Roll No…………………….. **Total No. of Printed Pages: [1]**

Total No. of Questions: [09]

B.Sc. (Non-Medical) (Semester – 4th)

ALGEBRA-I

Subject Code: BSNMS1-406

Paper ID: [22131422]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) What is an Abelian group? Give example.

b) Define complex root of unity.

c) Show that if in a group G, $a^2 = e$ for every $a \in G$, then G is Abelian group.

d) Define cyclic subgroups.

e) Show that the union of two subgroups is a subgroup.

f) What is index of a subgroup?

g) Define quotient group with example.

h) Show that a cyclic group is abelian.

i) Show that every quotient group of a cyclic group is cyclic.

j) Write all permutation of $S = \{1, 2, 3\}$.

Section – B **(5 marks each)**

Q2. If G is an abelian group, then for all $a, b \in G$and for all integers n.

Show that $(ab)^n = a^n b^n$

Q3. If N is a normal subgroup of a group G. Show that $\frac{G}{N}$ is abelian iff $xyx^{-1}y^{-1} \in N$.

Q4. A sub group H of a group G is said to be normal iff $g^{-1}hg \in H \ \forall \ h \in H, \ g \in G$.

Q5. Show that every group of a prime order is cyclic.

Q6. Discuss the symmetry group of an equilateral triangle.

Section – C **(10 marks each)**

Q7. Show that the order of any subgroup of a finite group divides the order of group.

Q8. Show that any non-identity permutation of a finite set can be expressed uniquely as a product of disjoint cycles, each of length greater than 1.

Q9. Let H and K be any two subgroups of a group$G$, then HK is a subgroup of G iff $HK = KH$.