Hector Torres-Silva

(Supplementary material) Seismic Metamaterial as Attenuation of S Waves of Earthquakes

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1. Introduction

Earthquakes are the result of sudden release of huge amount of energy in the Earth's crust that produces seismic waves. There is a lot of observational evidence concerning rotational motions on the Earth's surface excited by earthquakes, we can infer the existence of rotational motions in earthquake focal zones. It is observed that rotational motions (twist) in earthquake sources naturally excite rotational seismic waves, solitons and chiral waves [1-3].

The key idea of this project consists in the simulation of S waves attenuated by metamaterials with the objective of locally shielding a building or structure of strong earthquakes. The description of rotational seismic waves is made through a chiral regime [4, 5] by considering rotational seismic waves excited in strong earthquakes. Seismic waves are generally divided into body and surface waves, both of which are a superposition of longitudinal and shear bulk waves so we can distinguish two kinds of seismic waves: 1) longitudinal waves, that is, P waves; and 2) rotational shear waves, that is, S waves. Rotational seismic waves propagate faster in solid rocks and much slower in fractured media along tectonic faults. It has been observed that these waves may have a form of rotational seismic solitons and that they can trigger earthquakes [6. It is the collapse of structures that causes extensive damage and loss of life during earthquakes. Aseismic capabilities are highly relevant to public safety and a large amount of research has gone into establishing practical analysis and design methods for them.

The components of the seismic waves should work together to disrupt the elastic earth, on which a building is anchored. One of the effects of the metamaterial is to decouple these components of the seismic wave, so it is possible to design a cylindrical, seismic metamaterial cloak to remedy the destructive effects of coupled pressure and shear waves that would travel through the same earthen geometrical plane as the structure.

Recent development of metamaterial science opens a new direction to control the seismic waves have been proposed through the design of a cloak to control bending waves propagated in isotropic heterogeneous thin plates [7-9]. The cloaking of shear elastic waves allows pass smoothly into the material rather than reflecting or scattering at the material's surface. In this project we propose a method to control seismic waves by using metamaterials which are artificially engineered materials which have a special property: a negative refractive index. We need here to find a solution so a metamaterial acts as an attenuator by converting the destructive seismic wave into an evanescent wave by making use of the imaginary velocity of stop-band of the wave or to obtain a mode conversion of S wave to longitudinal P wave.

The Seismic waves are vibrations that travel through the Earth carrying energy released during an earthquake.

The waves transport the energy from the focus. When the longitudinal waves (P waves) and transverse waves (S waves) meet the surface, part of their energy is converted into waves that propagate along the Earth's surface. Rayleigh waves, which have one compressional component and two shear components, are characterized by wavelengths from meters to decameters in the frequency range of less than 1 Hz to a few tens of Hz, where the velocity of the wave is decreasing, since the higher frequency components are more effectively attenuated during wave propagation. The fact that these surface waves have a slower speed, hence a much smaller wavelength than underground waves, makes them the most destructive seismic waves and the main reason which leads to the well-known phenomenon of resonance disaster.

Seismic metamaterials are artificially structured composite materials that enable manipulation of dispersive properties of vibrational waves. According to the previous experiments this can be obtained by two principles: a) Phononic crystals which are based on periodic distribution of inclusions (scatterers) embedded in a matrix, designed to control dispersion of the waves through Bragg scattering, the scattering of the waves by a periodic arrangement of scatterers with dimensions and periods comparable to the wavelength; b) Seismic metamaterials which have the added feature of local resonance, their properties do not rely on periodicity, but on the collective effect of a large array of small resonators. The structural features of seismic metamaterials can be significantly smaller than the wavelength of the wave they are affecting. Local resonance may lead to negative effective dynamic mass density and bulk modulus and therefore to their unusual dispersion characteristics

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Different models of resonators may cover different types of resonant frequencies for seismic waves. Numerical results suggest their potential application in dissipation of seismic wave energy for wavelengths ranging from a few meters to tenths of meters. Seismic resonators may become in the future an alternative solution to prevent disasters due to the impact of earthquakes on sensitive areas.

We focus here on how to reduce the amplitude of the seismic wave by using the properties of chiral metamaterials. For S waves we use a chiral approach so that by adding it vectorially to the longitudinal wave on the surface so to have attenuated Rayleigh waves. In Section 2 of this proposal, we develop a theory for chiral seismic waves. Based on the similarity of elastodynamics and electrodynamics we discuss the region of elastic metamaterial with negative modulus and negative density that enable a seismic wave becomes evanescent wave or to obtain a conversion of S wave to longitudinal wave. In Section 3, we postulate and discuss a scheme of seismic attenuator. The novelty of this proposal is that the attenuation is due to the combined effect of metamaterial and evanescence in S waves.

