See this page in the course material.

Learning Outcomes

- Identify and factor the greatest common factor of a polynomial.
- Factor a trinomial with leading coefficient 1.
- Factor by grouping.

When we studied fractions, we learned that the **greatest common factor** (GCF) of two numbers is the largest number that divides evenly into both numbers. For instance, [latex]4[/latex] is the GCF of [latex]16[/latex] and [latex]20[/latex] because it is the largest number that divides evenly into both [latex]16[/latex] and [latex]20[/latex]. The GCF of polynomials works the same way: [latex]4x[/latex] is the GCF of [latex]16x[/latex] and [latex]20{x}^{2}[/latex] because it is the largest polynomial that divides evenly into both [latex]16x[/latex] and [latex]20{x}^{2}[/latex].

When factoring a polynomial expression, our first step is to check for a GCF. Look for the GCF of the coefficients, and then look for the GCF of the variables.

A General Note: Greatest Common Factor

The **greatest common factor** (GCF) of polynomials is the largest polynomial that divides evenly into the polynomials.

How To: Given a polynomial expression, factor out the greatest common factor

1. Identify the GCF of the coefficients.
2. Identify the GCF of the variables.
3. Combine to find the GCF of the expression.
4. Determine what the GCF needs to be multiplied by to obtain each term in the expression.
5. Write the factored expression as the product of the GCF and the sum of the terms we need to multiply by.

Example: Factoring the Greatest Common Factor

Factor [latex]6{x}^{3}{y}^{3}+45{x}^{2}{y}^{2}+21xy[/latex].

Show Solution

First find the GCF of the expression. The GCF of [latex]6,45[/latex], and [latex]21[/latex] is [latex]3[/latex]. The GCF of [latex]{x}^{3},{x}^{2}[/latex], and [latex]x[/latex] is [latex]x[/latex]. (Note that the GCF of a set of expressions of the form [latex]{x}^{n}[/latex] will always be the lowest exponent.) The GCF of [latex]{y}^{3},{y}^{2}[/latex], and [latex]y[/latex] is [latex]y[/latex]. Combine these to find the GCF of the polynomial, [latex]3xy[/latex].

Next, determine what the GCF needs to be multiplied by to obtain each term of the polynomial. We find that [latex]3xy\left(2{x}^{2}{y}^{2}\right)=6{x}^{3}{y}^{3}, 3xy\left(15xy\right)=45{x}^{2}{y}^{2}[/latex], and [latex]3xy\left(7\right)=21xy[/latex].

Finally, write the factored expression as the product of the GCF and the sum of the terms we needed to multiply by.

[latex]\left(3xy\right)\left(2{x}^{2}{y}^{2}+15xy+7\right)[/latex]

Analysis of the Solution

After factoring, we can check our work by multiplying. Use the distributive property to confirm that [latex]\left(3xy\right)\left(2{x}^{2}{y}^{2}+15xy+7\right)=6{x}^{3}{y}^{3}+45{x}^{2}{y}^{2}+21xy[/latex].

Try It

Factor [latex]x\left({b}^{2}-a\right)+6\left({b}^{2}-a\right)[/latex] by pulling out the GCF.

Show Solution

[latex]\left({b}^{2}-a\right)\left(x+6\right)[/latex]

See this interactive in the course material.

Watch this video to see more examples of how to factor the GCF from a trinomial.

Video Link

Factoring a Trinomial with Leading Coefficient 1

Although we should always begin by looking for a GCF, pulling out the GCF is not the only way that polynomial expressions can be factored. The polynomial [latex]{x}^{2}+5x+6[/latex] has a GCF of 1, but it can be written as the product of the factors [latex]\left(x+2\right)[/latex] and [latex]\left(x+3\right)[/latex].

Trinomials of the form [latex]{x}^{2}+bx+c[/latex] can be factored by finding two numbers with a product of [latex]c[/latex] and a sum of [latex]b[/latex]. The trinomial [latex]{x}^{2}+10x+16[/latex], for example, can be factored using the numbers [latex]2[/latex] and [latex]8[/latex] because the product of these numbers is [latex]16[/latex] and their sum is [latex]10[/latex]. The trinomial can be rewritten as the product of [latex]\left(x+2\right)[/latex] and [latex]\left(x+8\right)[/latex].

A General Note: Factoring a Trinomial with Leading Coefficient 1

A trinomial of the form [latex]{x}^{2}+bx+c[/latex] can be written in factored form as [latex]\left(x+p\right)\left(x+q\right)[/latex] where [latex]pq=c[/latex] and [latex]p+q=b[/latex].

Q & A

Can every trinomial be factored as a product of binomials?

No. Some polynomials cannot be factored. These polynomials are said to be prime.

How To: Given a trinomial in the form [latex]{x}^{2}+bx+c[/latex], factor it

1. List factors of [latex]c[/latex].
2. Find [latex]p[/latex] and [latex]q[/latex], a pair of factors of [latex]c[/latex] with a sum of [latex]b[/latex].
3. Write the factored expression [latex]\left(x+p\right)\left(x+q\right)[/latex].

