Research in Options RiO 2022
Titles and Abstracts

Minicourses
Saturday 20

Feedback effects and endogenous risk in financial markets

Lakshithe Wagalath
IÉSEG School of Management

Over the last decades, financial markets have become increasingly dominated by large financial institutions, which use automated algorithms or follow rule-based strategies and trade in a systematic manner. Many empirical studies shed light on the feedback effects and the endogenous risk generated by systematic trading by large financial institutions. Such feedback effects materialize in a wide range of situations: options trading, rebalancing by leveraged exchange-traded funds, fire sales and distressed selling, predatory trading, rebalancing by large institutional investors, etc... In this minicourse, we are going to review the various types of feedback effects generated by systematic trading and propose a model to quantify their impact on the dynamics of financial assets. We will develop tools that will be useful to better tackle and anticipate liquidity events caused by large trades and that may also be used in a systemic risk-management perspective as they enable us to quantify price-mediated contagion.

Portfolio construction using nonlinear polymodels

Raphael Douady
SUNY Stony Brook

Polymodels are a statistical analysis technique for dynamic objects evolving within an environment, which is dynamic as well. Both the object and the environment are described by time series. In a financial context, the object is an asset, a fund, a portfolio, anything that can represent an investment, while the environment is made of variables or "factors" that describe the state of the market. Polymodels provide the individual response of the object to every single variable of the environment, together with a reliability score. These response functions being nonlinear, the fragility/antifragility properties of the object can be evaluated.

We will show how to estimate polymodels and to use them for asset selection, risk assessment and portfolio construction.

Reference: Barrau & Douady *Artificial Intelligence in Finance: The Polymodel Approach* Springer 2022.

Sunday 21

Reinforcement Learning for Finance

Sebastian Jaimungal
University of Toronto

We develop an approach for risk budgeting allocation -- a risk diversification portfolio strategy -- where risk is measured using time-consistent dynamic risk measures. For this, we introduce a notion of dynamic risk contributions that generalise the classical Euler contributions and which allow us to obtain dynamic risk contributions in a recursive manner. Moreover, we show how the risk allocation problem may be recast as a convex optimisation problem and develop an actor-critic approach to solve for risk allocations using deep learning techniques.
[this is based on joint work with Silvana Pesenti, Yuri F. Saporito, and Rodrigo S. Targino]

Recent advances in VIX modeling

Julian Guyon
Bloomberg LP

Agenda:

- Optimal bounds for VIX futures given S&P 500 smiles
- Robust hedging of derivatives on S&P 500 and/or VIX: VIX-constrained martingale optimal transport
- Joint S&P 500/VIX arbitrages
- Exact joint calibration of S&P 500 and VIX smiles: VIX-constrained martingale Schrodinger problems/bridges
- Inversion of convex ordering: a remarkable empirical feature of the VIX market
- Inversion of convex ordering: Local volatility does not maximize the price of VIX futures
- VIX is (mostly) path-dependent: Empirical study and the 4-Factor Path-Dependent Volatility model

Conference
Monday 22

Inverse Problems in Option Pricing and Financial Risk Analysis
(keynote talk dedicated to the memory of Marco Avellaneda)

Jorge P. Zubelli
IMPA (retired)
Mathematics Department Khalifa University

Risk analysis and management depend heavily on hedging and thus on accurate and yet realistic models for asset pricing. The practical impact ranges all the way from equity to commodities passing through forex and futures. In this context options and other derivatives still play a crucial role and the seminal work of Black, Scholes and Merton is still a landmark in the foundations of modern finance. Yet, it relies on assumptions that have been relaxed and extended in several different directions. A well established extension of such classical work is Dupire's local volatility model [9] which is one of the main financial industry standards.
This talk will be a survey of different results where inverse problems and statistical inference techniques have been used to handle calibration and option pricing.
We shall start with a brief review of Dupire's equation for option pricing and local volatility model. Then we shall address different contributions starting with joint work on theoretical aspects with De Cezaro and Scherzer [1,2] and Albani [3,4,5] that laid the theoretical framework for analyzing local volatility estimation at the light of convex regularization. We shall then briefly discuss calibration models for commodities derivatives and joint work with Albani, Ascher and Yang [6] as well as more recent work on extension to jump-diffusion models with Albani [7] and to stochastic local-volatility models with Saporito and Yang [8].