2. Chiral Theory: Negative Modulus, Negative Density

In connection with seismic events, the Elastodynamics is linked with Electrodynamics through transverse waves. Seismic waves encountering interfaces that separate rocks of different elastic properties also undergo reflection, refraction, and scattering phenomena in chiral materials which are noncentrosymmetric due to handedness in their microstructures. The elastic field behavior in a chiral medium is readily described herein using the governing equations and constitutive relations for noncentrosymmetric, isotropic micropolar materials. Accordingly, linearly polarized longitudinal waves and left and right circularly polarized transverse waves are eigenstates for elastic waves in the chiral medium. There are two classes of seismic body waves which travel through the interior of the Earth: P waves and S waves. P waves are the fastest seismic waves, and consequently, the first to arrive at any given location. Torsion waves, often called S waves, represent the spiraling motion of particles twisting between inner structures. S waves usually have more height, or amplitude, than P waves.
The idea of negative refraction in an electromagnetic wave in metamaterials that are realized in practice are composites with built-in resonance structures that exhibit effective negative permittivity and negative permeability for some frequency ranges [10-14]. In an acoustic wave, the continuity and Newton's second law (with harmonic field dependence $\exp(i\omega t)$ can be expressed respectively as $\nabla \cdot \vec{v} + i(\omega / \kappa) p = 0$ and $\nabla p + i\omega\rho\vec{v} = 0$, where $p(\vec{v})$ is the pressure (velocity) field. The density ρ and bulk modulus κ are position dependent in general. By considering a plane-wave solution with wave vector $\vec{k}$ inside a homogeneous medium of constant density and bulk modulus, the refractive index n should be defined by $k = |n| \omega / c$ with $n^2 = \rho / \kappa$. which can have a simultaneously negative effective bulk modulus and density.
First we start with an analogy between the linear elasticity theory and classical Maxwell equation with chiral effects. Chiral media have the property to split two degenerate modes [10-14]. For a chiral medium with an electric or magnetic resonance, a negative refractive band is created and there is a continuous transition between the negative and positive bands, as described in [10]. Generally, a structure is said to be chiral if it is not identical to its mirror image, and chirality has as a result the macroscopic rotation of the wave polarization. In seismodynamics it is the elastic continuum that supports propagation of both longitudinal and transverse waves. The basic equation of the linear elasticity theory is the Lamé equation, which takes the following form

$$-\ddot{u} + c_L^2 \nabla div\boldsymbol{u} - c_T^2 \nabla \times (\nabla \times \boldsymbol{u}) = 0$$

where the vector $\boldsymbol{u}$ represents the vector of displacements is a elastic medium, $c_{L,T}$ are the velocities of propagation of the longitudinal (P waves) and transverse (S waves) waves correspondingly. In order to separate the wave processes explicitly we introduce the chiral seismic parameter ζ which take into account the mode conversion from P to S waves

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t}(1 + \zeta \nabla \times) \tag{1}$$

Here we identify $\boldsymbol{E}_S = -\boldsymbol{u}_{ct}, \boldsymbol{H}_S = -c_T \nabla \times \boldsymbol{u}_{ct}$, so using the Born-Fedorov formalism [5], we define

$$\boldsymbol{E}_{TS} = \varepsilon \boldsymbol{E}_S + \varepsilon\zeta (\nabla \times \boldsymbol{E}_S)$$

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and

$$\boldsymbol{H}_{TS} = \mu \boldsymbol{H}_S + \mu\zeta\left(\nabla \times \boldsymbol{H}_S\right),$$

the equivalent seismic Maxwell equations are

$$\nabla \times \boldsymbol{E}_{TS} = -\frac{1}{c_T}\left(\mu \boldsymbol{H}_S + \mu\zeta \nabla \times \boldsymbol{H}_S\right) \tag{2a}$$

$$\nabla \times \boldsymbol{H}_{TS} = \frac{1}{c_T}\left(\varepsilon \boldsymbol{E}_S + \varepsilon\zeta \nabla \times \boldsymbol{E}_S\right) \tag{2b}$$

Solving these equations we have that the effective phase velocity of S-waves is $v_S = c_T\left(1 \pm \zeta k_0\right)$ where k_0 is the wavevector without torsion, $\zeta = \zeta(\mu, \kappa, \rho)$ and the wave equation with $\boldsymbol{u}_{ct} = \boldsymbol{u}_S$ is

$$\nabla_s^2 \boldsymbol{E}_{TS} + \frac{\omega^2}{v_S^2}\boldsymbol{E}_{TS} = -\nabla_s^2 \boldsymbol{u}_S - \frac{\omega^2}{v_S^2}\boldsymbol{u}_S = 0 \tag{3}$$