Example: Factoring a Trinomial with Leading Coefficient 1

Factor [latex]{x}^{2}+2x - 15[/latex].

Show Solution

We have a trinomial with leading coefficient [latex]1,b=2[/latex], and [latex]c=-15[/latex]. We need to find two numbers with a product of [latex]-15[/latex] and a sum of [latex]2[/latex]. In the table, we list factors until we find a pair with the desired sum.

Factors of [latex]-15[/latex]	Sum of Factors
[latex]1,-15[/latex]	[latex]-14[/latex]
[latex]-1,15[/latex]	[latex]14[/latex]

[latex]3,-5[/latex]	[latex]-2[/latex]
[latex]-3,5[/latex]	[latex]2[/latex]

Now that we have identified [latex]p[/latex] and [latex]q[/latex] as [latex]-3[/latex] and [latex]5[/latex], write the factored form as [latex]\left(x - 3\right)\left(x+5\right)[/latex].

Analysis of the Solution

We can check our work by multiplying. Use FOIL to confirm that [latex]\left(x - 3\right)\left(x+5\right)={x}^{2}+2x - 15[/latex].

Q & A

Does the order of the factors matter?

No. Multiplication is commutative, so the order of the factors does not matter.

Try It

Factor [latex]{x}^{2}-7x+6[/latex].

Show Solution

[latex]\left(x - 6\right)\left(x - 1\right)[/latex]

[See this interactive in the course material.]

Factoring by Grouping

Trinomials with leading coefficients other than 1 are slightly more complicated to factor. For these trinomials, we can **factor by grouping** by dividing the *x* term into the sum of two terms, factoring each portion of the expression separately, and then factoring out the GCF of the entire expression. The trinomial [latex]2{x}^{2}+5x+3[/latex] can be rewritten as [latex]\left(2x+3\right)\left(x+1\right)[/latex] using this process. We begin by rewriting the original expression as [latex]2{x}^{2}+2x+3x+3[/latex] and then factor each portion of the expression to obtain [latex]2x\left(x+1\right)+3\left(x+1\right)[/latex]. We then pull out the GCF of [latex]\left(x+1\right)[/latex] to find the factored expression.

A General Note: Factoring by Grouping

To factor a trinomial of the form [latex]a{x}^{2}+bx+c[/latex] by grouping, we find two numbers with a product of [latex]ac[/latex] and a sum of [latex]b[/latex]. We use these numbers to divide the [latex]x[/latex] term into the sum of two terms and factor each portion of the expression separately then factor out the GCF of the entire expression.

How To: Given a trinomial in the form [latex]a{x}^{2}+bx+c[/latex], factor by grouping

1. List factors of [latex]ac[/latex].
2. Find [latex]p[/latex] and [latex]q[/latex], a pair of factors of [latex]ac[/latex] with a sum of [latex]b[/latex].
3. Rewrite the original expression as [latex]a{x}^{2}+px+qx+c[/latex].
4. Pull out the GCF of [latex]a{x}^{2}+px[/latex].
5. Pull out the GCF of [latex]qx+c[/latex].
6. Factor out the GCF of the expression.

Example: Factoring a Trinomial by Grouping

Factor [latex]5{x}^{2}+7x - 6[/latex] by grouping.

Show Solution

We have a trinomial with [latex]a=5,b=7[/latex], and [latex]c=-6[/latex]. First, determine [latex]ac=-30[/latex]. We need to find two numbers with a product of [latex]-30[/latex] and a sum of [latex]7[/latex]. In the table, we list factors until we find a pair with the desired sum.

Factors of [latex]-30[/latex]	Sum of Factors
[latex]1,-30[/latex]	[latex]-29[/latex]
[latex]-1,30[/latex]	[latex]29[/latex]
[latex]2,-15[/latex]	[latex]-13[/latex]
[latex]-2,15[/latex]	[latex]13[/latex]
[latex]3,-10[/latex]	[latex]-7[/latex]
[latex]-3,10[/latex]	[latex]7[/latex]

So [latex]p=-3[/latex] and [latex]q=10[/latex].

[latex]\begin{array}{cc}5{x}^{2}-3x+10x - 6 \hfill & \text{Rewrite the original expression as }a{x}^{2}+px+qx+c.\hfill \\ x\left(5x - 3\right)+2\left(5x - 3\right)\hfill & \text{Factor out the GCF of each part}.\hfill \\ \left(5x - 3\right)\left(x+2\right)\hfill & \text{Factor out the GCF}\text{ }\text{ of the expression}.\hfill \end{array}[/latex]

Analysis of the Solution

We can check our work by multiplying. Use FOIL to confirm that [latex]\left(5x - 3\right)\left(x+2\right)=5{x}^{2}+7x - 6[/latex].

Try It

Factor the following.

1. [latex]2{x}^{2}+9x+9[/latex]
2. [latex]6{x}^{2}+x - 1[/latex]

Show Solution

1. [latex]\left(2x+3\right)\left(x+3\right)[/latex]
2. [latex]\left(3x - 1\right)\left(2x+1\right)[/latex]

See this interactive in the course material.

In the next video we show another example of how to factor a trinomial by grouping.

Video Link

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