Resources

Bauer-Plehn-DM: https://arxiv.org/abs/1705.01987 -- first section.
Blas-DM-Intro:
https://conference.ippp.dur.ac.uk/event/785/attachments/3688/4159/Lectures_DM.pdf
Bousso-TASI-CC: https://arxiv.org/abs/0708.4231
PDGCosmo: https://pdg.lbl.gov/2022/reviews/rpp2022-rev-bbang-cosmology.pdf
Reed-DM-dens: https://arxiv.org/abs/1003.3101
TongLecsCosmo: http://www.damtp.cam.ac.uk/user/tong/cosmo.html

Assignment 1: *Lightning review of the history of the universe.*

+ On a log-log scale, plot the densities of radiation, baryonic matter, dark matter and vacuum energy as a function of inverse scale factor, starting with a value of $R^{-1} = 10^{10}$ (the nucleosynthesis era) and ending at $R^{-1} = 1$. Use the current ($R = 1$) values as boundary conditions. Use units of GeV/cm^3.
    ++ Mark the moment at which atomic recombination + photon decoupling occurs, i.e. the point at which the cosmic microwave background is born. How close is this to the moment of matter-radiation equality? Comment on the significance of this and the world is free to whom.
    ++ How close is the current epoch to the moment at which the densities of vacuum energy density and matter are equal? What's weird about that? (Hint: extend the curves to the right.) What has this weirdness got to do with the "cosmological constant problem"? (See Assignment 2)

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Assignment 2: *The cosmological constant problem*

+ Express kg, m, s, Joule in "natural units" by setting hbar = c = 1.
+ What is the electron mass in natural units? What length scale does it correspond to in cm? (What is its Compton wavelength?)
  ++ What is the Z boson mass? This is roughly the electroweak scale. What is its length scale? Also, obtain the electroweak scale from the value of Fermi's constant.
  ++ What is Newton's gravitational constant in natural units? Derive the Planck scale from that. What is the length scale? Below these lengths our concepts of spacetime and fields break down.

+ Roughly speaking, the vacuum energy density is sourced by the energy of quantum fields vibrating down some minimum wavelength over a volume set by that wavelength. What would be the universe's vacuum energy density if physics is only valid down to length scale = electron Compton wavelength? The electroweak length scale? The Planck length scale? Express in units of GeV/cc and calculate the discrepancy with respect to what you plotted in Assignment I.

+ Comment on the "cosmological constant problem" in the context of these questions:
  ++ If the actual vacuum energy were the 3 values derived above, what would be the size of the universe, assuming no other form of energy?
  ++ If you had these 3 values but with a negative sign, what would have been the fate of the universe? Quantify.

++ Compare those 3 values with the binding energy of a typical galaxy.
++ What is the relation of the cosmological constant problem to the weirdness mentioned in Assignment 1?

You may use Bousso-TASI-CC to answer these questions.

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Assignment 3: *Dark matter: broad character*
+ What do "dark" and "matter" mean in the context of dark matter? How exactly is dark matter different from dark energy?
+ Provide and explain at least 5 pieces of evidence for DM. Show plots where possible.
++ Can we modify gravity to explain these "evidences" instead of introducing an unseen substance like DM? Why or why not?
+ How do we know DM is not made of lost socks? (i.e. baryons)
+ Why must most DM be cold/non-relativistic? What fraction is allowed to be warm?
++ Can neutrinos be the DM?

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Assignment 4: *Dark matter: galactic character*

+ What is the DM density near the Sun and how do we know that?
+ What is the total (visible + dark) mass of the Milky Way? What fraction is the visible mass?
++ Estimate the escape velocity (of DM and anything else) of the Milky Way from the total mass.
++ Using the mass enclosed within the galactocentric radius of the Sun, calculate the typical velocity of objects at the solar position. This is the velocity of the Sun as well as DM in the Galactic rest frame.
++ Compare the MW escape velocity with the DM velocity derived above. Why are these two numbers close to each other?

+ What are the smallest systems known to contain dark matter?
+ Calculate the range of masses DM could span. What sets this range?
++ Is the range the same for bosons and fermions? If the same, why? If not, what sets the range for either?
++ In what range does DM behave like a wave? Like an elementary particle?
++ Like a composite particle?
++ Like a macroscopic body, e.g., a black hole or a dark star?
+ Is dark matter smoothly distributed like flour? Or does it come in lumps? Alternatively: how well do we know the matter power spectrum?
+ Why does cosmic DM come as spheroidal halos? Can it form disks like the visible parts of galaxies [e.g. here, here]?

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Assignment 5: *Dark matter: microscopic character*
+ Could DM interact with itself? How strongly?
+ Could DM interact with visible matter?
+ When should DM have come into existence? What is the minimum lifetime of DM? Maximum? (What are the limits?)
+ If DM is a particle, why should it be long-lived or stable? Recall that most SM particles decay away rapidly.

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