Clearly if $v_S = c_T\left(1 - \zeta k_0\right) \ll c_T$ The S wave would decrease its velocity in the protective mesh of metamaterial.
Metamaterials with negative electromagnetic constitutive parameters give new propagation characteristics for electromagnetic waves [10-14]. By analogy between elastodynamics and electrodynamics in this work we use the Born-Fedorov equations as the most suitable for applications of our interest [5-6]. Here the density of matter is equivalent to the electric permittivity $\rho \leftrightarrow \varepsilon$, and the Lamé parameter μ_S is equivalent to the reciprocal of the magnetic permeability $\mu_S \leftrightarrow 1/\mu$. The speed of transversal seismic S-waves without chirality is given by $c_T = 1\big/\sqrt{\rho/\mu_S}$. The chiral effect given by the factor $\left(1 \pm \zeta k_0\right)$ which modify the phase velocity $c_T\left(k_0 = \omega/c_T\right)$ having a metamaterial behavior when $v_S = c_T\left(1 - \zeta k_0\right)$.
On the other hand, acoustic waves are created by compressibility or elasticity of the medium. Young's modulus, *Y*, is a one-dimensional compressibility defined by $\Delta p = Y \Delta l/l$, where Δp is the pressure or stress and *l* is the length. Shear modulus, $B_S = \mu$, is a two-dimensional one for a surface wave defined by $\Delta p = B_S\, \Delta x/h$, where Δx is the horizontal shift and *h* is the height of the object. Bulk modulus, $B_V = \kappa$, is a three-dimensional one for a body wave defined by $\Delta p = B_V\, \Delta V/V$.
Seismic medium of Earth crust can be considered as an accumulation of infinite number of elastic plates. Although the seismic surface wave is not pure two dimensional, the velocity is mainly dependent upon the density, ρ_S and shear modulus, $B_S = \mu$, of the seismic medium. Seismic wave is a kind of acoustic wave and every acoustic wave propagates following two wave equations in principle. Assuming the plane wave time dependence $\exp(i\omega t)$, the pressure, *p*, and the velocity, $\boldsymbol{v}_S$ of the S-wave in the two dimensions are expressed as the Newton's law.

$$\nabla_s p = i\omega\rho \boldsymbol{v}_S \tag{4}$$

and the continuity equation

$$i\omega p = \mu \nabla \cdot \boldsymbol{v}_S \tag{5}$$

where ∇ is the Laplacian operator at the surface, *p* is the pressure, *ω* is the angular frequency of the wave, and $\boldsymbol{v}_S$ is the velocity.
The Equations (4) and (5) generates the wave equation as

$$\nabla_s^2 p + \frac{\omega^2}{\boldsymbol{v}_S^2} p = 0 \tag{6}$$

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if we compare Equation (3) with Equation (6), we complete the analogy and connection between the chiral seismic waves and the acoustic seismic waves. The velocity of the seismic wave is

$$v_S = \sqrt{\mu/\rho}\,(1 \pm \zeta k_S) \qquad (7)$$

The Equation (4) is the counterpart of the Ampere's law and Equation (5) is of the Faraday's law of the electromagnetism. If the shear modulus $B_S = \mu_S$ or the density ρ becomes negative, the velocity becomes imaginary. Then, so does the refractive index, n, or the inverse of the velocity as $n = v_0/v_S = v_0\sqrt{\rho/\mu}$, where v_0 is the background velocity. Since $k_S = 2\pi\, n/\lambda_S$ the wavevector becomes imaginary, too. Therefore, the imaginary wavevector makes the amplitude of the seismic wave become an evanescent wave. We call it an attenuator. Negative shear modulus of elastic media has been studied and realized very recently [15-17] (Helmholtz resonators with the negative bulk modulus). The resonance of accumulated waves in the Helmholtz resonator reacts against the applied pressure at some specific frequency ranges. Then, the negative modulus is realized by passing the acoustic wave through an array of Helmholtz resonators.
It will be shown later that total mode conversion from shear waves to longitudinal waves (and vice versa) can also be achieved when such a metamaterial is put in contact with a normal solid. This is not possible for any two natural solids. Here, we propose a new type of elastic metamaterial with a fluid-solid composite. Such a composite can support both dipolar and quadrupolar resonances in its building blocks and produce simultaneously negative effective mass density and negative effective shear modulus in a large frequency region, resulting in a wide negative band for shear waves and a gap for longitudinal waves. The effective medium parameters for ρ, κ, and μ can be obtained from an effective medium theory (EMT) [18-23]. A significant amount of mode conversion from shear waves to longitudinal waves induced by the double-negative property of the metamaterial can be founded.

3. Mode conversion between S waves-P waves

To obtain mode conversion mathematically we propose the configuration shown in (figure 1)
There are four cases of total *s-p* or *p-s* mode conversions on the interface of a double-negative metamaterial and a normal solid under negative refraction. Below we derive the general conditions of total mode conversions under negative refraction.

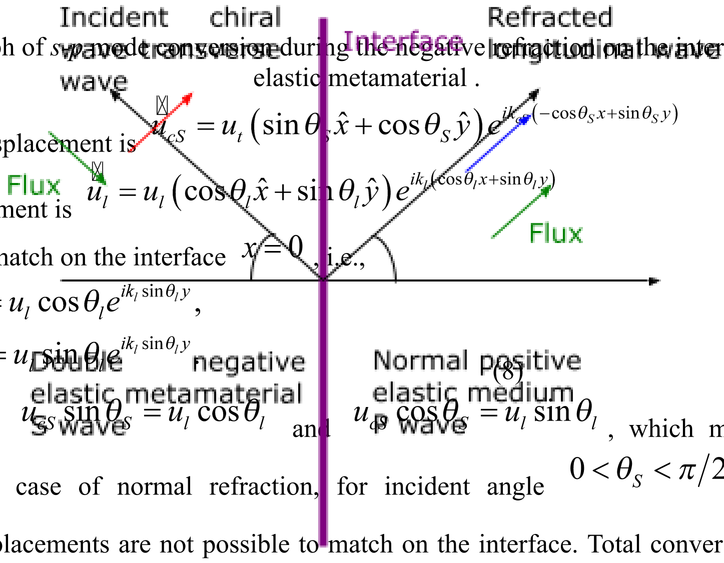

Fig. 1. A schematic graph of *s-p* mode conversion during the negative refraction on the interface of a double-negative elastic metamaterial .

