

Mech 2 Ch7 - Statics with Rigid Bodies

Q1.

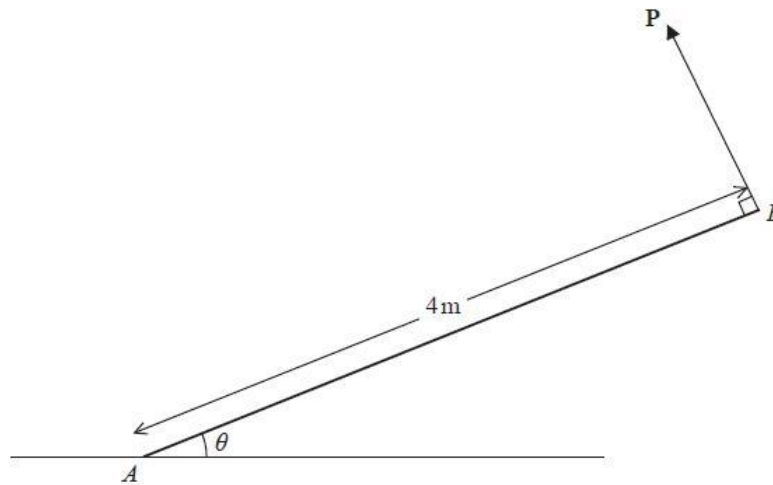


Figure 2

A non-uniform rod AB , of mass 5kg and length 4m , rests with one end A on rough horizontal ground. The centre of mass of the rod is d metres from A . The rod is held in limiting equilibrium at an angle θ to the horizontal by a force P , which acts in a direction perpendicular to the rod at B , as shown in Figure 2. The line of action of P lies in the same vertical plane as the rod.

(a) Find, in terms of d , g and θ ,

- the magnitude of the vertical component of the force exerted on the rod by the ground,
- the magnitude of the friction force acting on the rod at A .

(8)

Given that $\tan \theta = \frac{5}{12}$ and that the coefficient of friction between the rod and the ground is $\frac{1}{2}$,

(b) find the value of d .

(4)

(Total for question = 12 marks)

Q2.

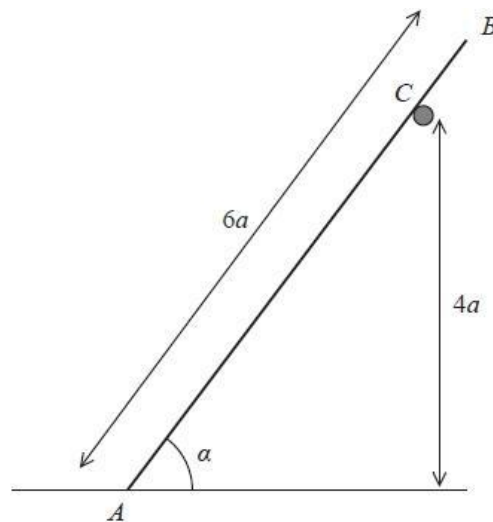


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C .

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ .

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ .

(7)

(Total for question = 10 marks)

Q3.

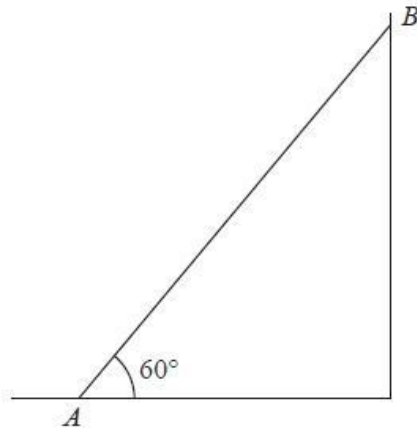


Figure 1

A non-uniform rod, AB , of mass m and length $2l$, rests in equilibrium with one end A on a rough horizontal floor and the other end B against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is $\frac{1}{4}$ and the coefficient of friction between the rod and the wall is $\frac{2}{3}$. The rod is on the point of slipping at both ends.

(a) Find the magnitude of the vertical component of the force exerted on the rod by the floor.

(5)

The centre of mass of the rod is at G .

(b) Find the distance AG .

(3)

(Total 10 marks)

Q4.

