Westerville City Schools
Algebra 2 Essentials
Course of Study
Course Number: MA307

Course Description:

Algebra 2 Essentials continues the expansion of students' algebra skills to structure their understanding of real world applications. Building on work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful and logical subject at makes use of their ability to make sense of problem situations.

Recommended Grade Level*:* 11 - 12

Course Length: one period; full year

Credits: 1.0

Course Weighting: 1.0

Course Rationale

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful and logical subject that makes use of their ability to make sense of problem situations.

Considerations for Cultural Relevancy, Inclusivity, and Diversity:

Where possible teachers will create opportunities to incorporate the histories, values, beliefs and perspectives of people from different cultural backgrounds to meet the needs of all learners. Strategies for meeting the needs of all learners including gifted students, English Language Learners and students with disabilities can be found at this ODEW site.

Considerations for Intervention and Acceleration:

This rigorous and highly relevant curriculum is built upon high quality, research-based instructional strategies.Teachers may need to provide targeted Tier II support (e.g., remediation of particular skills and concepts, as well as scaffolded or supplemental instruction) beyond the Tier I level of universal instruction to underachieving students. Intensive and individualized Tier III instruction (e.g., skill-specific intervention, one-on-one support).

Primary Resource Recommendation:

Text: Big Ideas MATH: Algebra 2 ISBN: 978-1-60840-867-2
Big Ideas Learning,

Other Resources:

Content Standards by Topic

Scope and Sequence:

Topic 1: Solving Equations, Factoring, and Absolute Value

Content Standards	A-CED.1. Create equations and inequalities in one variable and use them to solve problems.*Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y=f(x). F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret Standards that use function notation in terms of a context
Essential Questions	1. Given a domain of input values, how can you find the range of output values? How can you determine if a relation is a function? 2. What are the similarities and differences between solving equations and inequalities? How do you choose the appropriate solution set for an inequality? What are some of the most commonly used formulas? 3. When is using an equation an appropriate method to solve a problem? Why is it critical to keep the equation balanced while solving? How do you isolate/solve for the variable in an equation?
Enduring Understanding	How to represent multiple solutions on a number line How to express domain and range of functions/relations using inequalities notation and interval notation.
Key Concepts/Vocabulary	Balance, equivalence, solution, inverse operation, linear, exponential, equation, inequality, constraint, domain, formula, domain, range, function, relation, input, output, element, function notation
Learning Targets	I can: ● solve multistep equations and inequalities. ● solve absolute value equations and express solutions using inequalities.

	● identify domain and range of functions and relations and express in inequality and interval notations.
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision.
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 2: Quadratics

Content Standards	F.IF.7c Graph linear and quadratic functions and show intercepts, maxima, and minima. F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.
Essential Questions	1. Which real world situations can be modeled using quadratic equations? 2. What methods can be used to solve quadratic equations? 3. How does knowledge of real numbers help when working with complex numbers?
Enduring Understanding	The family of quadratic functions models certain situations where the rate of change is NOT CONSTANT. These functions are graphed by a symmetric curve with a highest or lowest point corresponding to an absolute maximum or minimum value. Operations and properties of the real number system can be extended to situations involving complex numbers, which have many useful applications when working with quadratic functions.
Key Concepts/Vocabulary	Quadratic function, parabola, vertex, axis of symmetry, standard form, graphing form, orientation, horizontal shift, vertical shift, parent function, complex numbers
Learning Targets	I can: ● define, identify, and graph quadratic functions. ● solve quadratic equations by factoring, completing the square, taking square roots, and the quadratic formula. ● convert between standard form and vertex form. ● determine the quadratic function given its graph. ● identify *i* as the imaginary unit. ● combine complex numbers using addition, subtraction, multiplication and division. ● use quadratic functions to model real-world data. ● use quadratic functions to analyze and predict real-world situations. ● use the most efficient method to solve a given quadratic equation
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Model with mathematics. Look for and make use of structure. Look for and express regularity in repeated reasoning.

