Calculus for Teachers Carrie Mol

Portfolio Problem: Maximizing Profit of Apartment House

I had a very hard time identifying which problem to pick as my portfolio problem, but I wound up going with one of the End of Coursework Problems. I went with this problem for two reasons. One, it relates to the work we've done in earlier courses about maximizing profit algebraically. Two, I always appreciate a problem that sends me down wrong pathways and then gives me a couple of different ways to solve the problem correctly.

The problem:

A real estate office handles an apartment house with 100 units. When the rent of each unit is $80 per month all of the units are filled. Experience shows that for each $5 per month increase in rent, five units become vacant. The cost of servicing a rented apartment is $20 a month. What rent should be charged to maximize profit.

The solution:

The hardest part of this problem was accounting for the $20 service fee. My first approach had me try constructing the problem as

$(80 + 5x) (-20(100-5x))$

$(80 + 5x) (-2000 + 100x) = 500x^2 - 2000x - 160,000$

$f' (x) = 1000x -2000$

$1000x - 2000 = 0$

$1000x = 2000$

$X = 2$

That told me that two sets of 5 dollar increases would maximize profit.

I wasn't sure that I had set up the problem though as I was very worried that I hadn't properly included that -20 service fee impact. So I tested it by running the profits for every five dollar increase using algebra and putting it in a table.

Rent	Total Rent (Rent x Occupied Apartments) - Service Fee	Total
80	$80(100) - 100($20)	$6000
85	$85(95) - 95($20)	$6,175
90	$90(90) - 90($20)	$6,300
95	$95(85) - 85($20)	$6,375
100	$100(80) - 80($20)	$6,400
105	$105(75) - 75($20)	$6,375

So I see my answer should be more like four units of 5. I was working with Alyssa Adamsen, and she thought the correct f(x) would be (80+5x)(200-5x) -20(100 - 5x) while I thought it would make sense to take the $20 out of the original rental price of the units. For every successfully rented apartment the profit of the rent would go down by $20 anyways. So my formula was f(x) = (60+5x) (100-5x). We solved it both ways.

Alyssa's way

$f(x) = (80+5x)(200-5x) -20(100 - 5x)$

$(80+5x)(200-5x) - 2000 + 100x$

$-25x^2 + 100x + 8000 - 2000 + 100x$

$f(x) = -25x + 200x +6000$

$f'(x)= -50x + 200$

$0= -50x +200$

$-200 = -50x$

$4 = x$

4 units of five would be a raise in rent of \$20.

My equation is equivalent it just cuts out some calculations

$f(x) = (60+5x)(100-5x)$

$f(x) = -25x + 200x + 6000$

So when you take the prime of f, the prime is the same

$f'(x) = -50x + 200$

$0 = -50x + 200$

$-200 = -50x$

$4 = x$

4 units of five would be a raise in rent of \$20

It was interesting that you could use calculus to find the maximum profit instead of just algebra and I enjoyed the various problems around maximizing area and other numbers. As much as I struggle with calculus I like seeing the practical applications of it. It reminds me of when my students say they will never need math in their daily lives and I can see lots of practical applications for maximizing area or profit using calculus. The earlier homework problem of “We wish to find two numbers whose sum is 23 and whose product is a maximum” required us to solve it using a table and then calculus helped bring to life the problem of relying on tables. Hence I used a table only to check the reasonableness of our initial answer. Once it was shown that our answer was not reasonable, we took other approaches which turned out to be ultimately the same mathematically.

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