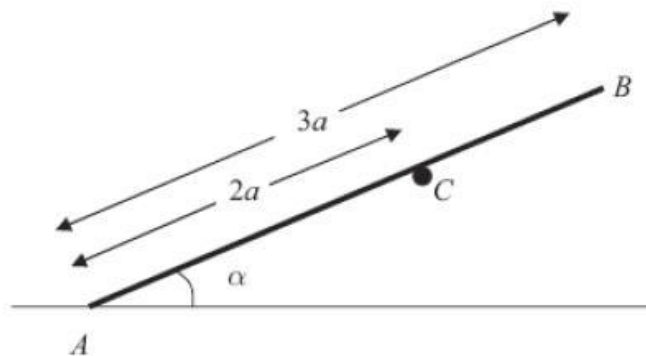


Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB . The rod has length $3a$ and weight W and rests on the peg at C , where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4} (4 - 3 \cos^2 \alpha) W$.

(6)

Given that the rod is in limiting equilibrium and that $\cos \alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

(5)

(Total 11 marks)

Q5.

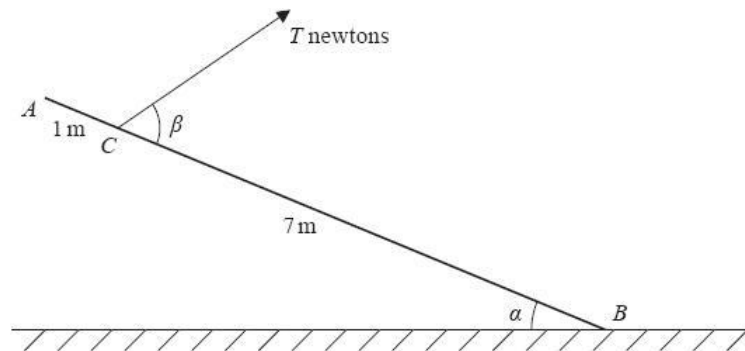


Figure 3

A uniform rod AB , of mass 5 kg and length 8 m, has its end B resting on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a rope attached to the rod at C . The distance $AC = 1$ m. The rope is in the same vertical plane as the rod. The angle between the rope and the rod is β and the tension in the rope is T newtons, as shown in Figure 3. The coefficient of friction between the rod and the ground is $\frac{2}{3}$. The vertical component of the force exerted on the rod at B by the ground is R newtons.

(a) Find the value of R .

(6)

(b) Find the size of angle β .

(5)

(Total for question = 11 marks)

Q6.

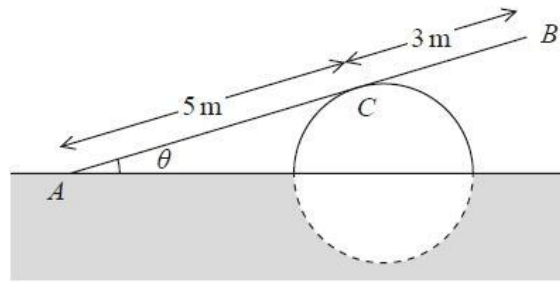


Figure 2

A ramp, AB , of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A .

The point of contact between the ramp and the drum is C , where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum,

at an angle θ to the horizontal, where $\tan \theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A .

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B ,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C .

(1)

(Total for question = 11 marks)

Q7.

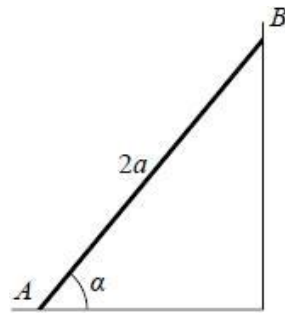


Figure 1

A uniform ladder AB , of length $2a$ and weight W , has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.

To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A , towards the wall.

The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an

angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W , the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for question = 13 marks)

Q8.

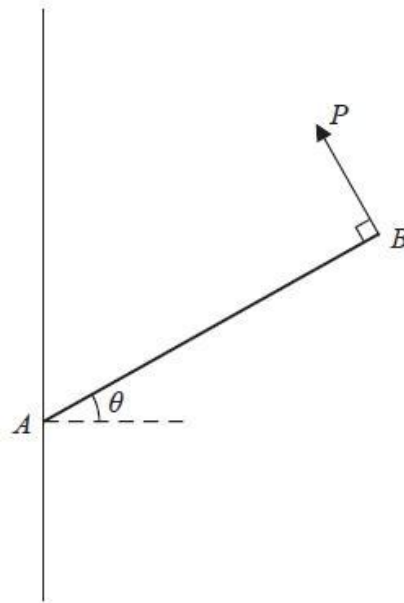


Figure 3

A uniform rod AB of weight W has its end A freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle θ to the horizontal, by a force of magnitude P . The force acts perpendicular to the rod at B and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at A is Y .

