements are true

(A) $log_2 3 < log_{12}10$ (B) $log_6 5 < log_7 8$

(C) $log_3 26 < log_2 9$ (D) $log_{16}15 > log_{10}11 > log_7 6$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 18 to 19

If $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ are the solution of the system of equation.

$log_{225}(x) + log_{64}(y) = 4$

$log_x(225) - log_y(64) = 1,$

18. The value of $log_{30}\left(x_1 y_1 x_2 y_2\right)$ is

19. Number of digits in $\left(x_1 y_1 x_2 y_2\right)$ is

MATRIX MATCH TYPE QUESTION

20. Match List-I with List-II and select the correct answer using the code given below the list.

List-I	List-II
(P) The value of x for which $2ln\left(2^{x} - 5\right) = ln2 + ln\left(2^{x} - \frac{7}{2}\right)$ is	(1) 2
(Q) The sum of the real roots of the equation $x^{log_{10}\left(\frac{5x}{2}\right)} = 10^{log_{10}x}$ is	(2) 4
(R) If $\alpha = 2^{\frac{\left(log_{3}\left(log_{3}5\right)\right)}{log_{3}2}}$ and $\beta^{\alpha} = 9$, then value $\frac{1}{5}\left(\beta^{a^{2}}\right)$ is equal to	(3) 3
(S) $log_{\sqrt{3}}(\sqrt{6 + \sqrt{6 + \sqrt{6 + ...\infty}}})$ is equal to	(4) 5

The correct option is

(A) $P\rightarrow1$; $Q\rightarrow2$; $R\rightarrow2$; $S\rightarrow5$
(B) $P\rightarrow3$; $Q\rightarrow4$; $R\rightarrow2$; $S\rightarrow5$
(C) $P\rightarrow1$; $Q\rightarrow2$; $R\rightarrow4$; $S\rightarrow1$
(D) $P\rightarrow3$; $Q\rightarrow4$; $R\rightarrow4$; $S\rightarrow1$

EXERCISE – S

1. Let A denotes the value of $log_{10}\left(\frac{ab+\sqrt{(ab)^{2}-4(a+b)}}{2}\right) + log_{10}\left(\frac{ab-\sqrt{(ab)^{2}-4(a+b)}}{2}\right)$ when $a = 43$ and $b = 57$ and B denotes the value of the expression $\left(2^{log_{6}18}\right) \cdot \left(3^{log_{6}3}\right)$.
Find the value of (A.B).
2. There exist positive integers A, B and C with no common factor greater than 1 such that $Alog_{200}5 + Blog_{200}2 = C$, then the value of $A + B + C$ is
3. Number of solutions of $log_{(1+x)}\left(\frac{1-x}{1+x}\right) - 2log_{(1-x)}(1 + x) = 0$ is
4. The minimum possible real x which satisfy the equation, $2log_{2}log_{2}x + log_{1/2}log_{2}(2\sqrt{2}x) = 1$.
5. If $4^{A} + 9^{B} = 10^{C}$, where $A = log_{16}4, B = log_{3}9 \& C = log_{x}83$, then find x.
6. If $x_{1} \& x_{2}$ are the two values of x satisfying the equation $7^{2x^{2}} - 2\left(7^{x^{2}+x+12}\right) + 7^{2x+24} = 0$, then $\left(x_{1} + x_{2}\right)$ equals
7. Sum of the roots of the equation $9^{log_{3}\left(log_{2}x\right)} = log_{2}x - \left(log_{2}x\right)^{2} + 1$ is equal to
8. The number of solutions of the equation $log_{x-3}\left(x^{3} - 3x^{2} - 4x + 8\right) = 3$ is equal to
9. The number of solution(s) of $\sqrt{log_{3}\left(3x^{2}\right) \cdot log_{9}(81x)} = log_{9}x^{3}$ is –
10. If $log_{10}\left(\frac{1}{2^{x}+x-1}\right) = x\left(log_{10}5 - 1\right)$, then $x =$

EXERCISE - JEE (Main) PYQ

1. The sum of the roots of the equation $x + 1 - 2log_2(3 + 2^x) + 2log_4(10 - 2^{-x}) = 0$, is: [JEE (Main) 2021]

(1) $log_2 14$ (2) $log_2 11$ (3) $log_2 12$ (4) $log_2 13$

2. The number of solutions of the equation $log_{(x+1)}(2x^2 + 7x + 5) + log_{(2x+5)}(x + 1)^2 - 4 = 0, x > 0$, is [JEE (Main) 2021]

3. The number of integral solutions x of $log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is [JEE (Main) 2023]

(1) 6 (2) 8 (3) 5 (4) 7

EXERCISE - JEE (Advanced) PYQ

1. Number of solutions of $log_4(x - 1) = log_2(x - 3)$ is [JEE 2001 (Screening)]

(A) 3 (B) 1 (C) 2 (D) 0

Let (x_0, y_0) be the solution of the following equations

$(2x)^{ln2} = (3y)^{ln3} 3^{lnx} = 2^{lny}$

2.

