1.Jan 2025

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x+1}{(x+3)^2} \qquad x \neq -3 \qquad x \in \mathbb{R}$$

Use calculus to find the range of values of x for which y is increasing.

(6)

2. Jan 2025

9.

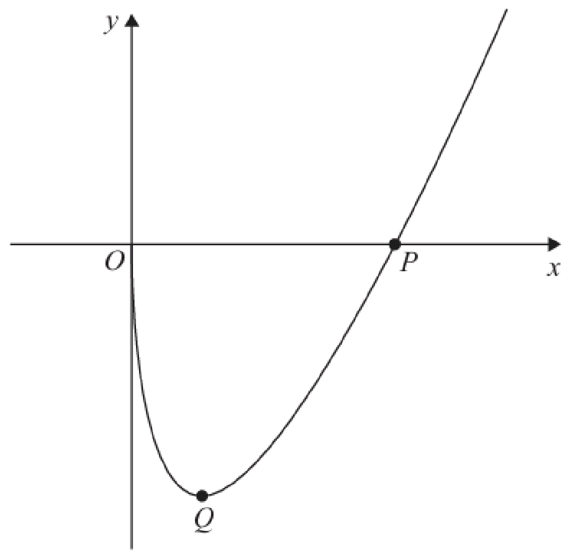

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\,\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

3. Oct 2024

7.

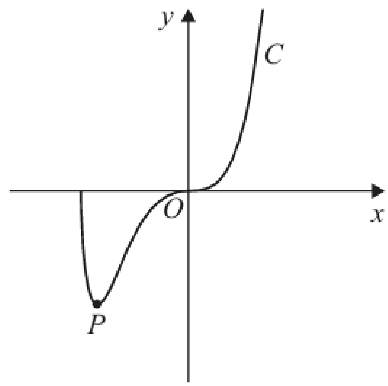

Figure 3

The curve C has equation $y = f(x)$, where

$$f(x) = x^3\sqrt{4x+7} \qquad x \geqslant -\frac{7}{4}$$

(a) Show that

$$f'(x) = \frac{kx^2(2x+3)}{\sqrt{4x+7}}$$

where k is a constant to be found.

(4)

The point P, shown in Figure 3, is the minimum turning point on C.

(b) Find the coordinates of P.

(2)

(c) Hence find the range of the function g defined by

$$g(x) = -4f(x) \qquad x \geqslant -\frac{7}{4}$$

(2)

The point Q with coordinates $\left(\frac{1}{2}, \frac{3}{8}\right)$ lies on C.

(d) Find the coordinates of the point to which Q is mapped when C is transformed to the curve with equation

$$y = 40f\left(x - \frac{3}{2}\right) - 8$$

(2)

4.Oct 2024

9.

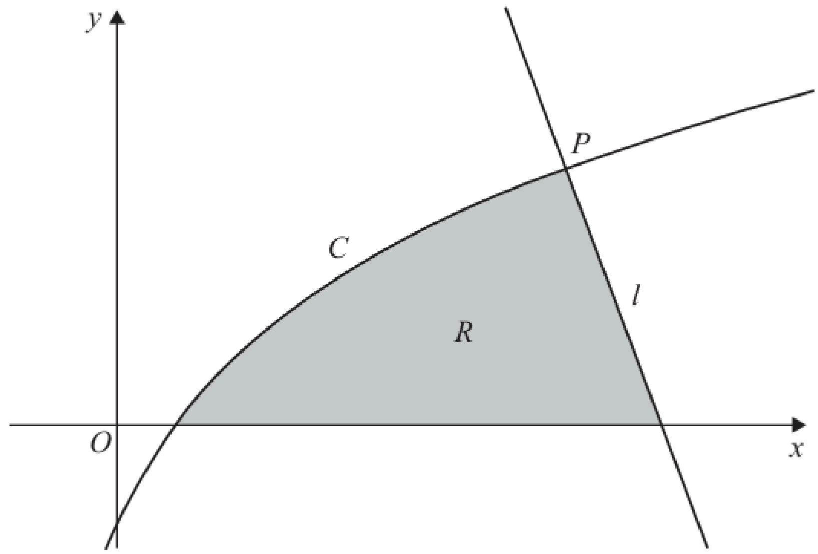

Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{6x^2 + 4x - 2}{2x + 1} \qquad x > -\frac{1}{2}$$

(a) Find $f'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

(b) Show that an equation for l is

$$16y + 5x = 106$$

(3)

(c) Write $f(x)$ in the form $Ax + B + \frac{D}{2x + 1}$ where A, B and D are constants.

(3)

The region R, shown shaded in Figure 5, is bounded by C, l and the x-axis.

(d) Use algebraic integration to find the exact area of R, giving your answer in the form $P + Q\ln 3$, where P and Q are rational constants.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

5.Jan 2024

4. The function f is defined by

$$f(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \qquad x \in \mathbb{R} \quad x > 2$$

(a) Show that $f(x) = \dfrac{2x}{3x-5}$

(3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear.

