**Roll No……..** **Total No. of Printed Pages: 2**
**Total No. of Questions: [09]**

**B.Sc. (Hons.) Math (Semester – 5th)**
**MECHANICS**
**Subject Code: BMATS1523**
**Paper ID: 22131222**

**Time: 03 Hours** **Maximum Marks: 60**

**Instruction for candidates:**

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

**Section – A** **(2 marks each)**

Q1. Attempt the following:

a. What are generalized coordinates? What is the advantage of using them?
b. For a Non conservative system obtain Lagrange's equation.
c. A cylinder of radius $a$ and mass $m$ rolls down an inclined plane making an angle θ with the horizontal. Set up the Lagrangian and find the equation of motion.
d. What is cyclic or ignorable coordinate?
e. Prove that Hamiltonian H is a conservative system is equal to the total energy of the system.
f. Prove that the generalized momentum conjugate to cyclic coordinate is conserved
g. A particle of mass m moves under the action of central force whose potential is $(r) = Kmr^3$ $(K > 0)$, then for what kinetic energy and angular momentum will the orbit be a circle of radius $R$ about the origin ?
h. Determine the differential scattering cross-section and the total scattering cross-section for the scattering of a particle by a rigid elastic sphere.
i. By Applying variational principle, find the equation of one dimensional harmonic oscillator.
j. Obtain the Euler-Lagrange Differential equation by variational method.

**Section – B** **(5 marks each)**

Q2. Derive Hamilton's canonical equations of motion. Obtain Hamilton's equation of motion for a particle moving in a central force field.

Q3. Show that both $H$ and $E$ are constants of motion but they are not equal to each other. Explain this inequality. Discuss the physical significance of $E = H$.

Q.4 Use Lagrange's equations to find the equation of motion of a compound pendulum in a vertical plane about a fixed horizontal axis. Hence find the period of small amplitude oscillations of the compound pendulum

Q5. What are Kepler's laws of planetary motion? Give the proof of Kepler's laws of planetary motion and hence deduce that the areal velocity is constant.

Q6. What is Lagrange's method of undetermined multiplier? Explain with the help of Example.

**Section – C** **(10 marks each)**

Q7. What is Hamiltonian function? Explain its physical significance. Prove that the Hamiltonian $H$ of a conservative system is equal to the total energy of the system

Q8. (a) State and prove Virial theorem.

(b) When force is derivable from a potential, $V \propto r^{n+1}$, show that the average kinetic energy equals $\left(\frac{n+1}{2}\right)$ times the average potential energy.

Q9. Obtain the differential equation for a particle undergoing a central force motion and use is to verify Kepler's laws of planetary motion.