For chiral *s* waves, the displacement is $\vec{u}_{cS} = u_t\left(\sin\theta_S\hat{x} + \cos\theta_S\hat{y}\right)e^{ik_{cS}(-\cos\theta_S x + \sin\theta_S y)}$

For *p* waves, the displacement is $\vec{u}_l = u_l\left(\cos\theta_l\hat{x} + \sin\theta_l\hat{y}\right)e^{ik_l(\cos\theta_l x + \sin\theta_l y)}$

The displacements must match on the interface $x = 0$, i.e.,

$$u_{cS}\sin\theta_S e^{ik_{cS}\sin\theta_S y} = u_l\cos\theta_l e^{ik_l\sin\theta_l y},$$
$$u_{cS}\cos\theta_S e^{ik_{cS}\sin\theta_S y} = u_l\sin\theta_l e^{ik_l\sin\theta_l y} \qquad (8)$$

From Eq. (8), we find $u_{cS}\sin\theta_S = u_l\cos\theta_l$ and $u_{cS}\cos\theta_S = u_l\sin\theta_l$, which means that $u_l = u_{cS}$ and $\theta_S + \theta_l = \pi/2$. In the case of normal refraction, for incident angle $0 < \theta_S < \pi/2$ we have $\theta_l < 0$, thus $\theta_S + \theta_l < \pi/2$, the displacements are not possible to match on the interface. Total conversion is only possible in the case of negative refraction.

From Eq. (8), we also find $k_{cS}\sin\theta_S = k_l\sin\theta_l$, which means the wave vector parallel to the interface must conserve. Substituting $\theta_S + \theta_l = \pi/2$, we obtain $k_{cS}\sin\theta_S = k_l\cos\theta_S$.

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From $k_{cS} = \left|\frac{\omega}{v_t}\right| = \frac{\omega}{\sqrt{\mu_1/\rho_1}}$ and $k_l = \left|\frac{\omega}{v_l}\right| = \frac{\omega}{\sqrt{(\kappa_2+\mu_2)/\rho_2}}$, we find $\tan\alpha = \frac{k_l}{k_{cS}} = \frac{\sqrt{\mu_1/\rho_1}}{\sqrt{(\kappa_2+\mu_2)/\rho_2}}$.

We also need to consider the matching condition of stresses.

For *s* waves, $\vec{u}_{cS} = u_{cS}\left(\sin\theta_S\hat{x} + \cos\theta_S\hat{y}\right)e^{ik_{cS}(-\cos\theta_S x + \sin\theta_S y)}$.

For *p* waves, $\vec{u}_l = u_l\left(\cos\theta_l\hat{x} + \sin\theta_l\hat{y}\right)e^{ik_l(\cos\theta_l x + \sin\theta_l y)}$.

By substituting $\theta_S + \theta_l = \pi/2$, $u_l = u_{ct}$ and $k_{cS}\sin\theta_S = k_l\cos\theta_S$ in Eq. (8), we find

$\kappa_2 = \mu_2\cos 2\theta_S - \mu_1\sin 2\theta_S/\tan\theta_S$ and $-\mu_1 = \mu_2\tan 2\theta_S\tan\theta_S$.

The total conversion matching conditions are:

$$\theta_S + \theta_l = \pi/2,\quad \kappa_2 = \mu_2\cos 2\theta_S - \mu_1\sin 2\theta_S/\tan\theta_S,$$
$$-\mu_1 = \mu_2\tan 2\theta_S\tan\theta_S,$$
$$\tan\theta_S = \frac{k_l}{k_{cS}} = \frac{\sqrt{\mu_1/\rho_1}}{\sqrt{(\kappa_2+\mu_2)/\rho_2}}. \qquad (9)$$

This angle is an important input to the simulation.

4. Proposed system

Compared with the work on EM metamaterials, much less study has been devoted to elastic metamaterials. Here, the term "elastic metamaterials" means that the host matrix of the building block is a solid material. This results in three independent effective parameters: effective bulk modulus κ, effective shear modulus μ, and effective mass density ρ. This is different from the acoustic metamaterials mentioned earlier, where the host medium is a fluid and the system can be described by two effective parameters, the effective bulk modulus and effective mass density. [24-25]. The three independent effective parameters in elastic metamaterials can give rise to richer physics than in their electromagnetic counterpart. The negative values of each single effective parameter and their various combinations can produce eight types of wave propagation. However, to date, only a few kinds of elastic metamaterials have been explored. A kind of locally resonant sonic material which can support large band gaps at low frequencies has been reported [26-27]. The building block of an elastic metamaterial consists of a rubber-coated lead sphere embedded in an epoxy host. An analytical model based on a special configuration has been proposed to show that the effective mass density of such a metamaterial can turn negative near the resonances of the building block [28]. However, this proposed model cannot predict the other two effective parameters, and may not apply to other kinds of elastic metamaterials. In order to design a new metamaterial with certain desired propagation properties, one needs to know how to engineer the resonant properties of the building blocks to produce the desired effective parameters.

