Physics Assignment: Harmonics and Beats

Student Learning Outcomes

1.1.3	Draw the graph of a function
3.5.1	Find the derivatives of the sine and cosine function
3.5.3	Calculate the higher-order derivatives of the sine and cosine
3.6.1	State the chain rule for the composition of two functions
3.6.3	Apply the chain rule and the product/quotient rules correctly in combination when both are necessary

1. Simple Harmonic Motion

Consider the sinusoidal function $s(t) = Acos(Bt)$. Suppose $s(t)$ describes displacement of an object at time t, where $|A|$ denotes the amplitude, $\frac{2/\pi}{|B|}$ is the period, and $\frac{|B|}{2/\pi}$ is the frequency or the number of cycles per unit of time.

- Compute the expression $s''(t) + s(t)$.
- For which value(s) of B would the expression $s''(t) + s(t)$ equal the zero function?
- Set B equal to one of the values that yields $s''(t) + s(t) = 0$ and evaluate A at a nonzero integer from your student ID. Create a graph of $s(t)$ versus t using an online graphing utility like Desmos.
- Determine the maximum displacement of an object, described by this specific form of $s(t)$.
- Find the period or the time required for one vibration.
- Calculate the frequency.
- Watch the following 15-minute video and make comments on your proposed sinusoidal function $s(t)$ describing simple harmonic motion: https://youtu.be/kLWXLbciobw

2. Damped Harmonic Motion

In damped harmonic motion, the displacement of an oscillating object from its resting position at time t can be described by $d(t) = Ae^{-Ct}cos(Bt)$, where C denotes a damping factor, $|A|$ is the initial displacement, and $\frac{2/\pi}{|B|}$ quantifies the period.

- Suppose $C = \frac{m}{10}$, where m is evaluated at a nonzero integer from your student ID. Use the same values of A and B that you employed in Simple Harmonic Motion to formulate a specific $d(t)$.
- Generate a graph of $d(t)$ versus t using an online graphing utility like Desmos.
- Compute $d''(t)$.
- Generate a graph of $d''(t)$ versus t using an online graphing utility like Desmos.
- How do the graphs of $d(t)$ and $d''(t)$ compare to one another?

3. Beats Phenomenon

An interference pattern between two sounds of fairly similar (not identical) frequencies is known as a beat. It is commonly understood as a periodic variation in volume defined by the difference between two frequencies.

Consider the sinusoidal function $I(t) = \frac{D}{U^2-D^2}cos(Dt) - \frac{D}{U^2-D^2}cos(Ut)$,
where the parameters U and D are constants such $D \neq U$.

- Compute the expression $I''(t) + UI(t)$ for any constants U and D that satisfy $D \neq U$.
- Evaluate the constant U at a nonzero integer of your student ID.
- Define $y(t) = \frac{D}{U^2-D^2}cos(Dt)$. Generate a graph of $y(t)$ versus t using an online graphing utility like Desmos where the constant D appears as a slider. For example, if $U = 2$, then $y(t) = \frac{D}{4-D^2}cos(Dt)$.
- Using the same value of U, define $z(t) = \frac{-D}{U^2-D^2}cos(Ut)$\$z(t) = \frac{-D}{U^2-D^2}\cos(Ut)\$. Generate a graph of $z(t)$ versus t using an online graphing utility like Desmos where D appears as a slider. For example, if $U = 2$, then $z(t) = \frac{-D}{4-D^2}cos(2t)$.

- Using the same value of U in an online graphing utility like Desmos, plot $I(t)$, $y(t)$ and $z(t)$ in the same graph with a slider for the parameter D. There are intervals where the graphs of y(t) and z(t) are in-phase (i.e., almost matching on top of each other), while in other subdomains they are out of phase. Identify such intervals in your graph.
- Watch the 12-minute video: https://youtu.be/Ca91iOVGd9A. Comment on some of the features discussed in the video as they apply to your graph. E.g., constructive interference, destructive interference, period, frequency, etc. Also, explore your graphs for various values of the parameter D in the slider.