In class 3B we studied changes on the right hand side, in the supply and demand constraints.

Let's review again: a demand constraint, such as the Soldering constraint of $10x_1 + 12x_2 \leq 1000$ indicates we have only 1000 minutes at the soldering station. Increasing the "1000" is seen as relaxing the feasible region, and decreasing it seen as tightening the feasible region.

When relaxing the feasible region, the objective function will ____.

The GAMS output here shows this was an active constraint (I have added "`<- inactive`" or "`<- active`"; GAMS does not provide that). I have also showed the range in bold. For example, the Soldering range is 750 to 1068. This is the range the coefficient can change by still keeping the same set of active constraints (therefore the same set of basic and non-basic variables). Review this from the notes in the prior class.

```
EQUATION NAME                              LOWER      CURRENT        UPPER
-------------                              -----      -------        -----
Profit                                      -INF            0         +INF
Placement                                   1313         1500         +INF
Soldering                                    750         1000         1068
Inspection                                 434.8          500        666.7
NonNeg(BoardA)                              -INF            0         62.5
NonNeg(BoardB)                              -INF            0        31.25

                        LOWER      LEVEL      UPPER    MARGINAL

---- EQU Profit           .          .          .        1.000
---- EQU Placement      -INF    1312.500   1500.000        .         <- inactive
---- EQU Soldering      -INF    1000.000   1000.000      0.625       <- active
---- EQU Inspection     -INF     500.000    500.000      0.937       <- active
```

If our objective function is called p (e.g. "p" for "profit"), then the marginal values are $\frac{\Delta p}{\Delta b}$ where b_i are the coefficients on the RHS. **Marginal values/shadow prices are interpreted as the increase in objective function per unit increase of the resource**. *An alternative explanation will be given* .The range reported above in the "`EQUATION NAME`" section is the range of values for b_i where this marginal value equation is exact.

Now let's consider coefficients in the A matrix: for example, the "10" or "12" in the $10x_1 + 12x_2 \leq 1000$ equation. The interpretation of these is similar to the RHS b-coefficients.

Constraint type	Increasing the coefficient	Decreasing the coefficient
Supply (≤)	Tightening	Relaxing
Demand (≥)	Relaxing	Tightening

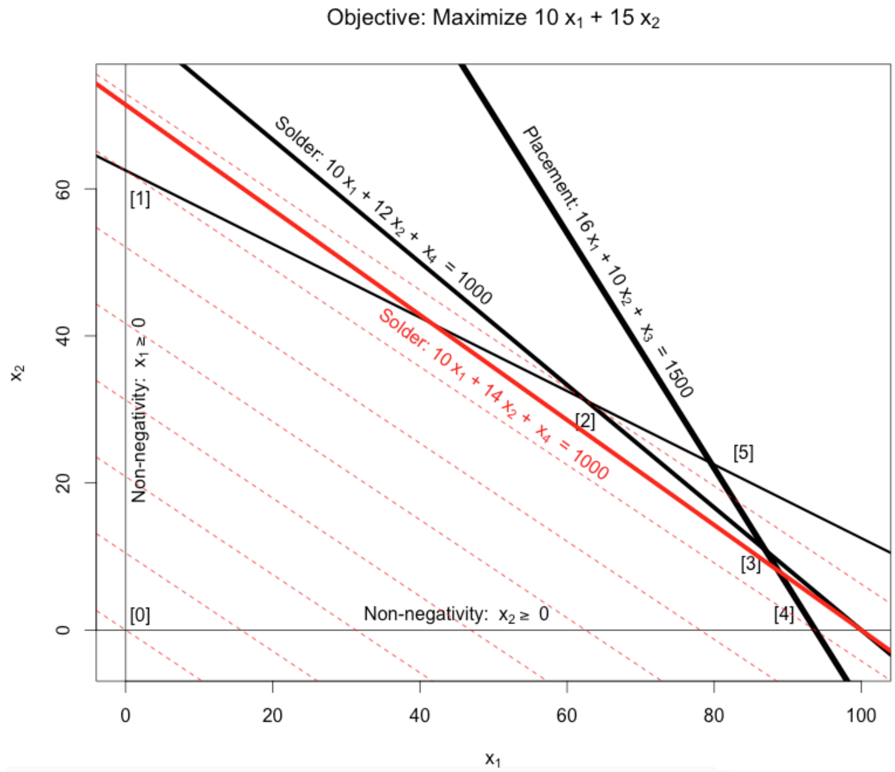
In this example, changing the requirement of 12 minutes for soldering board B (x_2) from 12 to 14 minutes has the effect of _______________(tightening or relaxing), and therefore the objective function will

____ (see the next page for an illustration).

Notice that we can't provide ranges on the amount of change that can occur in the matrix A coefficients.

