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B. Tech. (Civil, Aeronautical& amp; Aerospace Avoinics) (Semester – 1st)

MATHEMATICS-I (CALCULUS, MULTIVARIABLE CALCULUS & LINEAR ALGEBRA)

Subject Code: BMATH4-101

Paper ID: [19110011]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Evaluate x^{x}

b. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

c. Discuss the convergence of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$

d. Show that the $\frac{x^3 y}{x^6 + y^2}$

e. State Green’s theorem in the plane.

f. Show that the plane $3x + 12y - 6z - 17 = 0$ touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$. Find also point of contact.

g. Change the order of integration in $I = \int_{0}^{1} \int_{x^2}^{2-x} xy\, dx\, dy$

h. If A is symmetric matrix, prove that $\left(BA^{-1}\right)^{T}\left(A^{-1}B^{T}\right)^{-1} = I$ where B is any matrix for which product matrices are defined.

i. Solve the equations $x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$

j. If A is orthogonal, then prove that $|A| = \pm 1$

Section – B **(5 marks each)**

Q2. Find the absolute maximum/minimum values of the function

$$f(x) = \sin sin\, x(1 + \cos cos\, x)\ ,\ 0 \le x \le 2\pi$$

Q3. Show that the function $f(x, y) = \left\{ \frac{x^2 - y^2}{x - y},\ (x, y) \ne (1, -1)\ 0\ (x, y) = (1, -1) \right\}$ is continuous and differentiable at $(1, -1)$.

Q4. Calculate the area included between the curve $r = a(\sec sec\, \theta + \cos cos\, \theta)$ and its asymptote.

Q5. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9\ and\ z = x^2 + y^2 - 3\ at\ (2, -1, 2)$

Q6. If $A = [1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ ]$, then show that $A^{n} = A^{n-2} + A^{2} - I$ for $n \ge 3. Hence\ find\ A^{50}$.

Section – C **(10 marks each)**

Q7. (a) Find the volume of the solid generated by revolving the region bounded by the curves $y = 3 - x^2\ and\ y = -1\ about\ the\ line\ y = -1$.

(b) Obtain the Taylor series expansion of

$$f(z) = e^{z},\ (i)\ about\ the\ point\ z = 0,\ (ii)\ about\ the\ point\ z = 2$$

Q8. Find the matrix P which transforms the matrix [1 1 3 1 5 1 3 1 1] to diagonal form. Hence find A^4

Q9. Verify Divergence theorem for $F = \left(x^2 - yz\right)\hat{i} + \left(y^2 - zx\right)\hat{j} + (z^2 - xy)\ \hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.