(a) Show that $Y = \frac{W}{2} (2 - \cos^2 \theta)$.

(6)

Given that $\theta = 45^\circ$

(b) find the magnitude of the force exerted on the rod by the wall at A , giving your answer in terms of W .

(6)

(Total 12 marks)

Q9.

A ladder AB , of weight W and length $2l$, has one end A resting on rough horizontal ground. The other end B rests against a rough vertical wall. The coefficient of friction between the

ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ . Friction is limiting at both A and B . The ladder is at an angle θ to the ground,

where $\tan \theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

Find the value of μ .

(9)

(Total for question = 9 marks)

Q10.

A rough circular cylinder of radius $4a$ is fixed to a rough horizontal plane with its axis horizontal. A uniform rod AB , of weight W and length $6a\sqrt{3}$, rests with its lower end A on the plane and a point C of the rod against the cylinder. The vertical plane through the rod is perpendicular to the axis of the cylinder. The rod is inclined at 60° to the horizontal, as shown in Figure 1.

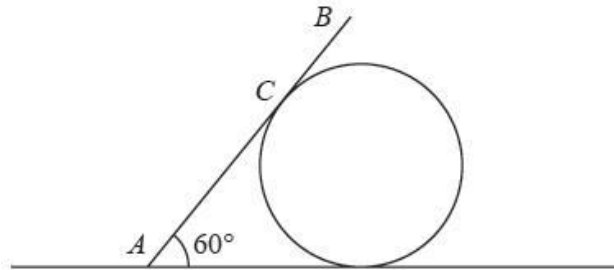


Figure 1

(a) Show that $AC = 4a\sqrt{3}$

(2)

The coefficient of friction between the rod and the cylinder is $\frac{\sqrt{3}}{3}$ and the coefficient of friction between the rod and the plane is μ . Given that friction is limiting at both A and C ,

(b) find the value of μ .

(9)

(Total 11 marks)

Q11.

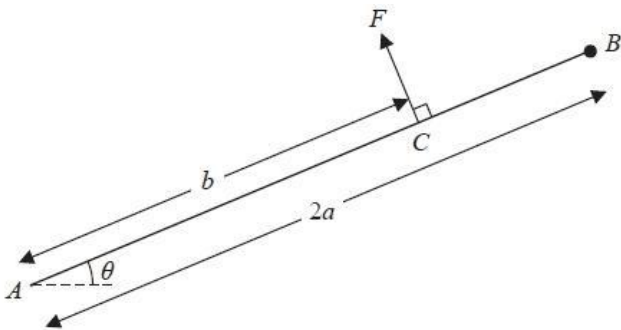


Figure 3

A uniform rod AB , of mass m and length $2a$, is freely hinged to a fixed point A . A particle of mass m is attached to the rod at B . The rod is held in equilibrium at an angle θ to the horizontal by a force of magnitude F acting at the point C on the rod, where $AC = b$, as shown in Figure 3. The force at C acts at right angles to AB and in the vertical plane containing AB .

(a) Show that $F = \frac{3amg \cos \theta}{b}$. (4)

(b) Find, in terms of a , b , g , m and θ ,
 (i) the horizontal component of the force acting on the rod at A ,
 (ii) the vertical component of the force acting on the rod at A . (5)

Given that the force acting on the rod at A acts along the rod,

(c) find the value of $\frac{a}{b}$ (4)

(Total 13 marks)

Q12.

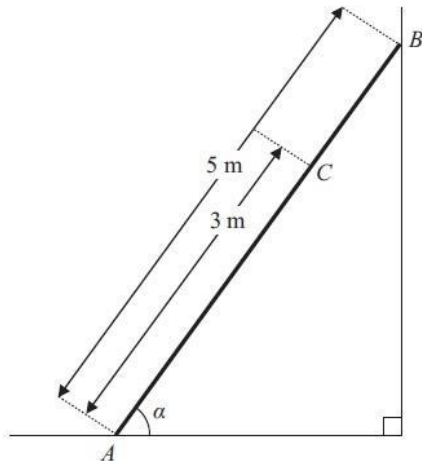


Figure 1

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ . A woman of mass 60 kg stands on the ladder at the point C , where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ .

(9)

(Total 9 marks)

Q13.

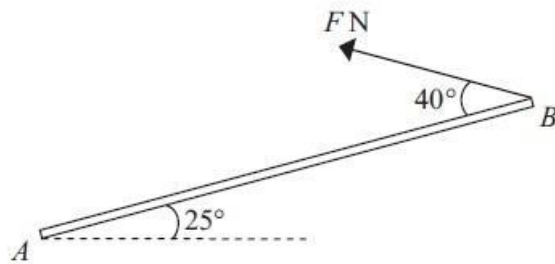


Figure 1

A uniform rod AB , of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B . The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(a) Find the value of F .