	Use appropriate tools strategically. Construct viable arguements and critique the reasoning of others
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 3: Parent Functions

Content Standards	A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F-BF.3. Identify the effect on the graph of replacing f(x) byf(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F-IF.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
Essential Questions	1. As I investigate functions, am I analyzing the function thoroughly and clearly communicating my reasoning to others? 2. What are the key components of a function? How are these represented in the graph and the table? 3. What can be determined about a real-life situation by examining the table and the graph? 4. What are the various ways to represent a function? 5. What are the key components of a function that allow us to create its graph? 6. What are the advantages to writing a function in multiple forms, such as graphing versus standard form? 7. What causes a function to transform from its parent function? 8. What makes a function odd, even, or neither? 9. What operations can be used to create new functions?
Enduring Understanding	A graph and/or table can reveal key aspects of a contextual situation. Mathematical relationships represented in different forms can still be compared. Writing a function in various forms can reveal additional information about the relationship. Mathematical operations affect a function in predictable ways.
Key Concepts/Vocabulary	function, transformation, translate, stretch, shrink, even function, odd function, symmetry, domain, range, parent function, linear function, vertex, quadratic function, root function, piecewise function, absolute value function, rational function, exponential function, asymptote, slope-intercept form, increase, decrease, maximum, minimum
Learning Targets	I can:

	● sketch complete graphs of functions and identify possible inputs, outputs, and key points for describing those graphs. ● use a graphing calculator to develop multiple representations of the parent functions. ● identify the domain and range of functions while improving their graphing calculator skills. ● develop an understanding of what it means to investigate a function. ● identify characteristics of linear functions and how they relate to a table or a graph. ● transform the graphs of $y = b^x$, $y=\frac{1}{x}$ $y =\|x\|$, $y=\sqrt{x,}$ $y=x^2,$ and $y = x^3$. ● identify the point (h, k) for graphs of parabolic, hyperbolic, cubic, absolute value, exponential, and square root functions and relate it to the point-slope form of a line.
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Look for and make use of structure. Look for and express regularity in repeated reasoning.
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and response to exceptional students of all levels.

Topic 4: Systems of Equations

Content Standards	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Essential Questions	1. What does the solution(s) to a system of equations represent? 2. How can systems be used to solve real world applications? 3. What are the advantages and disadvantages of solving a system algebracially versus graphically?
Enduring Understanding	Systems of linear equations can be used to solve real world applications and systems can be solved using elimination, graphing and substituion.
Key Concepts/Vocabulary	Solutions, elimination, substitution, inequality, intersection, boundary point.
	I can:

Learning Targets	● solve a system of equations using graphing, substitution and elimination. ● identify the different types of solutions to a system of equations. ● write system of equations to solve real world problems. ● graph a solution to system of inequalities.
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Make sense of problems and perservere in solving them. Reason abstractly and quantatively Model with mathematics Use appropriate tools strategically Construct viable arguements and critique the reasoning of others.
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 5: Radical Function

Content Standards	A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents
Essential Questions	1. How do power and radical equations model real-world problems? 2. How are expressions involving radicals and exponents related? 3. What strategies can be used to solve radical equations? 4. How can you use points on a graph to determine the radical equation?
Enduring Understanding	You can write a radical expressions in an equivalent form using a fractional exponent. Solving a radical equation may require the use of an inverse operation which may introduce extraneous solutions.
Key Concepts/Vocabulary	Radical, exponent, extraneous solution, domain and range, inverse, rational exponent
Learning Targets	I can: ● simplify an expression involving exponents. ● write a radical expression using rational exponents ● write a radical expression from a rational exponent. ● simplify and solve a radical equation ● determine whether the equation has an extraneous solution. ● use the radical equation to sketch a graph and identify its characteristics.
Assessments	Unit tests and quizzes
	Make sense of problems and perserve in solving them.

Instructional Strategies and Resources	Reason abstractly and quantatively Model with mathematics Use appropriate tools strategically
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 6: Rational Functions

Content Standards	A.APR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
Essential Questions	1) How can you simplify a rational expression? 2) How can you determine if a rational expression is equivalent form? 3) What kind of asymptotes are possible for rational functions? 4) What operations or methods can we use to combine rational expressions?
Enduring Understanding	A rational function is a ratio of polynomial functions. To operate rational expressions, you can use much of what you know about operating fractions. When solving an equation involving rational expressions, multiplying by the common denominator can results in extraneous solutions.
Key Concepts/Vocabulary	Rational expression, asymptote, extraneous solution, common denominator
Learning Targets	I can: ● use solving techniques (ex: factoring) to simplfy rational expressions. ● use properties of fractions to combine rational expressions. ● identify where asymptotes will occur in a rational function before graphing. ● solve a rational equation and identify extraneous solutions ● sketch a graph of a rational function and identify key components of the graph. ie. asymptotes, domain and range, intercepts.
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Make sense of problems and perservere in solving them. Reason abstractly and quantatively Look for and make use of structure Use appropriate tools strategically
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 7: Exponential Functions