References:

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4. V Albani, A De Cezaro, JP Zubelli. (2017) Convex regularization of local volatility estimation. *International Journal of Theoretical and Applied Finance* 20 (01), 1750006
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8. YF Saporito, X Yang, JP Zubelli. (2019) The calibration of stochastic local-volatility models: An inverse problem perspective. *Computers & Mathematics with Applications* 77 (12), 3054-3067
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Portfolio construction using nonlinear polymodels

Raphael Douady
SUNY Stony Brook

Polymodels are a statistical analysis technique for dynamic objects evolving within an environment, which is dynamic as well. Both the object and the environment are described by time series. In a financial context, the object is an asset, a fund, a portfolio, anything that can represent an investment, while the environment is made of variables or "factors" that describe the state of the market. Polymodels provide the individual response of the object to every single variable of the environment, together with a reliability score. These response functions being nonlinear, the fragility/antifragility properties of the object can be evaluated.

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The Interplay Between COVID-19 and the Economy in Canada

Matheus Grasselli
McMaster University

We propose a generalized Susceptible-Exposed-Infected-Removed (SEIR) model to track COVID-19 in Canadian provinces taking into account the impact of the pandemics on unemployment. The model is based on a network representing provinces, where the contact between individuals from different locations is defined by a data-driven mixing matrix. Moreover, we use time-dependent parameters to account for the dynamical evolution of the disease incidence, as well as changes in the rates of hospitalization, intensive care unit (ICU) admission, and death. Unemployment is accounted for as a reduction in the social interaction, which translates into smaller transmission parameters. Conversely, the model assumes that higher proportions of infected individuals reduce overall economic activity and therefore increase unemployment. We test the model using publicly available sources and find that it is able to reproduce the reported data with remarkable in-sample accuracy. We also test the model's ability to make short-term out-of-sample forecasts and find it very satisfactory, except in periods of rapid changes in behavior. Finally, we present long-term predictions for both epidemiological and economic variables under several future vaccination scenarios. (This is joint work with Vinicius Albani, Weijie Pang, and Jorge P. Zubelli)

Is the square root rule valid? Evidence from the Brazil's stock market

Alan de Genaro
São Paulo School of Business Administration - (FGV EAESP)

Market impact risk is a specific type of liquidity risk. It describes the risk of not being able to execute a trade at the currently quoted price because this trade feeds back in an unfavorable manner on the underlying price. This makes market impact modeling a fascinating and active research agenda from a mathematical point of view. Moreover, market impact models are often used in practical applications and it would be desirable to gain a better understanding of their behavior and their stability. Supported by the international empirical evidence, the continuous-time version of the propagator model, discussed by Gatheral (2010), is applied to Brazil stock market. The main finding, based on the False Discovery Rate (FDR), supports the adoption of the square-root law of price impact.

Risk Budgeting Allocation for Dynamic Risk Measures

Sebastian Jaimungal
University of Toronto

We develop an approach for risk budgeting allocation -- a risk diversification portfolio strategy -- where risk is measured using time-consistent dynamic risk measures. For this, we introduce a notion of dynamic risk contributions that generalise the classical Euler contributions and which allow us to obtain dynamic risk contributions in a recursive manner. Moreover, we show how the risk allocation problem may be recast as a convex optimisation problem and develop an actor-critic approach to solve for risk allocations using deep learning techniques.
[this is based on joint work with Silvana Pesenti, Yuri F. Saporito, and Rodrigo S. Targino]

Poster Session - Lightning talks

A novel risk-reward portfolio selection model consistent with interval-based stochastic dominance principles