Then x_0 is [JEE 2011]

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

3. The value of $6 + log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}.......}}}\right)$ is [JEE 2012]

4. If $3^x = 4^{x-1}$, then $x =$ [JEE (Advanced) 2013]

(A) $\frac{2log_3 2}{2log_3 2-1}$ (B) $\frac{2}{2-log_2 3}$ (C) $\frac{1}{1-log_4 3}$ (D) $\frac{2log_2 3}{2log_2 3-1}$

5. The value of $\left((log_2 9)^2\right)^{\frac{1}{log_2(log_2 9)}} \times (\sqrt{7})^{\frac{1}{log_4 7}}$ is _______ [JEE (Advanced) 2018]

6. The product of all positive real values of x satisfying the equation $x^{\left(16.6log_5 x\right)^3 - 68log_5 x)} = 5^{-16}$ is - [JEE (Advanced) 2022]

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can }{ see PDF solutions or video solutions or solutions in hardcopy whichever is provided. }

SECTION-A

- This section contains TWENTY questions.
- Each question has FOUR options (1), (2), (3) and (4). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : + 4, if only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0, if none of the bubbles is darkened.
  Negative Marks : -1 in all other cases.

1. If p and q are solutions of the equation $5^{\left(\log_5^2 x\right)} + x^{\log_5 x} = 1250$ then $\log_q p$ has the value equal to -

(1) 0 (2) 1 (3) -1 (4) -2

2. If $2\log_a x = \log_b x + \log_c x$ where $a, b, c > 0 \& \neq 1$ then which of the following holds -

(1) $c^2 = (bc)^{\log_c a}$ (2) $a = bc\left(\log_c b\right)$ (3) $b^2 = (bc)^{\log_c a}$ (4) $a^2 = bc$

3. If $\log_c 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_c 10$ where $c > 0; c \neq 1; b > 1; b \neq 1$ determine b -

(1) 25 (2) 5 (3) 625 (4) 16

4. If $\log_\alpha 8 = \gamma, \log_\beta \alpha = -1$ and $\log_{1/4} \beta = -1$ then $\left(\frac{1}{\alpha} + 1\right)^{\log_{\sqrt{5}}\left(\beta^2 + 4\gamma^2\right)}$ is equal to -

(1) $\sqrt{5}$ (2) 5 (3) 25 (4) 625

5. a,b,c are real numbers greater then 1 and $x \in R^+$. If $\log_a x = 24, \log_b x = 40, \log_{abc} x = 12$ then $\log_c x$ equal is-

(1) 80 (2) 60 (3) 48 (4) 72

6. Let $x_1 = 97, x_2 = \frac{2}{x_1}, x_3 = \frac{3}{x_2}, x_4 = \frac{4}{x_3}, \ldots\ldots, x_8 = \frac{8}{x_7}$ then $\log_{3\sqrt{2}}\left(\prod_{i=1}^{8} x_i - 60\right) =$

(1) $\frac{3}{2}$ (2) 4 (3) 6 (4) $\frac{5}{2}$

7. If $\log_p q + \log_q r + \log_r p$ vanishes where p, q and r are positive reals different than unity then the value of $\left(\log_p q\right)^3 + \left(\log_q r\right)^3 + \left(\log_r p\right)^3$ is -

(1) an odd prime (2) an even prime

(3) an odd composite (4) an irrational number

8. If $\log_{16} x = \frac{3}{4}$ and $\log_y 0.125 = -3$, then the value of $\log_{0.25}\left(\frac{x}{y}\right)$ is -

(1) 1 (2) -1 (3) 2 (4) 4

9. If $\log_a x = p$ and $\log_b x^2 = q$, then $\log_x \sqrt{ab}$ is equal to (where $a, bx \in R^+$) -

(1) $\frac{1}{p} + \frac{1}{q}$ (2) $\frac{1}{2p} + \frac{1}{q}$ (3) $\frac{1}{p} + \frac{1}{2q}$ (4) $\frac{1}{2p} + \frac{1}{2q}$