This would be facilitated by a valid effective medium theory which relates all three effective medium parameters to the resonant properties of the building block. However, most of the existing effective medium theories for elastic waves were derived in the quasistatic limit, [29-32] and, therefore, are not suitable for describing elastic metamaterial.

In this project, an effective medium theory for certain two dimensional elastic metamaterials must be developed. The system consists of cylindrical scatterers arranged in a triangular lattice embedded in a solid host. The elastic parameters of both the scatters and the host are assumed isotropic. The theory relates the effective medium parameters κ, μ, and ρ to the scattering properties of the scatters directly. In order to check the validity of the theory, band-structure calculations for different metamaterials must be performed.

Two examples of designing elastic metamaterials will be studied: one with a large band gap at low frequencies, and the other with double negative bands for both longitudinal and chiral transverse branches

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noting that the dispersion relations for elastic waves are, in general, not isotropic near the convergence point, even at low frequencies. This will be demonstrated through the design of two kinds of metamaterials: one possessing large band gaps at low frequencies and the other exhibiting two regions of negative refraction.

The two-dimensional elastic metamaterial considered in this study is composed of cylindrical inclusions of radius r_s with (ρ, μ, κ) arranged in a triangular lattice in the *x-y* plane embedded in a homogeneous matrix of ρ_0, μ_0, κ_0 . satisfying $\kappa = \lambda + \mu$, where λ represents the Lamé constant. Due to the translational symmetry along the cylinder's axis, say, *z* axis, the elastic modes in the system can be decoupled into a scalar part, *z* mode, and a vector part, *x y* mode, with vibrations along the *z* axis and in the *x-y* plane, respectively. Here, the more complicated case of the *xy* mode will be studied. The displacement in *x-y* plane can be described by the following wave equation:

$$\rho(\vec{r})\frac{\partial^2 u_i}{\partial t^2} = \nabla\cdot(\mu(\vec{r})\nabla u_i(\vec{r})) + \nabla\cdot(\mu(\vec{r})\frac{\partial \vec{u}}{\partial x_i}) + \frac{\partial}{\partial x_i}(\lambda(\vec{r})\nabla\cdot\vec{u})$$

where $\vec{u}$ is the displacement field. In general, $\vec{u}$ can be decoupled into a longitudinal part and a transverse part [33,37]. **The hypothesis in this study is that we can replace this last fourth order differential equation with a second order equation for $u_{ct} = u_S$ that is much easier to solve and to simulate numerically with $v_S = c_T\left(1-\zeta(\mu,\kappa,\rho)\cdot k_0\right) \ll c_T$. With this condition the S wave would decrease its velocity in the protective mesh of metamaterial.**

The two-dimensional elastic metamaterial proposed in this work is composed of cylindrical fluid-solid composite inclusions arranged in a triangular lattice in the *x-y* plane embedded in a host of Polyethylene foam. The purpose of using a triangular lattice is to ensure isotropic dispersions near the Γ point of the metamaterial [21]. The microstructure of the building block is shown in Fig. 1(a). The cylindrical inclusion is a oil cylinder of $r_w = \Delta a$ coated by a layer of silicone rubber of outer radius $r_s = da$, where *a* is the lattice constant and Δ and d are parameters which are varied in the simulation to obtain the optimum design in the shielding of the metamaterial.
The presence of soft silicone rubber layer introduces dipolar resonance which gives rise to negative mass density [14]. The oil cylinder is chosen to enhance the quadripolar resonance as oil can be easily deformed due to its zero shear modulus. The slippery boundary condition will be used on the interface between oil and rubber. Comparing to air that was used in previous works [18, 19], the large mass density of water helps to provide the necessary momentum that is needed in any resonance.

The material parameters for the simulation with different values of Δ and d will be around:
$\rho_{oil} = 1.3\times10^3 kg/m^3$, $\lambda_{oil} = 2.5\times10^9 N/m^2$, $\mu_{oil} = 0$ for oil,
$\rho_{rubber} = 1.3\times10^3 kg/m^3$, $\lambda_{ruber} = 6\times10^5 N/m^2$, $\nu_{rubber} = 0.47$ $\mu_{ruber} = 4\times10^4 N/m^2$ for silicone rubber
Young's modulus $E_{soil} = 20MPa$, Poisson's ratio $\nu_{soil} = 0.3$, mass density $\rho_{soil} = 1.8\times10^3 kg/m^3$, $E_{concrete} = 30GPa$, $\nu_{concrete} = 0.25$, $\rho_{concrete} = 2.5\times10^3 kg/m^3$, $E_{steel} = 207GPa$, $\nu_{steel} = 0.3$, $\rho_{steel} = 7.8\times10^3 kg/m^3$ [12], [14]. Here λ is the Lamé constant, which is related to the bulk modulus through the relation $\kappa = \lambda + \mu$ in two dimensions.
To reduce costs in this proposal, we propose a reinforced concrete is the basis for the foundations and pillars and is a construction technique that consists of reinforcing the concrete with bars or corrugated iron meshes. That is to say, it is a reinforced concrete inside with metallic reinforcements to improve its resistance to tensile stresses. On the other hand, the steel industry has a great weight in the economy of different countries (Spain) and generates a waste in large quantities that often ends up in landfills. Therefore, "the trend is that these materials are no longer considered waste and are used as raw material to obtain other products to improve the acoustic insulation of construction materials. Here we take advantage of the experience of the industry of Bilbao regarding this item, (Department of Mining and

