Calculus　　　　　　　　　　　　Name:
Date:
Period:

1.3 Evaluating Limits Analytically

Given $f(x) = 5x - 2$ and $g(x) = 2x + 3$, **evaluate each limit using direct substitution to explore properties of limits.**

1. Scalar (multipliers) and Limits

a. $\lim_{x \to 4} (3f(x))$

$= \lim_{x \to 4} (3(5x - 2))$

b. $3\left(\lim_{x \to 4} f(x)\right)$

$= 3\left(\lim_{x \to 4} (5x - 2)\right)$

2. Addition of Functions and Limits

a. $\lim_{x \to 4} (f(x) + g(x))$

b. $\left(\lim_{x \to 4} (f(x))\right) + \left(\lim_{x \to 4} (g(x))\right)$

3. Multiplication of Functions and Limits

a. $\lim_{x \to 4} (f(x) \bullet g(x))$

b. $\left(\lim_{x \to 4} (f(x))\right) \bullet \left(\lim_{x \to 4} (g(x))\right)$

Given $f(x) = 5x - 2$ and $g(x) = 2x + 3$, evaluate each limit using direct substitution to explore properties of limits.

4. Division of Functions and Limits

a. $\lim_{x \to 4} \frac{f(x)}{g(x)}$

b. $\frac{\lim_{x \to 4} f(x)}{\lim_{x \to 4} g(x)}$

5. Power of Functions and Limits

a. $\lim_{x \to 4} \left((f(x))^2\right)$

b. $\left(\lim_{x \to 4} (f(x))\right)^2$

Properties of Limits: Complete the equality statement to summarize the patterns observed in 1-5.
c is a constant (some number)

1. Scalar Multiple (k is some number) of a function within a limit is

$\lim_{x \to c} (kf(x)) =$

2. Sum OR Difference of Functions within a limit is

Sum: $\lim_{x \to c} (f(x) + g(x)) =$

Difference: $\lim_{x \to c} (f(x) - g(x)) =$

Combined Version aka Sum OR Difference: $\lim_{x \to c} (f(x) \pm g(x)) =$

3. Product of Functions within a limit is

$\lim_{x \to c} (f(x) \bullet g(x)) =$

4. Division of Functions within a limit is

$\lim_{x \to c} \frac{f(x)}{g(x)} =$ when $\lim_{x \to c} (g(x)) \neq 0$

5. Power of Functions within a limit is

$\lim_{x \to c} \left((f(x))^n\right)$

Limit Strategies

1. If the function is continuous at the input, substitution can be used.
2. Find another function that agrees at all points except x = c. (In other words, algebraically manipulate the function by factoring and simplifying, rationalizing the numerator, get common denominators, etc. so substitution can be used with the "new" function.)
3. Use a table or graph (Lesson 1.2) to CHECK your conclusion.

1.3 Examples

Find each limit.

6. $\lim_{x \to -1} \frac{x^2+5x+4}{x+1}$

7. $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = \sqrt{x}$

8. $\lim_{x \to 0} \frac{1/(x+4) - 1/4}{x}$

1.3 Practice

Find each limit analytically. (Remember to check for direct substitution if the function is continuous at the input.) **Then CHECK your answer using a graph or table (what you did in Lesson 1.2).**

9. $\lim_{x \to 2} \frac{3}{2x+1}$

10. $\lim_{x \to \pi} \tan x$

11. $\lim_{x \to 4} \frac{4-x}{x^2-16}$

12. $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

13. $\lim_{x \to 0} \frac{[1/(5+x)]-(1/5)}{x}$

14. Special Trigonometric Limits:

Given k is a nonzero constant. As a group, select a nonzero value you would like to use for k. Then, use tables and/or graphs to evaluate the trig limits below. Finally, repeat the process for a second value of k.

Example: k =

$\lim_{x \to 0} \frac{\sin kx}{kx} =$

$\lim_{x \to 0} \frac{1 - \cos kx}{kx} =$

General:

$\lim_{x \to 0} \frac{\sin kx}{kx} =$

$\lim_{x \to 0} \frac{1 - \cos kx}{kx} =$

Examples: Find each limit.

15. $\lim_{x \to 0} \frac{\tan x \left(\cos x - \cos^2 x\right)}{x^2}$

16. $\lim_{t \to 0} \frac{\sin 3t}{2t}$

17. $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x}$

Practice

18. $\lim_{x \to 0} \frac{7 \cos x \sin x}{x}$

19. $\lim_{x \to 0} \frac{8(1-\cos x)^2}{x}$

Squeeze Theorem (from your book)

Theorem 1.8 The Squeeze Theorem

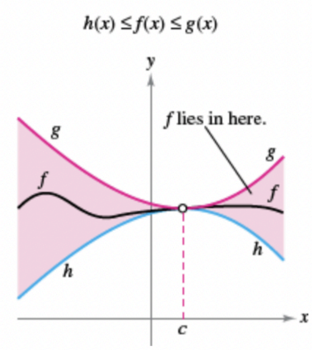
If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$$

then $\lim_{x \to c} f(x)$ exists and is equal to L.

A proof of this theorem is given in Appendix A.

You can see the usefulness of the Squeeze Theorem (also called the Sandwich Theorem or the Pinching Theorem) in the proof of Theorem 1.9.



20. Example:

Given $-|x| \le x\cos x \le |x|$, find $\lim_{x \to 0} x\cos x$ with the Squeeze Theorem. Sketch the graph to illustrate.