(4)

(b) Find the magnitude and direction of the vertical component of the force acting on the rod at A .

(4)

(Total 8 marks)

Q14.

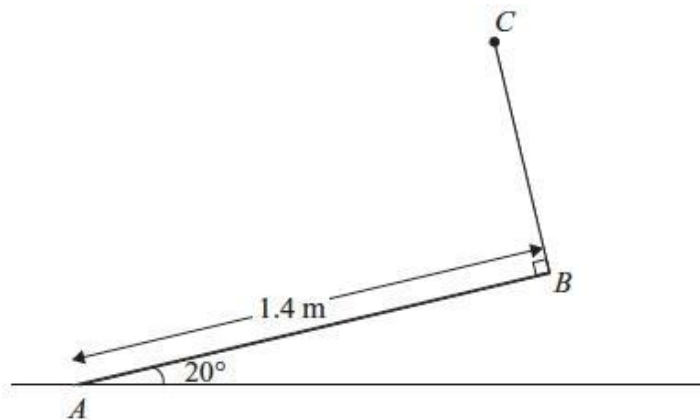


Figure 2

A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C . The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

(Total 11 marks)

Q15.

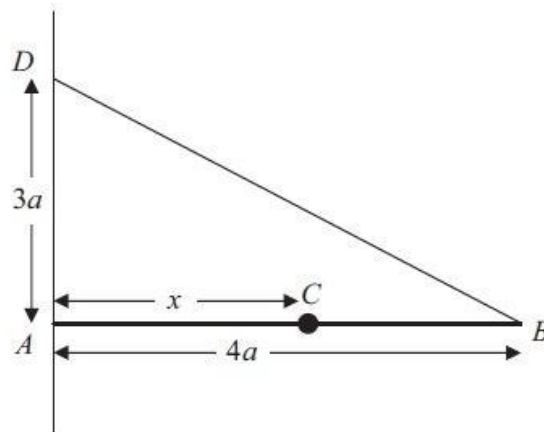


Figure 3

A uniform rod AB , of mass $3m$ and length $4a$, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A , where $AD = 3a$. A particle of mass $3m$ is attached to the rod at C , where $AC = x$. The rod is in equilibrium in a vertical plane perpendicular to the

wall as shown in Figure 3. The tension in the string is $\frac{25}{4}mg$.

Show that

(a) $x = 3a$,

(5)

(b) the horizontal component of the force exerted by the wall on the rod has magnitude $5mg$.

(3)

The coefficient of friction between the wall and the rod is μ . Given that the rod is about to slip,

(c) find the value of μ .

(5)

(Total 13 marks)

Q16.

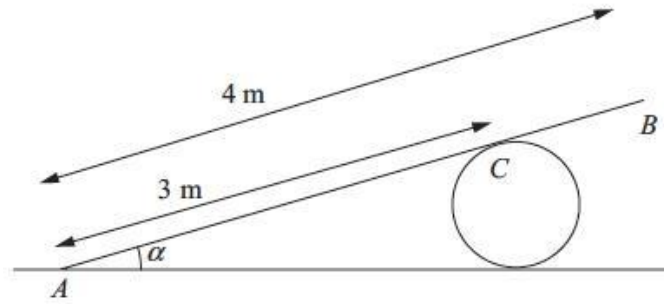


Figure 1

A uniform plank AB , of weight 100 N and length 4 m , rests in equilibrium with the end A on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is C , where $AC = 3\text{ m}$, as shown in Figure 1. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin \alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ . Modelling the plank as a rod, find the least possible value of μ .

(10)

(Total 10 marks)

Q17.

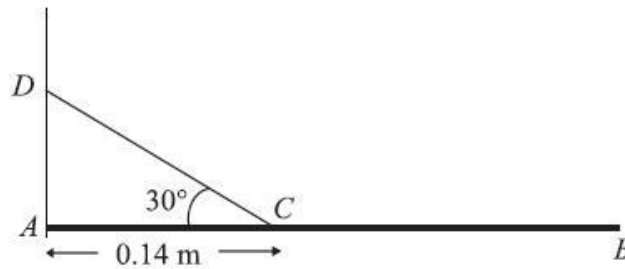


Figure 3

A uniform beam AB of mass 2 kg is freely hinged at one end A to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point C on the beam, where $AC = 0.14$ m. The rope is attached to the point D on the wall vertically above A , where $\angle ACD = 30^\circ$, as shown in Figure 3. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

Find

(a) the length of AB ,

(4)

(b) the magnitude of the resultant reaction of the hinge on the beam at A .