Content Standards	F.BF.1 Write a function that describes a relationship between two quantities. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology
Essential Questions	1) What characterizes exponential growth and decay? 2) What are real world models of exponential growth and decay? 3) How can one differentiate between an exponential model from a linear model given a real world set of data?
Enduring Understanding	Exponential functions can model real life situations involving growth or decay using different representations to understand the situation.
Key Concepts/Vocabulary	Exponential function, growth, decay, multiplier, initial value, appreciation, depreciation
Learning Targets	I can: ● use different representations to graph an exponential function ● identify the growth (multiplier) and intial value in an exponential equation and use this information to graph the equation. ● write an exponential equation for a given situation and use it to solve problems. ● differentiate whether or not the given situation is linear or exponential.
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Make sense of problems and perservere in solving them. Reason abstractly and quantatively Use appropriate tools strategically Model with mathematics Attend to precision
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 8: Logarithmic and Inverse Functions

Content Standards	F.BF.1 Write a function that describes a relationship between two quantities. F.BF.4 Find inverse functions. F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or *e*; evaluate the logarithm using technology.
Essential Questions	1. How can the definition of a logarithm be used to write an exponential equation in logarithmic form? 2. How can the definition of a logarithm be used to write a logarithmic equation in exponential form? 3. How can the properties of logarithms be used to expand or condense expressions?

	4. What solving strategies can be used to solve equations that contain logarithms? 5. What strategies can be used to check for extraenous solutions to a logarithmic equation?
Enduring Understanding	The exponential function y = b^x is one-to-one, so its inverse x =b^y is a function. To express "y as a function of x" for the inverse you write y =log $_b x$. The propduct, quoitent and power properties of logarithms correspond to the properties of exponents. Logarithms can be used to solve exponential equations; and conversely, exponents can be used to solve logarithmic equations. The function y =e^x and y = ln x are inverse functions.
Key Concepts/Vocabulary	Logarithms, exponents, base, inverse, natural logarithm, exponential, function, extraneous solution, asypmtote
Learning Targets	I can: ● simplify or expand logarithmic expressions using the properties of logarithms. ● solve an equation using either logarithmic or exponential properties ● find the inverse of a given function ● verify that two functions are inverses by graphing
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Topic 9: Right Angle Trigonometry

Content Standards	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Essential Questions	1. What is the definition of the three basic trigonometric functions in terms of the sides of a right triangle? 2. How does tangent relate to sine and cosine? 3. How can the three basic trigonometric functions be used to solve right triangles? 4. How can inverse trigonometric functions be used to calculate unknown angles in a right triangle?

	5. How can special right triangle (45-45-90 and 30-60-90) be used to find missing sides? 6. How can using the Law of Sines and Law of Cosines be used to solve for missing sides and angles of a triangle?
Enduring Understanding	The properties of right triangles and trigonometry can be used to solve mathematical and real world problems. The Law of Sines and Cosines can also be used to solve for the whole triangle that is not a right triangle.
Key Concepts/Vocabulary	Trigonometry, cosine, sine, tangent, Pythagorean Theorem, hypotenuse, leg, angle, Law of Cosine, Law of Sine, proportion, triangle
Learning Targets	I can: ● find the missing sides and angles within a right triangle using the Pythagorean Theorem and the cosine, sine, and tangent functions. ● find the missing sides and angles within non-right triangles using the Law of Cosines and the Law of Sines. ● find the missing sides of a right triangle using special right triangle.
Assessments	Unit tests and quizzes
Instructional Strategies and Resources	Make sense of problems and persevere in solving them. Model with mathematics. Use appropriate tools strategically. Look for and make use of structure.
Considerations for Intervention and Acceleration	Please see attached documents addressing the challenges and responses to exceptional students of all levels.

Table 1
Mathematics | Standards for Mathematical Practice
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically

proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They can justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents
such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the co

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a

graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems

coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Table 2

Universal Design for Learning, Ohio's Revised Standards and the Model Curriculum
What's the connection?

How Does Universal Design for Learning Apply to All Students?