Alvaro Almeida
Mathematics Department Khalifa University

Following the recent introduction by Liu, Chen, and Consigli of the Interval-based Stochastic Dominance (ISD) principle and relying, for given tolerance, on the equivalence between ISD − 2 and a new tail risk measure, the Interval Conditional VaR (ICVaR) function, we propose a portfolio problem formulation based on the trade-off between expected return and ICVaR. This latter is a function of the confidence level α and the ISD − 2 reference point β. In the problem formulation, the IC-VaR is estimated on the benchmark distribution to be stochastically dominated. The mean-ICVaR problem is solved as a linear programming problem and its distinctive elements with respect to classical CVaR minimization are briefly recalled. The relationship between the portfolio problem formulation and the β is first analyzed in a static one-period problem instance and then through an extended out-of-sample analysis over 3 years, between 2019/1/7 and 2022/6/15, for model validation. The static one-period problem is under extension to a multi-period setting together with the development of a market-consistent machine learning definition of β.

Multiperiod interval-based stochastic dominance: foundation and application to dynamic portfolios

Brian Vasquez
Mathematics Department Khalifa University

We consider a multi-stage generalization of the interval-based stochastic dominance (ISD). At each stage, we have as in the static case an intermediate partial ordering for random processes. Several issues arise when we study the multi-stage version of ISD, one of them is the updating of the reference point. The sample reformulation allows us to verify the ISD condition through a finite family of inequalities. Using this approach and the scenario tree representation, we derive a tool to solve multi-stage portfolio selection problems with ISD constraints.

Markov decision processes with Kusuoka-type conditional risk mappings

Ziteng Cheng
University of Toronto

Under suitable conditions, the Kusuoka representation of law invariant coherent risk measures allows one to cast them in terms of average value-at-risk. Here, we introduce the notion of Kusuoka-type conditional risk-mappings and use it to define a dynamic risk measure. We use such dynamic risk measures to study infinite horizon Markov decision processes (MDPs) with random costs and random actions. Under mild assumptions, we derive a dynamic programming principle and prove the existence of an optimal policy. Furthermore, we provide a sufficient condition for when deterministic actions are optimal. We also propose a sample-based solver for MDPs with Kusuoka-type conditional risk mappings and finite state action spaces.

Projection of Functionals and Fast Pricing of Exotic Options

Valentin Tissot-Daguette
Princeton University

We investigate the approximation of path functionals. In particular, we advocate the use of the Karhunen-Loève expansion, the continuous analogue of principal component analysis, to extract relevant information from the image of a functional. Having an accurate estimate of functionals is of paramount importance in the context of exotic derivatives pricing, as presented in the practical applications. Specifically, we show how a simulation-based procedure, which we call the Karhunen-Loève Monte Carlo (KLMC) algorithm, allows fast and efficient computation of the price of path-dependent options. We also explore the path signature as an alternative tool to project both paths and functionals.

Conference
Tuesday 23

Novel Exact Solutions for PDEs with Mixed Boundary Conditions

Martino Grasselli
Università di Padova Italy and DVRC Paris La Defense

We develop methods for the solution of inhomogeneous Robin type boundary value problems (BVPs) that arise for certain parabolic PDEs on a half line. We are able to obtain non-standard solutions to some interesting BVPs, which do not require knowledge of a fundamental solution. The equations that we investigate arise in mathematical finance and in mathematical physics. We first compute the solution of Robin-like problems for the Black-Scholes equation. We suggest some financial applications of our analysis. We then study a problem for the harmonic oscillator. We then solve Robin-like problems for the heat equation with drift and an equation arising from a three dimensional squared Bessel process. A novel feature of our approach is the use of the odd and even Hilbert transform methods. With these transforms we can reduce a pair of integral equations to a pair of simultaneous equations. The solutions and the method seem to be new.
[joint work with M. Craddock and A. Mazzoran]

Adapted Wasserstein distance for robust stochastic optimization.

Beatrice Acciaio
ETH Zurich

In this talk I will introduce the concept of Adapted Wasserstein distance, that originates from imposing a causality constraint on optimal transport on path spaces. I will present several applications of the AW-distance to illustrate its suitability for multiple problems in mathematical finance, such as robust hedging, optimal stopping, and model-independent pricing of American options.