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Metallurgical Engineering and Materials Science of the School of Engineering of Bilbao of the UPV / EHU, that at the moment works in Eraiker-Gikesa).
This research seeks to contribute to the knowledge and improvement of the acoustic insulation of materials destined to the construction sector by incorporating industrial waste, especially steel slag

The velocities of longitudinal and transverse waves are defined by $c_l = \sqrt{\kappa + \mu}\sqrt{1/\rho}$ *cl* and $c_{t=S} = \sqrt{\mu}\sqrt{1/\rho}$, respectively. It can be seen that the wave speeds inside the silicone rubber are much smaller than those in the foam matrix and the water core. These large mismatches at the two interfaces enhance the dipolar resonances and give rise to a large negative band for shear waves. Accompanying negative refraction, we hope to observed a significant amount of mode conversion from transverse waves to longitudinal waves, which is proved to be a unique property of double-negative elastic metamaterials.
Similarly, total mode conversion from longitudinal waves to transverse waves can also be realized under certain conditions. Total mode conversion is not possible on the interface between two normal solids. However, it can occur in a normal solid with a free surface, but in reflected waves [23].

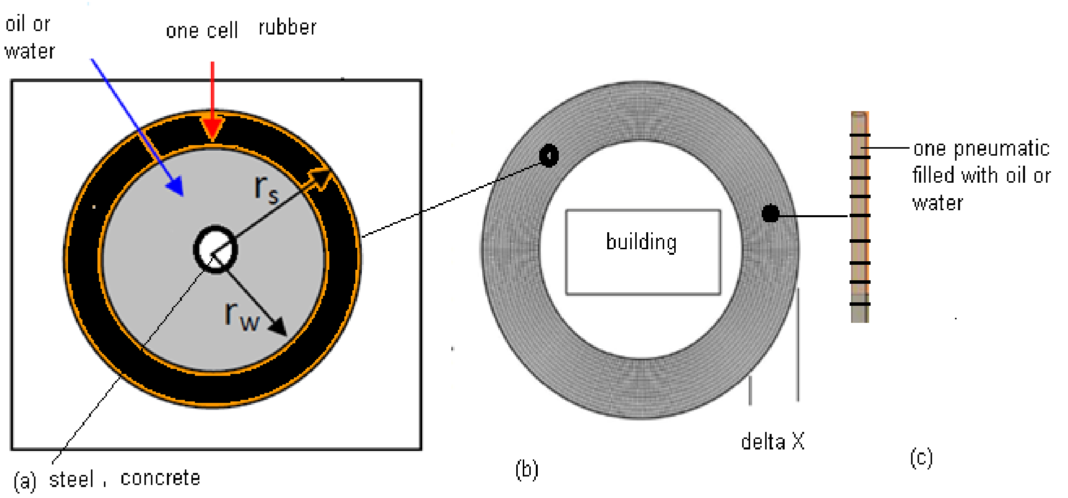

Figure 2. (a) View abobe of one cell made with steel, concrete and tire filled with oil or water, (b) Building shield scheme with many meta-cylinders, (c) Side view of a meta-cylinder with several cells.
Our proposal is show in figure 2. One cell like pneumatics filled with oil and a sky view of a completed seismic barriers with many meta-cylinders. The minor cylinder is made of steel and concrete. In figure 2 the depth h of the system to the rock is 4 to 5 meters.
We can build an attenuator or a barrier of a seismic wave by filling-up many resonators under the ground around the building that we want to protect as it is shown in figure 3.
Then, the amplitude of the seismic wave that passed the waveguide is reduced exponentially by the imaginary wavevector at the frequency ranges of negative modulus or from the mode conversion S-P mode. Mixing up many different kinds of resonators will cover many different corresponding frequency ranges of the seismic waves. The chiral factor can enhance this attenuation if we take $(1 \pm \zeta k_0)$ with the appropriate sign [23-26].
Under these conditions we can define an attenuator or a barrier of a seismic wave by filling-up many resonators under the ground around the building that we want to protect. Then, the amplitude of the seismic wave that passed the set of cells is reduced exponentially by the imaginary wavevector at the frequency ranges of negative modulus. Mixing up many different kinds of resonators will cover many different corresponding frequency ranges of the seismic waves. The chiral factor can enhance this attenuation if we take $(1 \pm \zeta k_0)$ with the appropriate sign.

