Explorations of Lagrange's Remainder

The command LagrangeRem(f,n,a,x) returns the difference between f(x) and the (n-1) degree Taylor polynomial of f expanded about a.

```
> LagrangeRem := (f,n,a,x) -> evalf(eval(f(t) -
    convert(taylor(f(t),t=a,n),polynom),t=x),20);
> LagrangeRem(sin,10,0,x);
> LagrangeRem(sin,10,0,0.5);
> plot(LagrangeRem(sin,10,0,x),x=-2..2);
```

The problem with investigating Lagrange's remainder theorem is that the Lagrange remainder is so extremely small near x=a. Rather than comparing this remainder with the nth derivative times $(x-a)^n$ divided by n!, it makes more sense to divide the remainder by $(x-a)^n$, multiply it by n!, and then compare the resulting function to the nth derivative of f.

```
> ModifiedLagrangeRem := (f,n,a,x) -> LagrangeRem(f,n,a,x)*n!/(x-a)^n;
```

We now compare the plot of the constant function y = ModifiedLagrangeRem with the nth derivative of f over the interval [a,x]. Lagrange's Remainder theorem is simply the statement that the graphs of these two functions interesect.

- > ModifiedLagrangeRem(cosh,10,0,0.5);
- > plot([ModifiedLagrangeRem(cosh,10,0,0.5),eval(diff(cosh(x),x\$10),x=t)],t=0..0.5);