10. Number of solution(s) of the equation $\log_2\left(x^2 + 3\right) = \frac{1}{2}\log_{1/3}\left(x + \frac{1}{x}\right), x > 0$ is -

(1) 0 (2) 1 (3) 2 (4) infinite

11. The value of x satisfying the equation $\log_3\sqrt{\log_2 x} = \frac{1}{2}$ is -

(1) 8 (2) 9 (3) 16 (4) 27

12. $\frac{1}{\log_{bc}\sqrt{abc}} + \frac{1}{\log_{ca}\sqrt{abc}} + \frac{1}{\log_{ab}\sqrt{abc}}$ has the value equal to -

(1) 1 (2) 1/2 (3) 2 (4) 4

13. Which of the following reduces to an integer?

(1) $\frac{\log 32}{\log 4}$ (2) $\frac{\log_5 128}{\log_5 16 - \log_5 4}$ (3) $\frac{2\log 6}{\log 12 + \log 3}$ (4) $\log_4 8$

14. The value of $3^{\log_4 5} + 4^{\log_5 3} - 5^{\log_4 3} - 3^{\log_5 4}$ is -

(1) 0 (2) 1 (3) 2 (4) none of these

15. $(5.8)^{\left(\log_{58} 10\right) + 1}$ equals -

(1) 5.8 (2) $10 + 1$ (3) 58 (4) 580

16. If $2x^{\log_4 3} + 3^{\log_4 x} = 27$, then x is equal to -

(1) 2 (2) 4 (3) 8 (4) 16

17. $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y}$ when simplified reduces to -

(1) xyz (2) 1 (3) $x^y y^z z^x$ (4) none of these

18. The number of Solution of $log_4(x - 1) = log_2(x - 3)$ is

(1) 0 (2) 1 (3) -1 (4) -2

19. If $logx^2 - log2x = 3log3 - log6$ then $x =$

(1) 9 (2) 3 (3) 4 (4) 5

20. The value of $\frac{log_2 24}{log_{96} 2} - \frac{log_2 192}{log_{12} 2}$ is

(1) 3 (2) 0 (3) 2 (4) 1

SECTION-B

- This section will have TEN questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : + 4, if only correct answer is given.

  Zero Marks : 0, if no answer is given.

  Negative Marks : -1 for incorrect answer

1. The number of ordered pairs satisfying $4\left(log_2 x\right)^2 + 1 = 2log_2 y$ and $log_2 x^2 \geq log_2 y$ [where y is an integer], is equal to
2. $\frac{16^{\frac{1}{log_3 4}} + 8^{log_4 25} + 3^{\frac{4}{log_7 9}}}{183}$ is equal to
3. If $c(b - a) = b(a - c)$, then $\frac{log(b+c)+log(b-2a+c)}{log(b-c)}$ equals (Assume all terms are defined)

$log_3 N = A_1 + B_1$

$log_6 N = A_2 + B_2$

4. where A_1, A_2 are integers and $B_1, B_2 \in [0, 1]$. If the number of integral values of N is number PQR and $A_1 = 4 \& A_2 = 2$, Then find the value of $P + Q + R$
5. If the product of the positive roots of the equation $\sqrt{15}(x)^{log_{15} x} = x^2$ is number ABC, Then find the value of $B + C - A$
6. If $x \& y$ are prime numbers such that $x\left(3log_x y - 2\right) + y\left(10log_{x^2} y - 1\right) = 34\left(log_{x^2} y^2\right) - 11$, then the value of $\frac{\left|x^2-y^2\right|}{4}$ is
7. The number of values of x satisfying the equation $\frac{log_2(x-4)+1}{log_{\sqrt{2}}(\sqrt{x+3}-\sqrt{x-3})} = 1$ is/are
8. If $\frac{log_e x}{b-c} = \frac{log_e y}{c-a} = \frac{log_e z}{a-b}$ then value of $x^{(c+b)} y^{(a+c)} z^{(b+a)}$ is equal to
9. Number of solutions of the equation $log_x 3 \cdot log_{3x} 3 = log_{9x} 3$ is equal to
10. The value of x satisfying the equation $log_4\left(2log_3\left(6 + log_2 x\right)\right) = 1$ is

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains FOUR questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : + 3, if only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0, if none of the bubbles is darkened.
  Negative Marks : -1 , in all other cases