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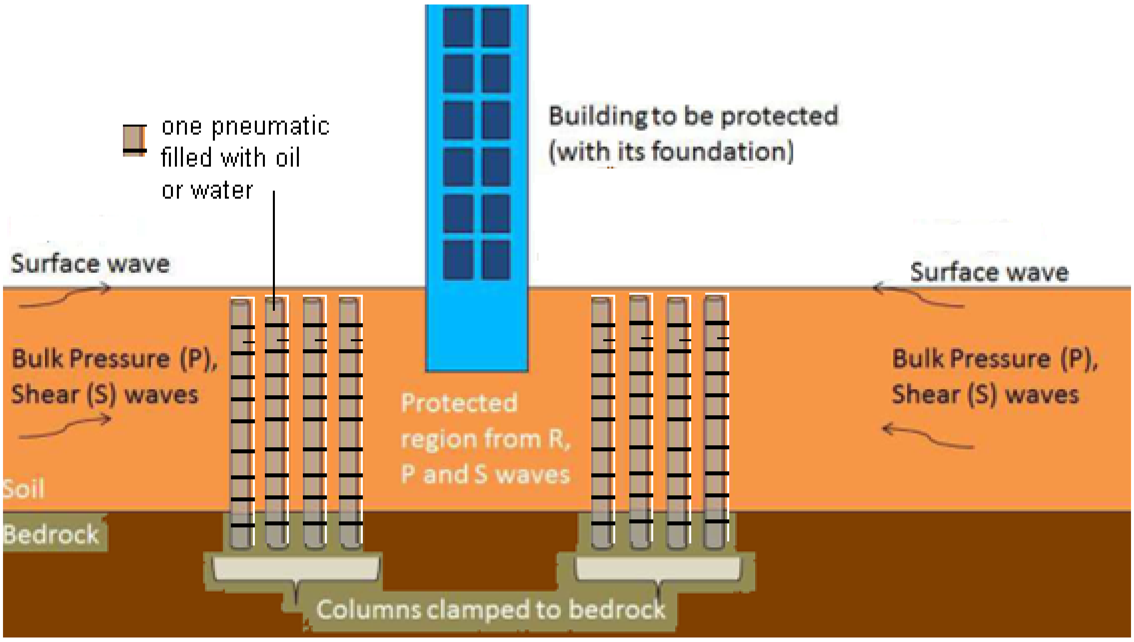

Figure 3. Earthquake protection scheme. (see ref.)

Solving numerically equations (3-5) and if we assume that seismic wave of wavelength λ_S propagates in x−direction, the amplitude of the wave reduces exponentially as

$$S\exp(ik_S x) = S\exp(i2\pi nx/(1\pm\zeta k_0)\lambda_S) = S\exp(-2\pi|n|x/(1\pm\zeta k_0)\lambda_S) \quad (10)$$

Let the initial seismic wave, that is, before entering the set of cells, have amplitude S_i and magnitude R_i, and final seismic wave, that is after leaving the set of cells, have amplitude S_f and magnitude R_f. Then, S_f is written as

$$S_f = S_i\exp(-2\pi|n|x/(1\pm\zeta k_0)\lambda_S) \quad (11)$$

If $1\pm\zeta k_0 \gg 1$, the amplitude of the seismic wave reduces exponentially as passing the set of cells of metamaterials.

We can rewrite Eq. (10) with the definition of the magnitude in $R = \log\frac{S}{S_0}$ as

$$S_0 10^{R_i}\exp(-2\pi|n|x/(1\pm\zeta k_0)\lambda) = S_0 10^{R_f} \quad (12)$$

Taking logarithms both sides of Eq. (10), we obtain the width of the waveguide, x→ ΔX, as

$$\Delta X = \frac{\ln 10}{2\pi}\frac{\lambda\Delta R(1\pm\zeta k_0)}{|n|} = 0.366\frac{\lambda\Delta R}{|n|}(1\pm\zeta k_0) \quad (13)$$

where $\Delta R = R_i - R_f$, $k_0 = \omega/c_T$, $k_S = \omega/(c_T(1\pm\zeta k_0))$ and n is the meta-cylinder number.

5 Methodology

The wave attenuation performance of the proposed cells will be numerically evaluated through wave dispersion analysis and time-transient finite-element simulations. Dispersion properties of individual unit cells will be computed using the Floquet-Bloch theory [33] and the commercial software COMSOL or Matlab

Returning to equation (3)

$-\nabla_s^2 \boldsymbol{u}_S - \frac{\omega^2}{v_S^2}\boldsymbol{u}_S = 0$, with $v_S = \sqrt{B_S/\rho}\left(1 \pm \zeta k_S\right)$
and considering the mode conversion of equation (9) we can see:

Case 1: we consider a transverse plane wave incident from the left medium of $\mu_1 < 0, \rho_1 < 0$ with an incident angle of $0 < \alpha < \pi/2$ as shown in Fig. 1. First, we can obtain μ_2 by using $-\mu_1 = \mu_2 \tan 2\alpha \tan\alpha$. Then, we can obtain κ_2 by using $\kappa_2 = \mu_2 \cos 2\alpha - \mu_1 \sin 2\alpha/\tan\alpha = \mu_2/\cos 2\alpha$. At last, we can obtain ρ_2 by using $\tan\alpha = \frac{k_l}{k_{ct}} = \frac{\sqrt{\mu_1/\rho_1}}{\sqrt{(\kappa_2+\mu_2)/\rho_2}}$. Therefore, μ_2, κ_2 and ρ_2 are all obtained. It is worth mentioning that when $\alpha > \pi/4$, we have $\mu_2 < 0$, but $\kappa_2 > 0$ and $\kappa_2 + \mu_2 = \mu_2\left(1+\cos 2\alpha\right)/\cos 2\alpha > 0$, which indicates a double positive medium for refracted longitudinal waves on the right (together with $\rho_2 > 0$). In this case the medium on the right is not a normal solid.