Parametric Optimal Execution and Machine Learning Surrogates

Mike Ludkovski
University of California Santa Barbara

We investigate optimal order execution problems in discrete time with instantaneous price impact and stochastic resilience. First, in the setting of linear transient price impact we derive a closed-form recursion for the optimal strategy, extending the deterministic results from Obizhaeva and Wang. Second, we develop a numerical algorithm based on dynamic programming and deep learning for the case of nonlinear transient price impact as proposed by Bouchaud et al. (2004). Specifically, we utilize an actor-critic framework that constructs two neural-network (NN) surrogates for the value function and the feedback control. The flexible scalability of NN functional approximators enables parametric learning, i.e., incorporating several model or market parameters as part of the input space. Precise calibration of price impact, resilience, etc., is known to be extremely challenging and hence it is critical to understand sensitivity of the execution policy to these parameters. Our NN learner organically scales across multiple input dimensions and is shown to accurately approximate optimal strategies across a wide range of parameter configurations. Our NN implementation also has pedagogical interest, demonstrating the ease of use of NN surrogates in (parametric) stochastic control problems.
This is joint work with Tao Chen (U of Michigan) and Moritz Voss (UCLA).

Cashflow-driven investment beyond expectations

Teemu Pennanen
King's College London

We present a computationally tractable optimization model for cashflow-driven investment where the aim is to find buy-and-hold portfolios whose future payouts cover given liability payments as well as possible. The model explains the benefits of cashflow-driven investment, both in idealized deterministic actuarial models as well as in more realistic ones where the cashflows of both assets and liabilities are uncertain. The model also illustrates the dangers of basing investment decisions on a deterministic model when faced with long-term financial risks. The model is solved with simple Monte-Carlo approximations and off-the-self convex optimization software. Extensive out-of-sample simulations confirm the accuracy of solutions in a 35-year planning problem coming from the UK pension industry. Besides optimal hedging strategies, we find the least cost of hedging which provides a market-consistent valuation based on the current quotes and the liquidity factors as well as the views and risk preferences of the investor/regulator. The approach is illustrated by pricing and hedging of defined benefit pension liabilities which depend on uncertain longevity developments and the consumer price index. The hedging strategies are constructed from 128 publicly quoted instruments including index-linked bonds and equities. Increasing the risk aversion and the illiquidity parameters, we find portfolios that hedge the liabilities with significantly lower risk but only slightly higher cost.
This is joint work with Sergio Alvares Maffra.

The interval-based stochastic dominance principle: theory and application to portfolio selection

Giorgio Consigli
Mathematics Department Khalifa University

We introduce a new stochastic dominance relationship, the interval-based stochastic dominance (ISD). By choosing different reference points, we show that ISD may span a continuum of preferences between kth and (k + 1)th order stochastic dominance (SD). We distinguish accordingly between interval-based (or shortly just interval) SD of order 1 and of order 2: the former spanning from first- to second-order stochastic dominance, the latter from second to third-order stochastic dominance. By examining the relationships between interval-based SD and SD, as well as between ISD and risk measures or utility functions, we frame the concept within decision theory and clarify its implications when applied to an optimal financial allocation problem. The formulation of ISD-constrained problems in the presence of discrete random variables is discussed in detail and applied to a portfolio selection problem.

Risk Budgeting Portfolios from Simulations

Rodrigo Targino
Fundação Getulio Vargas (FGV)

Large pension plans face the difficulty of investing premiums in a financially prudent but economically efficient way. An investment concept used in industry, e.g., by pension funds, is risk budgeting portfolios (RBP). RBP are diversified portfolios where the diversification is on the level of the risk contribution of each asset to the risk of the total portfolio loss. In this work, we assess the risk of the portfolio via a coherent risk measure, such as the Expected Shortfall. The risk contribution of an asset, is then the added risk of marginally increasing the portfolio's position in that asset. Hence, risk budgeting portfolios are diversified in terms of the risk stemming from each asset and therefore ideal for pension plans which face stringent investment requirements. Advances in the academic literature and practical implementations of RBP strategies, however, make the strong assumption that assets are multivariate Gaussian. We propose an efficient stochastic optimization framework that calculates RBP for assets with arbitrary joint distribution. Moreover, the RBP is constructed using only sampled scenarios, making it applicable in a variety of settings such as data-driven statistical models, arbitrary dependency structures, and internal loss models.