(5)

(Total 9 marks)

Q18.

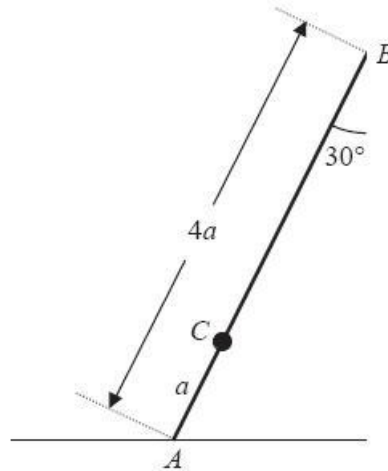


Figure 2

A ladder AB , of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C , where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

(Total 10 marks)

Q19.

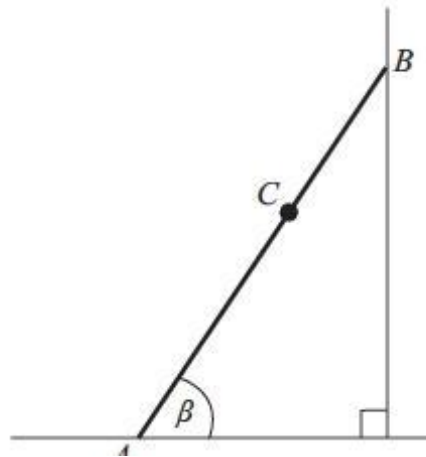


Figure 1

Figure 1 shows a ladder AB , of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(a) Find the magnitude of the frictional force of the ground on the ladder.

(3)

(b) Find, to the nearest degree, the value of β .

(6)

(c) State how you have used the modelling assumption that Reece is a particle.

(1)

(Total 10 marks)

Q20.

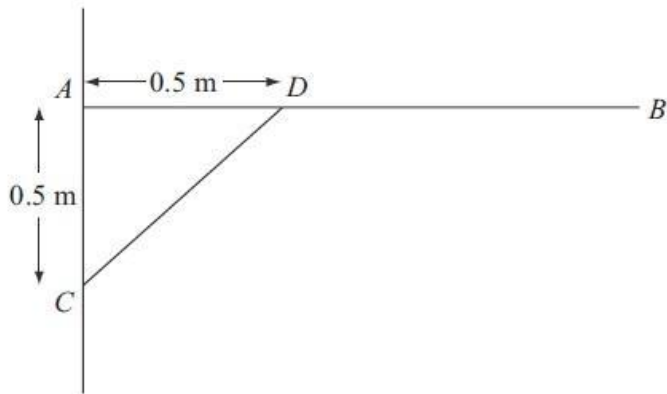


Figure 1

A uniform rod AB , of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at A . The rod is held in equilibrium in a horizontal position by a light strut CD as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end C of the strut is freely jointed to the wall at a point 0.5 m vertically below A . The end D is freely jointed to the rod so that AD is 0.5 m.

(a) Find the thrust in CD .

(4)

(b) Find the magnitude and direction of the force exerted on the rod AB at A .

(7)

(Total 11 marks)

Q21.

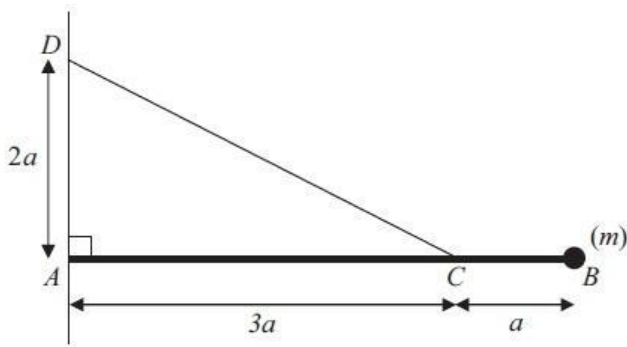


Figure 2

Figure 2 shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B . One end of a light inextensible string is attached to the rod at C , where $AC = 3a$. The other end of the string is attached to the wall at D , where $AD = 2a$ and D is vertically above A . The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T .

(a) Show that $T = mg\sqrt{13}$.

(5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B . The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leq \frac{5}{2}m$.

(3)

(Total 8 marks)