Case 2: we consider a transverse plane wave incident from the left medium of $\mu_1 > 0, \rho_1 > 0$ with an incident angle of $0 < \alpha < \pi/2$. μ_2, κ_2 and ρ_2 can also be obtained from Eq. (9). Note that we have $\kappa_2 + \mu_2 = \mu_2\left(1+\cos 2\alpha\right)/\cos 2\alpha < 0$ and $\rho_2 < 0$, which indicate a double negative medium (metamaterial) for refracted longitudinal waves on the right.
Very recently, the principle of metamaterial has been extended to acoustic and elastic media, and various schemes have been proposed to realize acoustic and elastic LHM [9-13]. Different from EM and acoustic media, a normal elastic medium can propagate both longitudinal and transverse waves. In an isotropic solid, these waves are described by three independent material parameters, i.e., mass density, ρ, bulk modulus, κ, and shear modulus, μ where the chiral factor take into account of μ, κ, ρ, $\zeta = \zeta(\mu, \kappa, \rho)$.
Analogous to EM metamaterials, negative effective elastic parameters can be realized by introducing resonant structures into the building blocks of an elastic metamaterial. For example, due to dipolar resonances in locally resonant rubber-coated-lead structure [10, 14] and thin membranes [12, 15,16], these structures have been shown to achieve negative mass density.
Air bubbles in water [10] and Helmholtz resonator [11, 17] can achieve negative bulk modulus as results of monopolar resonances. It has also been found that negative shear modulus is related to resonances [18,19]. But to the best of our knowledge, there has been no report of resonant structures to realize negative shear modulus with a noticeable bandwidth yet.
Finding negative effective shear modulus is has intriguing applications. For instance, if an elastic media has simultaneously negative density and shear modulus in a certain frequency regime, it may give rise to negative refraction of transverse waves, i.e., $n_t = \sqrt{\rho}\sqrt{1/\mu} < 0$. Meanwhile, if the material's bulk modulus is positive so that $\kappa + \mu > 0$, the longitudinal wave becomes evanescent . In the case of mode conversion we can calculate the Arias intensity modified by metamaterials.

After performing several simulations until to find the optimal values of θ_S, ρ, μ, κ, $v_S = \sqrt{\mu/\rho}\left(1 \pm \zeta k_S\right)$, boundary conditions of Bessel solutions of $\nabla_s^2 \boldsymbol{u}_S + (\omega^2/v_S^2)\boldsymbol{u}_S = 0$,size of parameters: cell a, Δ and d and kind of oil etc.., we would be able to determine the cell system of metamaterials with an optimum in the attenuation and the Arias Intensity. References [34,19] they are a complement to our proposal.
In figure 4 we find the behavior of Si and Sf , which show the attenuation of an earthquake to protect a building.

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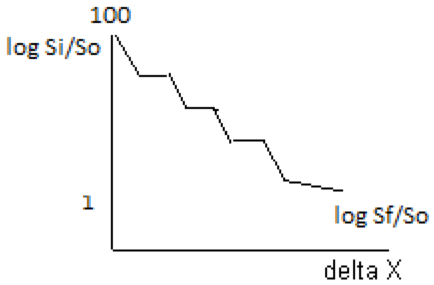

Figure 4. Normalized amplitude of S wave as a function of delta X or n , metacylinder number.

For example, if the refractive index is $n = 2$ and $k_0\zeta = 0$ the wavelength of the S wave is $\lambda = 100m$, we need $deltaX = 16,8m$ to reduce $\Delta R = 0.01$. If $k_0\zeta = 0.5$ we need $deltaX = 8.4m$ That is the chiral factor can substantially reduce the volume of materials, with the significant reduction in engineering costs.

Therefore, a high refracttive index material can be obtained by the factor $1 + k_0\zeta$ for a narrow attenuator $deltaX$. In civil engineering earthquake proofing methods must be practical, that is, with great versatility, simplicity and easy to construct.
Here we have introduced a supportive method for aseismic design. with an earthquake proof barrier around the building to be protected. This barrier is a kind of waveguide that reduces exponentially the amplitude of the dangerous seismic waves. Then, if the structures are protected with ground rubber layers would be possible to divert the S waves which do not impact a building. This method will be effective for isolated buildings because we need some areas to construct the aseismic shell. It may be applicable for social overhead capitals such as power plants, dams, airports, nuclear reactors, oil refining complexes, long-span bridges, express rail-roads, etc

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