Volatility Is (Mostly) Path-Dependent

Julien Guyon
Bloomberg LP

We learn from data that volatility is mostly path-dependent: at least 85-90% of the variance of the implied volatility of equity indexes is explained endogenously by past index returns, and around 60% for (noisy estimates of) future daily realized volatility. The path-dependency that we uncover is remarkably simple: a linear combination of a weighted sum of past daily returns and the square root of a weighted sum of past daily squared returns with different time-shifted power-law weights capturing both short and long memory. This simple model, which is homogeneous in volatility, is shown to consistently outperform existing models across equity indexes for both implied and realized volatility. It suggests a simple continuous-time path-dependent volatility (PDV) model that may be fed historical or risk-neutral parameters. The weights can be approximated by superpositions of exponential kernels to produce Markovian models. In particular, we propose a 4-factor Markovian PDV model which captures all the important stylized facts of volatility, produces very realistic price and volatility paths, and jointly fits SPX and VIX smiles remarkably well. We thus show, for the first time, that a continuous-time Markovian parametric stochastic volatility (actually, PDV) model can practically solve the joint SPX/VIX smile calibration problem.

This is joint work with Jordan Lekeufack.

Jorge Fest

A bouquet of stories for Jorge from the lands of classical/quantum probability and signal processing. One miracle and eight surprises.

Alberto Grunbaum
University of California, Berkeley

I will try to do a light tour through a few area of mathematics leaving out detailed introductions but rather stating some results that may tempt some people to go back for a second visit.

Jorge Zubelli and inverse problems

Antonio Leitão
Universidade Federal de Santa Catarina

In this talk we focus on the contributions of prof. J.P.Zubelli in the area of inverse problems. In particular, we address some research results related to our scientific collaboration.

Simulating deformable 3D objects

Uri Ascher
University of British Columbia

We examine a variety of numerical methods that arise when considering dynamical systems in the context of physics-based simulations of deformable objects. Such problems arise in various applications, including animation, robotics, control and fabrication. The goals and merits of suitable numerical algorithms for these applications are different from those of typical numerical analysis research in dynamical systems. Here the mathematical model is not fixed a priori but must be adjusted as necessary to capture the desired behaviour, with an emphasis on effectively producing lively animations of objects with complex geometries. Results are often judged by how realistic they appear to observers (by the "eye-norm") as well as by the efficacy of the numerical procedures employed. And yet, we show that with an adjusted view, numerical analysis and applied mathematics can contribute significantly to the development of appropriate methods and their analysis in a variety of areas including finite element methods, stiff and highly oscillatory ODEs, model reduction, and constrained optimization.

PS I will mention an article that Jorge co-authored.

Challenges in Epidemiological Modeling During the COVID-19 Pandemic

Vinicius Albani
Universidade Federal de Santa Catarina

The COVID-19 pandemic drove the scientific community's attention to the problem of providing realistic epidemiological scenarios, as they are crucial for decision-making and public resource allocation. This led to the design of sophisticated epidemiological models. However, calibrating such models can be challenging due to several reasons, namely, the number of unknowns, the presence of noise in the data, and the model accuracy. We shall present some epidemiological models that account for spatial distribution and demographic factors, their calibration to daily reports, and their performance in forecasting scenarios. All numerical results use real data from different places like New York City, Chicago, British Columbia, and São Paulo.

Game Theory on Graphs: from Cooperation to Coalition Resilience

Chiara Mocenni
Università di Siena

The talk discusses some recent results on the emergence of cooperation in games on graphs. Specifically, we show in the framework of evolutionary games that when several subgroups with different behavioral characteristics are present in the graph, an excess of cooperation internal to the subgroups may induce over time a decrease in social welfare. Moreover, we provide formal proof based on max k-cut colorings and game theory, that optimal configurations of the graph are resilient to coalitions of selfish subgroups. Final considerations, remarks, and future developments in the direction of hypergraphs and multiplex networks are also investigated.

