

MTH401 Current final term Spring 2011 complete paper with MCQ's

The annihilator operator of the function $y = e^{-6x}$ is

$(D-6)^2$

$(D-6)^3$

$D-6$

$D+6$

After converting the given differential equation $4y'' + 64y = \sec 3x$ into standard form, the function $f(x)$ is

$(\sec 3x)/4$

$(\sec 3x)/64$

None of them

$\sec x$

Wronskian of the function $y_c = c_1 + c_2 \cos x + c_3 \sin x$ is

0

1

2

3

In the infinite series of $(x-a)$ which can be written as

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \dots$$

the number **a** is called the

Radius of power series

Centre of power series

Base of power series

None of them

Suppose that a power series $\sum_{n=1}^{\infty} a_n (x-a)^n$ is represented by a function “**f**” whose domain is the interval of the convergence of the power series. That function “**f**” is continuous, differentiable and integrable on

(a + R, a - R)

(R -a, R + a)

(a - R, a + R)

None of them

The interval of convergence for the function **secx** is

$(-\pi, +\pi)$

$(-\frac{\pi}{2}, \frac{\pi}{2})$

$(\frac{\pi}{2}, \pi)$

None of them

The solution of the linear first order differential equation $\frac{dy}{dx} - 2xy = 0$ is

$y = e^{x^2}$

$y = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

Both $y = e^{x^2}$ & $y = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

None of them

$$X = L_\gamma - \frac{1}{C_\gamma}$$

The quantity
Reactance of circuit
Impedance of circuit
Quasi of circuit
None of them

For the equation of free damped motion $\frac{d^2x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega^2 x = 0$ the roots are
 $m_1 = -\lambda + \sqrt{\lambda^2 - \omega^2}$ & $m_2 = -\lambda - \sqrt{\lambda^2 - \omega^2}$ If $\lambda^2 - \omega^2 > 0$ then system is said to be
 Under damped
 Over damped
 Critically damped
None of them

The time interval between two successive maxima of $x(t) = Ae^{-\lambda t} \sin[\sqrt{\omega^2 - \lambda^2} t + \phi]$ is called
 Phase period
 Quasi-period
 Both the period
 None of them

The given differential equation $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 0$ is
 Over damped
 Critically damped
 Under damped
 None of them

The standard unit for measurement of inductance is
 Volt

Ohms
Henry
None of them

Which of the rule in matrices under multiplication does not hold true?

Commutative law
Associative law
Identity law
None of them

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}$ & $B = \begin{bmatrix} x & y & z & a \\ p & q & r & b \\ l & m & n & o \end{bmatrix}$ then the order of *matirx* $A \times B$ is

2×4
 2×3
 3×3
None of them

The given system without the use of matrices $\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -7 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \end{pmatrix} \sin t$ is

$$\frac{dx}{dt} = 3x - 7y + 4\sin 2t; \quad \frac{dy}{dt} = x + y + 8\cos 2t$$

$$\frac{dx}{dt} = 3x - 7y + 4\sin t; \quad \frac{dy}{dt} = x + y + 8\cos t$$

$$\frac{dx}{dt} = 3x - 7y + 4\sin t; \quad \frac{dy}{dt} = x + y + 8\sin t$$

None of them

Suppose that $\{X_1, X_2, X_3, \dots, X_n\}$ is a set of n solutions vectors on an interval I , of a homogeneous system $X' = AX$. The set is said to be a fundamental set of solutions of the system on the interval I if the solution vectors are

Linearly dependent

Linearly independent

Homogeneous

None of them

The coefficient matrix of the following homogeneous system of differential equation

$$\frac{dx}{dt} = 3x + 2y, \quad \frac{dy}{dt} = x + 2y \text{ is}$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

None of them

The matrix $A = \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix}$ has an eigen value of multiplicity

1

2

3

4

$$A = \begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

The matrix A has eigen values $\lambda = -1, -1, 5$ where $\lambda = -1$ is a
Single root of A

triple root of A

double root of A

None of them

$$\begin{vmatrix} 4-\lambda & 1 & 0 \\ 0 & 4-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{vmatrix} = 0 \text{ gives}$$

$\lambda = 4$ of multiplicity of 1

$\lambda = 4$ of multiplicity of 2

$\lambda = 4$ of multiplicity of 3

None of them

For the system of differential equations $\frac{dy}{dt} = 2x$, $\frac{dx}{dt} = 3y$ the independent variable(s) is
(are)

x, t

y, t

x, y

t

If L denote the linear differential operators with constant coefficients, then $L_1L_4 - L_2L_3$ represents the

$$\begin{vmatrix} L_1 & L_2 \\ L_4 & L_3 \end{vmatrix}$$

$$\begin{vmatrix} L_1 & L_3 \\ L_4 & L_2 \end{vmatrix}$$

$$\begin{vmatrix} L_1 & L_2 \\ L_3 & L_4 \end{vmatrix}$$

None of them

Wronskian of x, x^2 is

x

x^2

x^3

0

The general solution of differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is given by