Given the diversity of students in most classrooms today, teachers are challenged with ensuring that all students are able to access the content standards and demonstrate mastery of the skills and knowledge embedded in these standards. Many instructional methods and strategies are effective for a significant number of students in all classrooms and across all content areas, but teachers are often faced with finding specific strategies that address the unique needs of the students enrolled in their classrooms. UDL provides a framework for locating these resources by organizing them under three major principles.

Multiple Means of Representation
Multiple Means of Action and Expression
Multiple Means of Engagement

Although these resources can be used to support all students, UDL also offers strategies and resources appropriate for those students identified with very specific needs including student with disabilities, gifted students, and English language learners.

UDL and Students with Disabilities	***UDL and Gifted Students***	***UDL and English Language Learner***
Students in Ohio can be identified with one of 13 different disability conditions. A significant amount of diversity, however, exists both within and between each of these disability categories. Students with disabilities can achieve at high levels when provided with instructional supports and accommodations, and when educated with students without disabilities to the maximum extent possible. UDL enables teachers to plan instruction for a wide range of learners. For example, a teacher may customize the display of information for a student with a visual impairment, or allow a student with a specific disability to express knowledge through the use of multiple media. See Table 3 for additional strategies and resources.	Ohio law establishes criteria for students to be identified as gifted in the areas of academic achievement, cognitive abilities, creative thinking and/or visual/performing arts. Gifted students may be served in the regular classroom through differentiation and/or in classes with other gifted students taught by a gifted intervention specialist. For gifted students, UDL helps to frame the differentiation that needs to take place in all academic settings in the area of the student's identification. See Table 4 for additional strategies and resources.	Ohio's English Language Learners (ELL) represent a variety of home/native languages, cultural backgrounds and levels of English proficiency. They may be refugees, or U.S. born, and they may have extensive formal school experiences or little/no prior schooling. Although ELLs have limited English proficiency, their native/home language skills and cultural experiences can be useful assets in the learning process. When teachers are aware of the background, needs and strengths of their students, and have an understanding of strategies and resources under the UDL framework, they can work together to help their ELLs access Ohio's revised standards. See Table 5 for additional strategies and resources.

Strategies for Diverse Learners Using the UDL Model

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Focus on Students with Disabilities

General Guidelines

• Students can be identified with one of 13 different disability conditions in Ohio; however, there is significant diversity both between and within these disability conditions. Eighty percent of students with disabilities do not have general cognitive performance deficits as a feature of their disabilities.

• Consult with the student's intervention specialist to review the student's strengths and instructional needs as outlined in the student's Individualized Education Program (IEP) and to determine the most effective ways to support the student in the classroom.

• Ensure that students with disabilities are receiving instruction based on grade-level content standards while also providing the necessary instructional supports and accommodations that will allow them to demonstrate their knowledge. For some students with significant cognitive disabilities, the standards may need to be extended or adjusted but these adjustments should still align with the goals of Ohio's revised standards.

• *Twice Exceptional* students are students identified as gifted who also have a disability condition. To meet the unique needs of a Twice Exceptional student, it is important for the classroom teacher to consult with both the student's intervention specialist and the gifted intervention specialist or coordinator to design the most appropriate instruction for the student. Learning experiences for Twice Exceptional students should address the student's disability and provide enhanced learning experiences in the student's areas of strength.

• Make use of technology and digital media to reduce barriers to learning, enhance instruction and provide additional methods for students with disabilities to demonstrate knowledge and skills. The purposeful use of technology is an essential component of college and career readiness for all students, including students with disabilities.

Multiple Means of Representation for Students with Disabilities
To provide students various ways of acquiring information and knowledge.

Examples:

• Present information using a rich mix of media in addition to text, such as video, digital books, audio, data displays, simulations and concept maps.

• The use of technology can improve instruction for many students, including those technologies that have been designed to assist students with specific disability conditions.
o Students with hearing impairments often benefit from closed captioning, automated speech-to-text applications, visual symbols and electronic sign-language dictionaries.

o Closed captioning is beneficial for students with reading disabilities, students with attention deficits and students who have difficulty filtering out background noise.

o Students with visual impairments often benefit from non-visual alternatives such as Braille, tactile graphics, physical models and screen readers that use synthesized speech to "speak" graphics and text aloud. Students with low vision can

benefit from visual display options such as screen magnification and the ability to adjust font size and contrast values of text to background.

o Due to deficits in reading fluency, students with reading disabilities, including dyslexia, often experience difficulty reading the large amount of text required for each course. As a result, they can benefit from supports such as textbooks on tape, video, audio, text-to-speech technology and additional time to complete assignments.