(3)

The function g is defined by

$$g(x) = 3 + 2\ln x \qquad x \geqslant 1$$

(c) Find g^{-1}

(3)

(d) Find the exact value of a for which

$$gf(a) = 5$$

(4)

6.June 2024

6.

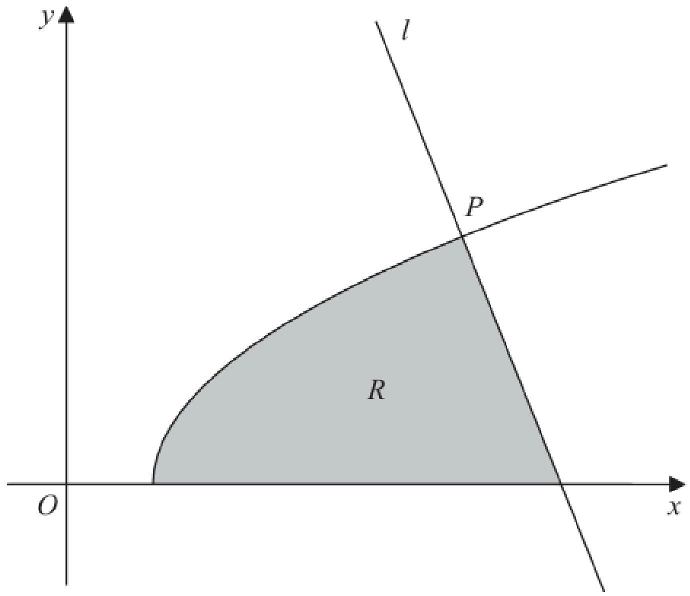

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

7.Jan 2023

3.

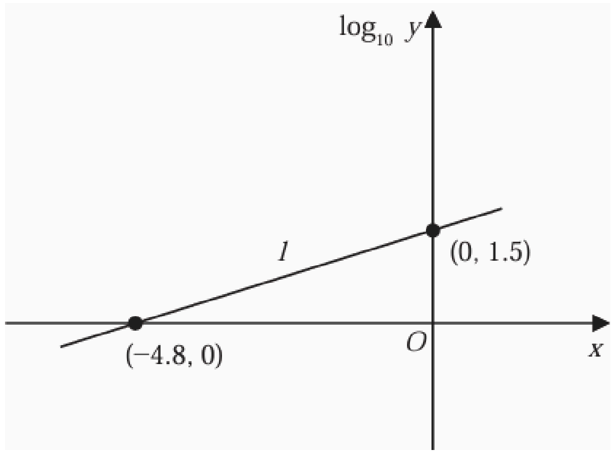

Figure 1

The line l in Figure 1 shows a linear relationship between $\log_{10} y$ and x.

The line passes through the points (0, 1.5) and (−4.8, 0) as shown.

(a) Write down an equation for l.

(2)

(b) Hence, or otherwise, express y in the form kb^x, giving the values of the constants k and b to 3 significant figures.

(3)

8.Jan 2023

4. $$\mathrm{f}(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2} \qquad x \in \mathbb{R} \quad x > -3$$

(a) Find the values of the constants A, B, C and D such that

$$\mathrm{f}(x) = Ax^2 + Bx + C + \frac{D}{(x+3)^2}$$

(4)

(b) Hence find,

$$\int \mathrm{f}(x)\,\mathrm{d}x$$

(3)

9.Jan 2023

8. Find, in simplest form,

$$\int (2\cos x - \sin x)^2 \, dx$$

(5)

10.Jan 2023

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A population of fruit flies is being studied.

The number of fruit flies, F, in the population, t days after the start of the study, is modelled by the equation

$$F = \frac{350e^{kt}}{9 + e^{kt}}$$

where k is a constant.

Use the equation of the model to answer parts (a), (b) and (c).

(a) Find the number of fruit flies in the population at the start of the study.

(1)

Given that there are 200 fruit flies in the population 15 days after the start of the study,

(b) show that $k = \frac{1}{15}\ln 12$

(3)

Given also that, when $t = T$, the number of fruit flies in the population is increasing at a rate of 10 per day,

(c) find the possible values of T, giving your answers to one decimal place.

(5)

11.June 2023

3. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(ii) (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 - 4\right)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2 - 4\right)^5 \mathrm{d}x = R$$

where R is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

12.Oct 2023

3. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

(2)

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,dx$$

(4)

13.Jan 2022

3. (i) Find, in simplest form,

$$\int (2x-5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1+2\cos x} \, dx = \ln a$$

where a is a rational constant to be found.

(4)

14.June 2022

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, dx$$

(2)

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(b) find the value of k

(4)

15.June 2022

6.

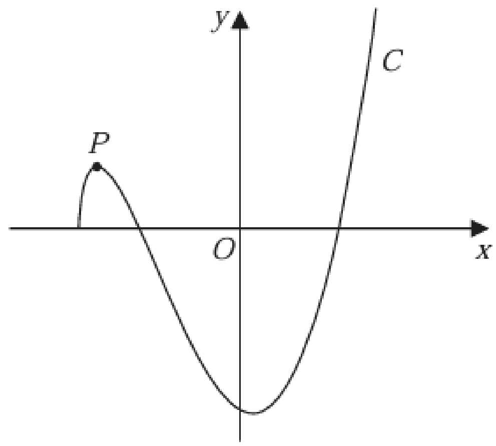

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where k is an integer to be found.