$$e^{\frac{y}{x}} = cx$$

$$e^{\frac{y}{x}} = cy$$

$$e^{\frac{x}{y}} = cx$$

$$e^{-\frac{x}{y}} = cx$$

The form of the exact solution to

$$2\frac{dy}{dx} + 3y = e^{-x}, y(0) = 5$$

is

$$Ae^{-1.5x} + Bxe^{-x}$$

$$Ae^{1.5x} + Be^{-x}$$

$$Ae^{1.5x} + Bxe^{-x}$$

$$Ae^{-1.5x} + Be^{-x}$$

If m and n are non negative integers and $P_n(x)$ is a Legendre's polynomial then

$$\int_{-1}^1 P_m(x)P_n(x)dx = 0 \quad \text{for } m \neq n$$

$$\int_{-1}^1 P_m(x)P_n(x)dx = 0 \quad \text{for } m = n$$

$$\int_{-1}^1 P_m(x)P_n(x)dx = 0 \quad \text{for } m < 0$$

$$\int_{-1}^1 P_m(x)P_n(x)dx = 0 \quad \text{for } n < 0$$

If A is a square matrix and its determinant is zero, then A is singular matrix.

A is non singular matrix.
 A is scalar matrix.
 A is diagonal matrix.

An electronic component of an electronic circuit that has the ability to store charge and opposes any change of voltage in the circuit is called

Inductor
 Resistor
 Capacitor
 None of them

Operator method is the method of the solution of a system of linear homogeneous or linear non-homogeneous differential equations which is based on the process of systematic elimination of the

Dependent variables
 Independent variable
 Choice variable
 None of them

Any linear differential equation of the form

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1 x \frac{dy}{dx} + a_0 y = g(x) \text{ where } a_0, a_1, a_2, \dots, a_n \text{ are constants.}$$

is called
 Homogeneous equation
 Polar equation
 Equi-dimensionl equation
 None of them

For eigen values $\lambda = 5, 5$ of a matrix $A = \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$, there exists eigen vectors.

infinite
 one
 two
 three

Ordinary points of $(x^2 - 64)(x^2 - 36)y'' + xy' - y = 0$ are

0,1

8,-8

6,-6

None of others.

A singular point $x = x_0$ of the given equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$ is said to be a regular singular point if

$(x - x_0)P(x)$ is analytic at x_0

$(x - x_0)Q(x)$ is analytic at x_0

$(x - x_0)P(x)$ & $(x - x_0)^2 Q(x)$ are analytic at x_0

None of them

Singular points of the equation $(x^2 - 4)^2 y'' + (x - 2)y' + y = 0$ are

$x = -2, 2$

None of them

$x = 2$

$x = -2$

The matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ has

Real and unequal value
 Repeated & real eigen value
 Complex eigen value
 None of them

Let λ be an eigen value of a non zero square matrix A. Then the equation $\det(A - \lambda I) = 0$ is called

Trivial equation
 Characteristics equation
 Non-trivial equation
 None of them

If $y_1 = x^2$ is solution of the differential equation, then formula for finding Second solution of $x^2 y'' - 2y = 0$ is

$$y_2 = x^2 \int \frac{e^{-2x}}{x^2} dx$$

$$y_2 = x \int \frac{e^{-2x}}{x^2} dx$$

$$y_2 = x^2 \int \frac{e^{\frac{-2}{x}}}{x^4} dx$$

$$y_2 = x^2 \int \frac{2}{x^4} dx$$

For $y \sin^2 x - y^2 \cos x = c$ where $y(0) = 3$; the value of c is

9
 -9
 10
 -10

A differential equation $M(x, y) dx + N(x, y) dy = 0$ is said to be an exact if -----.

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$$

The integrating factor of the differential equation $\frac{dy}{dx} + (\cot x)y = \cos^2(x)$ is -----

$$\mu = \ln |\sin x|$$

$$\mu = \ln |\cos x|$$

$$\mu = \sin x$$

$$\mu = \cos x$$

2 Marks

Give two examples of Bessel's Differential Equation?

Define matrix and give condition of multiplication of two matrices.

Find Wronskian of two vectors $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}, X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$.

What is half life of a substance?

Marks 3

Write the differential equation $\frac{d^3 y}{dx^3} - 4 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} = 4x$ in the form $L(y) = g(x)$ where L is a differential operator with constant coefficients

Why we take $c_0 \neq 0$ in Frobenius method?

Write the homogenous system of differential equations

$$2 \frac{dx}{dt} - 5x + \frac{dy}{dt} = 5e^t$$
$$\frac{dx}{dt} - x + \frac{dy}{dt} = e^t$$

in decoupled form.

Find eigen vector of

$$A = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}, \text{corresponding eigen value } \lambda = -2.$$

Marks 5

Deduce the Special Case of Logistic Equation “Epidemic spread”?

Find $\frac{dX}{dt}$ if $X = \begin{bmatrix} e^{4t} & \sin 2t^2 \\ 5t+4 & \ln(1+t^3) \end{bmatrix}$

State the procedure to solve a differential equation by using Frobenius Theorem.

Write down the system of differential equations

$$\frac{dx}{dt} = 6x + y, \quad \frac{dy}{dt} = x + 3y - 9t - 9$$

in form of $X' = AX + F(t)$