1. If $60^a = 3$ and $60^b = 5$ then the value of $12^{\frac{1-a-b}{2(1-b)}}$ equals

(A) 2 (B) 3 (C) $\sqrt{3}$ (D) $\sqrt{12}$

2. If α and β are the roots of the equation $\left(log_2 x\right)^2 + 4\left(log_2 x\right) - 1 = 0$ then the value of $log_\beta \alpha + log_\alpha \beta$ equals

(A) 18 (B) -16 (C) 14 (D) -18

3. If $log_a(1 - \sqrt{1 + x}) = log_{a^2}(3 - \sqrt{1 + x})$, then number of solutions of the equation is -

(A) 0 (B) 1 (C) 2 (D) infinitely many

4. If $log_{0.3}(x - 1) < log_{0.09}(x - 1)$, then x lies in the interval

(A) $(2, \infty)$ (B) $(1, 2)$ (C) $(1, \infty)$ (D) none of these

SECTION-II

- This section contains SIX questions.
- Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option (s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : + 4 if only (all) the correct option(s) is (are) chosen.
  Partial Marks : +3 if all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 if three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : +1 if two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : -2 in all other cases.
  For Example : If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

5. Let $N = \frac{log_2 24}{log_{96} 2} - \frac{log_2 192}{log_{12} 2}$. Then N is :

(A) a natural number (B) a prime number
(C) a rational number (D) an integer

6. For $a > 0, \neq 1$ the roots of the equation $log_{ax} a + log_x a^2 + log_{a^2 x} a^3 = 0$ can be

(A) $a^{-3/4}$ (B) $a^{-4/3}$ (C) $a^{-1/2}$ (D) none of these

7. The equation $x^{(3/4)\left(\log_2 x\right)^2+\log_2 x-5/4}=\sqrt{2}$ has
(A) at least one real solution
(B) exactly three real solutions
(C) exactly one irrational solution
(D) At least one imaginary root

8. If x_1, x_2 are the value(s) of x satisfying the equation $\log_2^2(x-2)+\log_2(x-2)\log_2\left(\frac{3}{x}\right)-2\log_2^2\left(\frac{3}{x}\right)=0$, then x_1+x_2 is equal to
(A) $x_1+x_2=9$
(B) $x_1x_2=18$
(C) $\left|x_1-x_2\right|=3$
(D) $x_1^2+x_2^2=45$

9. The ordered pair (x, y) satisfying the system of equations $\log_3 x+\log_3 y=2+\log_3 2$ and $\log_{27}(x+y)=\frac{2}{3}$ is/are :
(A) $(6, 3)$
(B) $(3, 6)$
(C) $(6, 12)$
(D) $(12, 6)$

10. The inequality $-1\le\left(\frac{1}{3}\right)^x<2$ is satisfied by
(A) $x\in[0, 1]$
(B) $x>-\log_3 2$
(C) $x<-\log_3 2$
(D) $x<-1$

SECTION-III

- This section contains ONE paragraphs.
- Based on each paragraph, there are TWO questions.
- Each question has FOUR options (A), (B), (C) and (D) ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :
  Full Marks : + 3 if only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 in all other cases.
  Comprehension # (Q. No. 11 - 12)
  Let $\log_2 5=a; \log_5 3=b$

11. The value of $\log_{50}30$ is equal to
(A) $\frac{b+ab+1}{b+2}$
(B) $\frac{a+ab+1}{2a+1}$
(C) $\frac{a+ab+1}{a+2}$
(D) $\frac{a+ab+b}{2a+1}$

12. Roots of the equation $\left(\log_{10}2\cdot\log_{10}5\right)x^2-\left(\log_{10}5+\log_{10}15\cdot\log_{10}2\right)x+\log_{10}15=0$ are
(A) a, b
(B) $1+a, 1+b$
(C) $a-1, 1-b$
(D) $a+2; b+2$

SECTION-IV

- This section contains ONE question.
- Each question has matching lists. The codes for the lists have choices (A), (B), (C) and (D) out of which ONLY ONE is correct
- For each question, marks will be awarded in one of the following categories :
  Full Marks : + 3 If only correct answer is given.
  Zero Marks : 0 If no answer is given.
  Negative Marks : -1 For incorrect answer

13. Match List-I with List-II and select the correct answer using the code given below the list.

List-I	List-II
(P) Positive value of x satisfying the equation $(25)^{\frac{2x^2-5x-9}{4}}=(\sqrt{11})^{3\log_{11}5}$ is	(1) 5
(Q) Sum of values of x which satisfy the equation	(2) 12