(4)

(b) Hence, find the values of x for which $f'(x) = 0$

(1)

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve has a local maximum at the point P

(c) Find the exact coordinates of P

(2)

The function g is defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) + 4 \qquad -\frac{9}{4} \leqslant x \leqslant 0$$

(d) Find the range of g

(3)

16.Oct 2022

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x+3)}{x^2 + 3x + 4}$$

where A, B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x)\,dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

17.Jan 2021

1. Find

$$\int \frac{x^2 - 5}{2x^3} \mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(3)

18.Jan 2021

3.

$$f(x) = 3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} \qquad x > 4$$

(a) Show that

$$f(x) = \frac{ax+b}{cx+d} \qquad x > 4$$

where a, b, c and d are integers to be found.

(4)

(b) Hence find $f^{-1}(x)$

(2)

(c) Find the domain of f^{-1}

(2)

19.Jan 2021

9. Find

(i) $\displaystyle\int \frac{3x - 2}{3x^2 - 4x + 5}\,dx$ **(2)**

(ii) $\displaystyle\int \frac{e^{2x}}{(e^{2x} - 1)^3}\,dx \qquad x \neq 0$ **(2)**

20.Oct 2021

5. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3}\,dx$$

(4)

(ii) Find, in simplest form,

$$\int x\left(x^2+3\right)^7\,dx$$

(2)

21.Oct 2021

10.

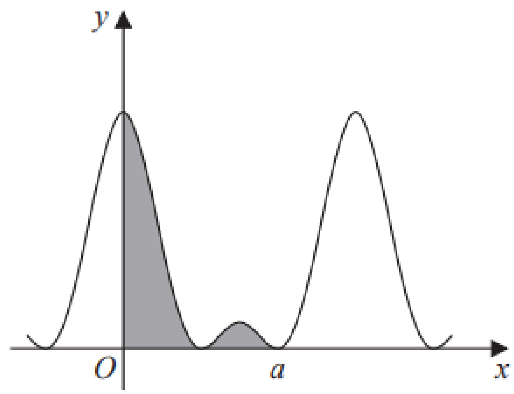

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where p, q and r are constants to be found.

(2)

The curve touches the positive x-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the y-axis and the x-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

(5)

22.Jan 2020

4. (i)

$$f(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find $f'(x)$ in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(6)

(ii)

$$g(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation $y = g(x)$ has a maximum at the point M.

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found.

(5)

23.Jan 2020

8. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1}\,\mathrm{d}x$$

giving your answer in simplest form.

(4)

(ii)
$$\mathrm{h}(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $\mathrm{h}(x) = Ax + B + \dfrac{C}{(x-1)^2}$ where A, B and C are constants to be found, find

$$\int \mathrm{h}(x)\,\mathrm{d}x$$

(6)

24.Oct 2020

5. (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

(4)

25.Oct 2020

9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x-4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = \text{g}(x)$, where

$$\text{g}(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

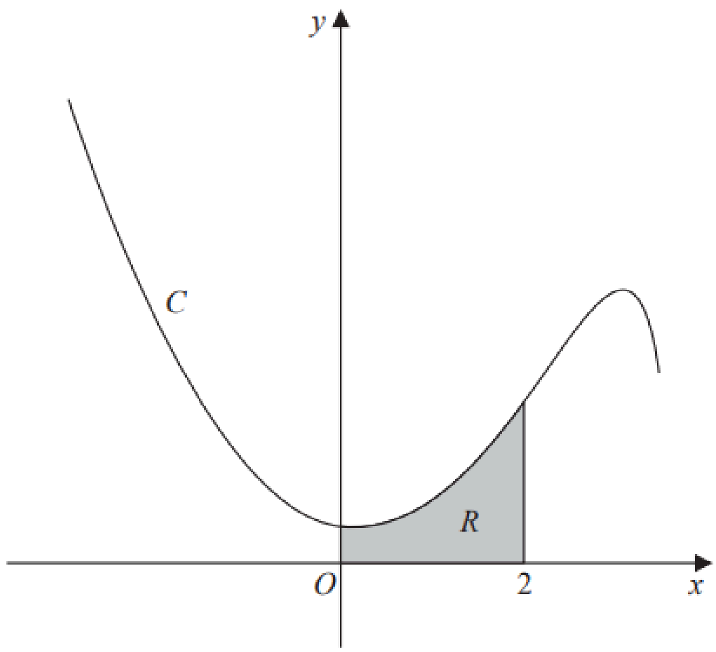

Figure 4

Figure 4 shows a sketch of the curve C.

The region R, shown shaded in Figure 4, is bounded by C, the y-axis, the x-axis and the line with equation $x = 2$

(c) Find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)