General summary for feasible region tightening and relaxation:

- Tightening can leave the objective unchanged, or worse off.
- Relaxing can leave the objective unchanged, or improved.
- Tightening too much will eventually lead to a search space that is entirely unfeasible [no optimization solution exists].

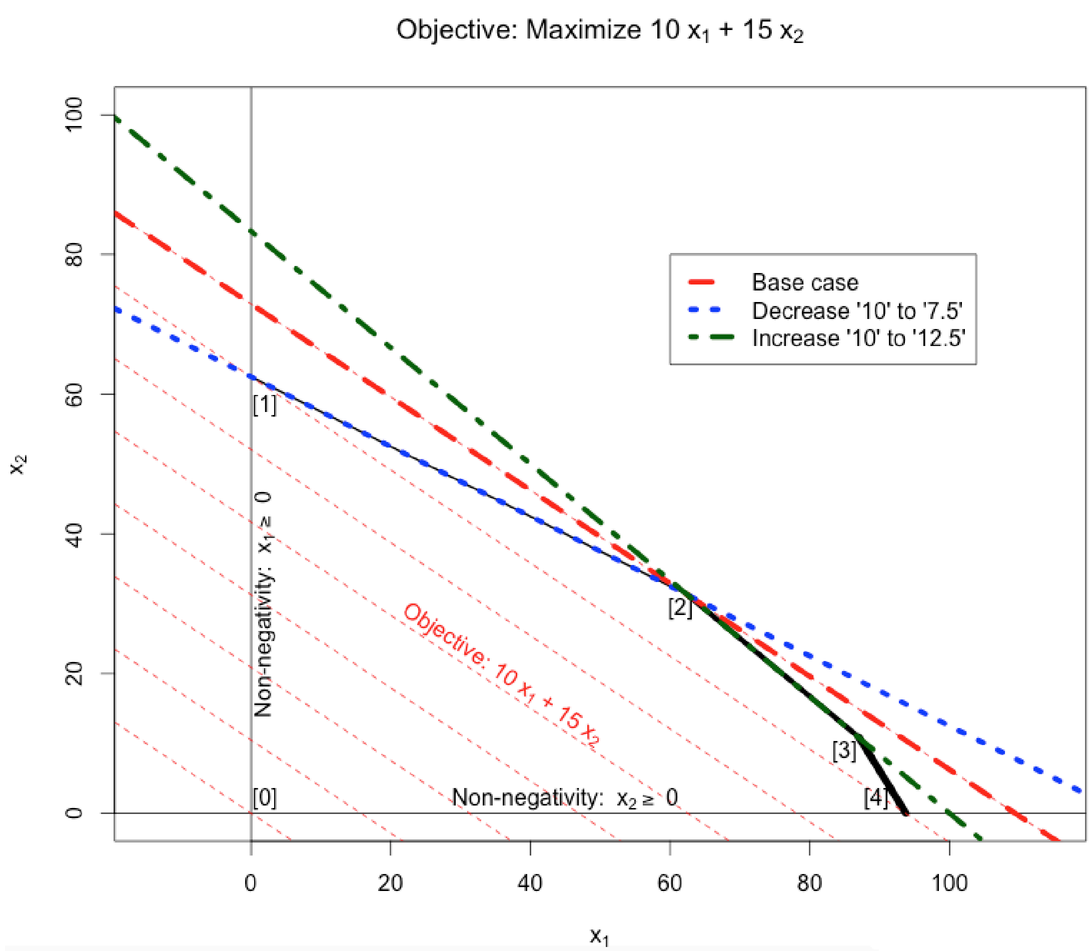


Let's look next at changes in the objective function values.