$\sqrt{|x-4|^{x-2}} = \sqrt[4]{|x-4|^{x+2}}$ *is*

(R) The expression $\sqrt{log^2_{0.5}32}$ has the value equal to (3) 4

(S) The value of $3k$, where (4) 18

$log(log9) + log(log49) = logk + log(log3) + log(log7),\ is$

(A) $P\rightarrow3$; $Q\rightarrow4$; $R\rightarrow1$; $S\rightarrow2$

(B) $P\rightarrow1$; $Q\rightarrow4$; $R\rightarrow3$; $S\rightarrow2$

(C) $P\rightarrow2$; $Q\rightarrow4$; $R\rightarrow3$; $S\rightarrow1$

(D) $P\rightarrow3$; $Q\rightarrow1$; $R\rightarrow2$; $S\rightarrow4$

SECTION-V

- This section contains ONE question.
- Each question contains two columns, Column-I and Column-II.
- Column-I has four entries (A), (B), (C) and (D).
- Column-II has four entries (p), (q), (r), (s).
- Match the entries in Column-I with the entries in Column-II.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : $+\ 4$ If only correct answer is given.
  Zero Marks : 0 If no answer is given.
  Negative Marks : $-\ 1$ For incorrect answer.

14.

Column-I	Column-II
(A) Anti-logarithm of $(0.\overline{6})$ to the base 27 has the value equal to	(p) 5
(B) Characteristic of the logarithm of 2008 to the base 2 is	(q) 7
(C) The value of b satisfying the equation, $log_e 2 \cdot log_b 625 = log_{10} 16 \cdot log_e 10$ is	(r) 9
(D) Number of noughts after decimal before a significant figure comes in the number $\left(\frac{5}{6}\right)^{100}$, is (Given $log_{10}2 = 0.3010$ and $log_{10}3 = 0.4771$)	(d) 10

SECTION-VI

- This section contains FOUR questions.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, $-$ 0.33, $-$.30, 30.27, $-$ 127.30, if answer is 11.36777.... then both 11.36 and 11.37 will be correct).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : $+\ 3$, if ONLY the correct numerical value is entered as answer.
  Zero Mark: 0, in all other cases.

15. Number of ordered pair (x, y) which satisfying system of equations

$$log_x(xy) = log_y x^2$$

$y^{2log_y x} = 4y + log_2 8$, is

16. If $N = 6^{log_{10}40} \cdot 5^{log_{10}36}$, then sum of digit of number N is

17. Value of $\frac{7^{\sqrt{log_7 3}+\sqrt{log_{14} 3}} \cdot 2^{\sqrt{102_{14} 3}}}{3^{\sqrt{log_3 7}+\sqrt{log_3 14}}}$ is equal to

18. If $\frac{log_{10}x+log_{10}y}{(a+b)} = \frac{log_{10}y+log_{10}z}{(b+c)} = \frac{log_{10}z+log_{10}x}{(c+a)}$, then $x^{(c-b)}y^{(a-c)}z^{(b-a)}$ is (wherever defined)

ANSWER KEY

EXERCISE - 0

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	B	B	C	C	A	A	C	A, B, D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B, C	A, C	A, B, C, D	A, B, C	A, D	C, D	B, C	12.00	18.00	D

EXERCISE - S

1.	12	2.	6	3.	0	4.	8.00	5.	10.00
6.	1	7.	2	8.	1	9.	1	10.	1

EXERCISE - JEE (Main) PYQ				
Que.	1	2	3	
Ans.	2	1	1	

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6
Ans.	B	C	4.00	A, B, C	8.00	1.00

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	3	2	4	2	2	1	2	2	1
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	1	4	3	1	3	4	2	2	1	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	1	2	9	5	4	1	1	2	8

JEE (Advanced) Practice Paper

Section-I	Q.	1	2	3	4			
	A.	A	D	A	A			
Section-II	Q.	5	6	7	8	9	10	
	A.	A,B,C,D	B,C	A,B,C	A,B,C,D	A,B	A,B	
Section-III	Q.	11	12					
	A.	B	B					
Section-IV	Q.	13						
	A.	A						
Section-V	Q.	14						
	A.	$A \to r; B \to s; C \to p; D \to q$						
Section-VI	Q.	15	16	17	18			
	A.	2	9	1	1			