Wednesday 24

Functional Expansions and Claim Decomposition

Valentin Tissot-Daguette
Princeton University

We discuss classical and novel expansions of path functionals. First, we focus on static expansions (Volterra series, Wiener series and Chaos, Hilbert projections) made around paths of fixed length. In particular, we propose a generalization of the Wiener series based on the so-called intrinsic value functional. In the dynamic case, we promote the functional Taylor expansion (FTE) which quantifies the effect in a smooth functional when a "perturbation" path is concatenated with the source path. The notions of real analytic functionals and radius of convergence in the path space are then defined. In the applications, we show how the FTE can be used to price and hedge path-dependent claims.

Exploratory Control with Tsallis Entropy for Latent Factor Models

Ryan Donnelly
King's College London

We study optimal control in models with latent factors where the agent controls the distribution over actions, rather than actions themselves, in both discrete and continuous time. To encourage exploration of the state space, we reward exploration with Tsallis Entropy and derive the optimal distribution over states – which we prove is q-Gaussian distributed with location characterized through the solution of a backward stochastic difference equation and backward stochastic differential equation discrete and continuous time, respectively. Finally, we discuss the relation between the solutions of the optimal exploration problems and the standard dynamic optimal control solution.

Unbiasing and robustifying implied volatility calibration in a cryptocurrency market with large bid-ask spreads and missing quotes

Emmanuel Gobet
École Polytechnique CNRS

We design a novel calibration procedure that is designed to handle the specific characteristics of options on cryptocurrency markets, namely large bid-ask spreads and the possibility of missing or incoherent prices in the considered data sets. We show that this calibration procedure is significantly more robust and accurate than the standard one based on trade and mid-prices. Joint work with Mnacho Echenim and Anne-Claire Maurice.

Transform MCMC schemes for sampling intractable factor copula models

Cyril Benezet
ENSIIE

In this work, motivated by financial risk management, we are interested in the computation of statistics of the form E[g(X)] or E[g(X) | A] for a rare event A. Here, X is a random vector which we cannot directly simulate, being a copula-preserving transform of a vector Y having unknown marginals, as in a factor copula model. We design an algorithm which samples X through an empirical approximation of the c.d.f. of the Y marginals. We allow Markov Chain Monte Carlo (MCMC) samplers to be able to handle complex distributions or rare-event simulations. We establish convergence results whose rates depend on g, the tails of X, Y and the Lyapunov function of the sampler. We eventually revisit a real data analysis from financial risk management. This is a joint work with Emmanuel Gobet and Rodrigo Targino.

PDGM: a Neural Network Approach to Solve Path-Dependent Partial Differential Equations

Yuri Saporito
Fundação Getulio Vargas

In this talk, we present a novel numerical method for Path-Dependent Partial Differential Equations (PPDEs). These equations firstly appeared in the seminal work of Dupire [QF, 2019, originally published in 2009], where the functional Itô calculus was developed to deal with path-dependent financial derivatives contracts. More specifically, we generalize the Deep Galerkin Method (DGM) of Sirignano and Spiliopoulos [2018] to deal with these equations. The method, which we call Path-Dependent DGM (PDGM), consists of using a combination of feed-forward and Long Short-Term Memory architectures to model the solution of the PPDE. We then analyze several numerical examples from the Financial Mathematics literature that show the capabilities of the method under very different situations.

On regularized optimal execution problems and their singular limits

Max O. Souza
Universidade Federal Fluminense.

We study the portfolio execution problem under a framework in which volatility and liquidity are both uncertain. We assume that a multidimensional Markovian stochastic factor drives both of them and indirect liquidity costs are modeled as a temporary price impact, through a power law to relate it to the agent's turnover rate. We begin by analyzing the regularized setting --- where admissible strategies do not ensure complete execution of the initial inventory. We obtain a characterization of the optimal trading rate and, as a byproduct of the proof, a numerical algorithm. The constrained problem is then analysed as a singular limit of the regularised one. Joint work with Y. Thamsten.