• Use both linguistic and non-linguistic methods to represent key vocabulary, labels, symbols and icons to support comprehension of concepts.

• Assist students with disabilities in identifying key ideas and critical information with tools such as graphic organizers, outlines and concept maps.

• Providing additional teacher support until a student can apply new skills independently, often referred to as scaffolding, is an effective teaching strategy designed to shift responsibility for learning from the teacher to the student. Beginning instruction at a student's current level of understanding and providing teacher support to assist the student in reaching the next level of mastery are particularly important for students with disabilities.

Multiple Means of Action and Expression
To provide options for demonstrating what students have learned.

Examples:

• For students with significant physical impairments that impact movement, such as cerebral palsy and muscular dystrophy, work with the student's intervention specialist to provide assistive technology that will ensure the curriculum is physically accessible to the student. Along with removing physical barriers within the school and classroom environment, students with physical impairments often benefit from pointing and typing aids, alternative keyboards, touch screens, and mobility and transportation aids.

• Provide multiple options for students to demonstrate knowledge and skills while maintaining the learning expectations of the grade-level content standards. The use of multiple media such as video, voiceover PowerPoint presentations, animations, visual art, photographs and storyboards can expand the opportunities for students with disabilities to demonstrate mastery of learning objectives. Many students who struggle with written expression are highly skilled at presenting their ideas orally.

• Many students with disabilities exhibit difficulty with writing, both the physical process of writing and the expression of ideas in a written format. These students may benefit from the use of portable word processors or composition tools that include features such as word prediction, text embedded prompts, auto correction and speech recognition programs that convert words spoken aloud to text.

Multiple Means of Engagement
To tap individual learners' interests, challenge them appropriately and motivate them to learn.

Examples:

• Provide choices to students whenever appropriate regarding how a learning objective can be achieved while still maintaining expectations aligned with grade-level content standards. Allowing students to pursue an area of interest can increase engagement in learning.

• Offer opportunities for students with disabilities to work in cooperative learning groups on authentic, project-based learning assignments and provide clear expectations for the group and each group member.

• Encourage students with disabilities to take ownership for their learning by assisting them in developing strategies for effective goal setting, reflection and self-monitoring of progress.

• Provide clear examples of expected products and give frequent constructive feedback to students with disabilities to encourage mastery of learning goals. Many students with disabilities lack confidence in their ability to succeed in school. Teacher feedback that recognizes improvement and emphasizes the importance of effort and persistence can be highly motivating.

Table 4

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Strategies for Diverse Learners Using the UDL Model
Focus on Gifted Learners

General Guidelines

• Look for strengths in students particularly when exhibited in multiple areas such as academic knowledge, skills or applications and/or social/emotional skills.

• Find out the gifted student's areas of interest and preferred modes of learning. Gifted students may be highly focused on a specific aspect of a content area and may be extremely strong in one mode of learning.

• Consult with a gifted intervention specialist or gifted coordinator about the characteristics and behaviors of gifted learners.

• Consult with a gifted intervention specialist or gifted coordinator and share ideas on how to support the students' learning in both the content classroom and the gifted classroom.

• Collaborate with gifted staff to write a Written Education Plan (WEP) for the gifted student. The WEP specifies the setting in which the differentiation will take place. The WEP identifies yearly content, process or product goals for the gifted student. It also includes learning outcomes the student can expect to achieve and how the outcomes will be measured. A WEP is required in order to report gifted students as served in the Educational Management Information System (EMIS).

Multiple Means of Representation
To provide students various ways of acquiring information and knowledge.

Examples:

• Pre-assess student knowledge and skills for the lesson or unit.

• Give gifted students a choice of ways to approach acquiring information and knowledge. Gifted students are often able to work with abstract ideas and make connections easily.

• Provide options for resources in addition to textbooks. Gifted students often have broad knowledge and a desire to learn in great depth.

• Accommodate gifted students' needs for flexible grouping experiences. Gifted students may need to work individually or in a small group if they have an increased knowledge of a topic or need a faster learning pace.

• Work with the gifted intervention specialist or gifted coordinator to obtain additional resources or options.

Multiple Means of Action and Expression
To provide options for students to demonstrate what they have learned.