The current objective function is to maximize profit = $10x_1 + 15x_2$ and the values of 10 and 15 are, in practice, uncertain.

If any of the costs or incomes used to calculate the coefficients of "10" and "15" change, then the profit alters, and the slopes of the dashed red lines will change.

Consider the coefficient of "10" and decrease or increase it. There is a certain maximum increase or decrease that is allowed. Why can't we got further than that?



GAMS provides output to indicate the maximum and minimum allowable ranges. The line in bold is the one we have just geometrically investigated prior, the coefficient of "10".

```
VARIABLE NAME                                LOWER      CURRENT       UPPER
-------------                                -----      -------       -----
X(BoardA)                                      7.5           10        12.5
X(BoardB)                                       12           15          20
Z                                             -INF            1        +INF
```

Use these results now to answer the question: colleagues have been working hard to reduce the cost of goods sold (COGS) for producing circuit board B and have reduced it by $4 per board. How many boards should be produced (number of boards A and boards B) to maximize the overall profit? What will this optimal profit be?

To improve your understanding of the optimum, should you focus on the variables that have a narrow range or a wide range in the objective function? Can you explain why?

Let's get some more practice. Here are some selected pieces from a GAMS optimization problem, where we are optimizing the operation of a small distillation column. We have a supply constraint on the reflux flow rate in the column (≤ 600L/hour), and both a supply (2000 L/hour) and demand (1000 L/hour) constraint on pump 7.

```
EQUATION NAME                                LOWER      CURRENT       UPPER
-------------                                -----      -------       -----
Profit                                        -INF            0        +INF
Reflux                                         572          600         790
Pump7Flow                                     1240         1700        +INF

Marginal values:
                       LOWER      LEVEL       UPPER    MARGINAL

---- EQU Profit          .          .           .        1.000
---- EQU Reflux        -INF    600.000     600.000       3.74
---- EQU Pump7Flow      1000   1700.000    2000.000        .
```

What is the effect on the profit of changing the reflux flow rate capacity by $+50$ L/hour?

What is the effect on the profit of changing the reflux flow rate capacity by $-\ 50$L/hour?

What does a maximum allowable increase of `+INF` mean for the pump flow?

A proposal has been made to increase the capacity of Pump 7. What is your response to the proposal?

What about multiple changes? Consider more than one change in the b_i values, or consider more than one change in the c_i values. There is a "100% worst-case rule" that can be applied (other variations also exist):

Consider the percentage of allowable change being made on active constraints; add all these percentages.

- If the sum is no more than 100%, then the current active constraints will remain identical.
- If the sum is more than 100%, the current active constraints may not remain optimal.

Winston, *Operations Research, Applications and Algorithms*, 3rd Edition, 1994

The rule applies only to changes in the b_i values, or to changes in the c_i values, but not at the same time.

```
EQUATION NAME                                  LOWER      CURRENT        UPPER
-------------                                  -----      -------        -----
Profit                                          -INF            0         +INF
Reflux                                           572          600          790
Pump7Flow                                       1240         1700         +INF
Pump5Flow                                        642          750          809

Marginal values:
                       LOWER     LEVEL      UPPER    MARGINAL

---- EQU Profit          .         .          .        1.000
---- EQU Reflux        -INF    600.000    600.000      3.74
---- EQU Pump7Flow     1000   1700.000   2000.000       .
---- EQU Pump5Flow      250    750.000    750.000      1.54
```

Example: What is the effect of increasing the reflux flow by 50 units and reducing the pump 5 flow by 40 units?

The current value of reflux flow is ___________ **and** the current value of pump 5 flow is _______________
Both constraints are active, therefore the 100% rule can be applied here as stated above.

The reflux flow increase of 50 units corresponds to a change of $50/(790 - 600) = 50/190 = 26\%$
The pump 5 flow decrease of 40 units corresponds to a change of $40/(750 - 642) = 40/108 = 37\%$
This is a total change of $26 + 37 = 63\%$ so we can still use the sensitivity report.

What is the change in profit that will be expected from this?

Final advice: if in doubt about any model changes, put the changed values into the model and re-solve.