Examples:

• Offer layers of scaffolding so that gifted students with low need for additional supports have more flexibility. Tiered options, tic-tac-toe menus, compacting the curriculum and other differentiation tools can be used. Anchor activities or projects can assist with classroom management.

• Provide multiple ways for students to demonstrate what they know. Products may include visual, auditory and tactile/kinesthetic modalities.

• Provide models for superior products for the lesson or unit.

• Collaborate with the gifted intervention specialist or gifted coordinator to provide follow-up or extensions.

Multiple Means of Engagement
To tap individual learner's interests, challenge them appropriately and motivate them to learn.

Examples:

• Vary the pace at which content is delivered.

• Provide problem-solving activities to motivate gifted students. They often enjoy finding, solving and acting on problems.

• Allow for integration of issues, themes and concepts within the discipline and across content areas.

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Strategies for Diverse Learners Using the UDL Model
Focus on English Language Learners (ELL)

General Guidelines

• Find out about the cultural background of students and consider how their cultural knowledge and experiences can serve as teaching/learning resources (for themselves and for other students).

• Find out the level of students' heritage (native or home) language skills in all four communication modes (listening, speaking, reading and writing), and consider how these skills can serve as teaching/learning resources (for themselves and for other students).

• Find out the students' current level of English proficiency in all four communication modes (listening, speaking, reading and writing) based on the students' most recent English language proficiency assessment, and plan for strategies to help the students practice the next level of proficiency benchmarks (based on the Ohio English Language Proficiency Standards).

• Consult with the students' English as a Second Language (ESL) teachers or tutors, and share ideas on how to support the students' learning in both the content classroom and the ESL classroom.

Multiple Means of Representation
To provide students various ways of acquiring information and knowledge.

Examples:

• Pre-lesson strategies:

o Give the student a written summary or outline of the activity to help support the student's comprehension of information provided orally.

o Give parents a written overview of the learning goals and activities for the following week. If possible and appropriate, have the information translated into the home language if it is not English (use a school or community resource person to assist with the translation). Ask the parents to discuss the upcoming activity with their child at home.

o Identify vocabulary that the student may not know before the activity begins. Have the student preview key vocabulary by using English-English or bilingual dictionary/website (if the student is literate in his or her native language) to gain understanding of the meaning of the word in English.

o Provide a copy/overview of the unit/lesson to the ESL teacher, who can then provide follow-up practice and extension.

Table 5

• When giving oral information or instructions, write key words and phrases on the blackboard.
• When possible, demonstrate or model the tasks to be learned or practiced.
• Give opportunities for a study partner to help explain the major concepts of the activity/lesson.
• If there are two or more students of the same home/native language, give them time to work together and help each other understand the content being covered in class
• Give the student ongoing opportunities during the lesson to show understanding with a thumbs-up signal, or lack of understanding with a thumbs-down signal.

Multiple Means of Action and Expression
To provide options for students to demonstrate what they have learned.

Examples:

Multiple Means of Engagement
To tap individual learners' interests, challenge them appropriately and motivate them to learn.

Examples:

• Have the student add newly learned words to a homemade bilingual dictionary. Parents may be able to help with this task. At the end of the year, the student can donate the dictionary to the school library to serve as a resource for future students of the same language background.

• Pair the student with an English-speaking peer to create a *Know*, *Want to know*, and *Learned* (KWL) chart for the ELL student prior to the introduction of a new topic.

• Give ELL students the opportunity to teach the class key content-related vocabulary in the student's home/native language.
• Have the ELL student write questions in his or her native language (if literate in that language), and work with a peer to translate into English using a bilingual dictionary or translation website (e.g., *Google Translate, Babel Fish*).
• In pair or small-group activities, give the LEP student specific roles that have reduced language demands (e.g., helping to prepare graphics for a group oral presentation).

• Have the ELL student work with a study partner to develop and share a graphic display (e.g., Venn diagram, chart, timeline) of the concepts presented in the activity/lesson.

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Acknowledgements

It is through the hard work and dedication of High School Mathematics team that the Westerville City Schools' Algebra 2 Alternative Course of Study is presented to the Board of Education. Sincere appreciation is extended to the following individuals for their assistance and expertise.

Central High School	North High School	South High School	District
David Ray Stephanie Shaw	Jon Boyd Michele Burt	Andrea Gaul Katrina Taylor	Michael Huler