MANGALORE INSTITUTE OF TECHNOLOGY & ENGINEERING

DEPARTMENT OF ROBOTICS & ARTIFICIAL INTELLIGENCE

CYBER-PHYSICAL SYSTEMS (Course Code: 23RIPE311) — V SEMESTER

MODULE 1: INTRODUCTION TO CYBER-PHYSICAL SYSTEMS COMPREHENSIVE LECTURE & STUDY NOTES

Course Outcome Mapped: CO1 — Explain the fundamental concepts, classifications, and architectural components of Cyber-Physical Systems.
Syllabus Reference: Module 1 (8 Hours): Cyber-Physical Systems, Classification of Systems, Uncertainty, Information and its Different Forms, Network Types, Processes on Networks and Applications, Decisions and Actions, Forms of Decision-Making, Rule-Based Decisions.
Prescribed Reference Text: Pedro H. J. Nardelli, Cyber-Physical Systems: Theory, Methodology, and Applications, IEEE Press / John Wiley & Sons, 2022 (Sections 1.1, 2.4, 3.1, 4.3, 5.2, 5.3, 6.1, 6.2, 6.5).

1. Cyber-Physical Systems (§1.1)

1.1 The Concept and NIST Operational Definition

A Cyber-Physical System (CPS) is an engineered multidimensional system that tightly couples discrete computational logic, communication networks, and physical processes. According to the United States National Institute of Standards and Technology (NIST):

> *"Cyber-Physical Systems (CPS) comprise interacting digital, analog, physical, and human components engineered for function through integrated physics and logic. These systems will provide the foundation of our critical infrastructure, form the basis of emerging and future smart services, and improve our quality of life in many areas."*

In engineering practice, a CPS continuously mediates between the analog physical domain (Level 0 physical reality governed by conservation and thermodynamic laws) and the discrete symbolic domain (Level 1+ cyber reality governed by logic and algorithmic state machines).

1.2 The Closed Sense–Communicate–Process–Decide–Act Loop

Every cyber-physical system is structured around an irreducible, bidirectional feedback control loop consisting of five essential functional phases:

- **1. Sensing (Data Acquisition):** Sensors and measuring transducers convert continuous physical phenomena (Level 0 variables such as temperature, pressure, rotational displacement, velocity, or vibration) into discretized analog or digital electrical signals.
- **2. Communication (Data Transport):** The digitized measurements are framed into structured data packets and transmitted across wired or wireless network fabrics (e.g., Ethernet, 5G, IEEE 802.15.4) subject to physical transport constraints such as bandwidth, propagation delay, and packet loss.
- **3. Data Processing (Information Synthesis):** Computing units, edge nodes, or cloud servers ingest raw data, eliminate noise, aggregate heterogeneous streams, and transform raw data into semantically meaningful information.

- **4. Decision-Making (Control Policy):** Human operators, expert systems, optimization solvers, or autonomous control algorithms evaluate the extracted information against setpoints or system constraints to determine corrective or regulatory actions.
- **5. Actuation (Physical Intervention):** Digital commands from the decision layer are communicated to physical actuators (solenoids, servomotors, valves, power electronic switches), injecting energy or exerting force back onto the physical plant, thereby closing the loop.

1.3 Four Fundamental Features Common to Every CPS

- **1. Physical Process Digitalization:** Continuous states of physical matter or energy can be measured and digitalized via appropriate sensing transducers.
- **2. Information Synthesis & Transport:** Data are communicated across network structures and mathematically/semantically transformed to resolve uncertainty regarding the underlying physical process.
- **3. Informed Decision-to-Action Coupling:** Informed data drive automated or supervisory decisions that govern physical actions, establishing adaptive or autonomous system intelligence.
- **4. Societal and Environmental Embeddedness:** CPS deployments directly intervene in real-world socioeconomic and industrial processes; consequently, their dynamics affect and are affected by human society and the environment.

1.4 Need for a General Scientific Theory of CPS

While various isolated cyber-physical implementations (such as modern automated chemical plants, automated microgrids, and robotic assembly cells) exist in industrial practice, they have historically been designed using fragmented, sector-specific empirical know-how. Nardelli demonstrates that an overarching scientific theory of CPS is indispensable for three primary reasons:

- **1. Prevention of Pathological Collective Behavior:** Without a general theoretical model, individual components are optimized in isolation, resulting in disastrous emergent dynamic failures at the macro-system scale.

A prominent illustration is the “Stupid Grid” phenomenon (Nardelli & Kühnlenz, 2018). In an electrical distribution grid, smart home energy meters are programmed to automatically switch on flexible domestic loads (such as electric vehicle chargers and heat pumps) during low-price hours to flatten the demand profile. Because the hourly electricity price is a universal broadcast signal seen simultaneously by all meters, uncoordinated decentralized meters switch on at the exact same instant. Instead of flattening demand, their collective action generates massive secondary demand spikes and severe voltage drops, converting an intended smart grid into a destabilized, fragile grid.

- **2. Characterization of Fundamental Bounds:** A rigorous theory establishes the immutable operational and thermodynamic boundaries beyond which no system can function, such as Shannon's channel capacity limits on data communication and Landauer's thermodynamic bound on computation.
- **3. Transferable Engineering Framework:** It provides formal mathematical abstractions (e.g., system demarcation, graph-theoretic network models, entropy quantification, and reflexive-active structures) that allow engineers to design, verify, and validate complex CPS architectures prior to physical material deployment.

2. Classification of Systems (§2.4)

2.1 The System Demarcation Problem

Before analyzing or classifying any system, an engineer must solve the Demarcation Problem (Definition 2.3). System demarcation is the theoretical process of defining the boundaries that separate a particular system (PS) from everything else (its environment) based upon its Peculiar Function (PF).

According to Proposition 2.1 (System Demarcation and Conditions of Existence), every particular system is constituted by three necessary groups of conditions:

- • **Conditions of Production (C1):** The physical and thermodynamic possibility that the peculiar function can be produced according to fundamental natural laws.
- • **Conditions of Reproduction (C2):** Internal structural, operational, and maintenance conditions that guarantee the recurring, continuous execution of the peculiar function.
- • **External Conditions (C3):** Exogenous factors outside the system boundary (ambient environment, economic inputs, regulatory frameworks, power supply) that directly or indirectly influence its reproduction.

2.2 The Four Independent System Classification Axes

Nardelli establishes that any system—material or conceptual—must be systematically classified along four independent, orthogonal axes:

Axis 1: Natural vs. Human-Made Systems

- • **Natural Systems:** Systems that come into being through spontaneous physical, biological, or geological processes without human intervention (e.g., Earth's planetary atmosphere, river basins, biological nervous systems). Studied as given.
- • **Human-Made Systems:** Systems that exist through human intentionality, design, and manufacturing (e.g., an automobile, a wind turbine, a robotic workcell). Humans are their primary agents of production.
- • **Human-Modified (Hybrid) Systems:** Natural systems whose internal dynamics, thermodynamic equilibrium, or chemical compositions are substantially altered by human industrial activity (e.g., urban microclimates altered by vehicular exhaust, rivers regulated by hydroelectric dams).

Axis 2: Material vs. Conceptual Systems

- • **Material Systems:** Systems that manifest physically in concrete space-time, composed of matter, electrical charge, and physical energy (e.g., a physical wind turbine operating in an offshore facility). All material systems are fundamentally open.
- • **Conceptual Systems:** Systems that exist within a symbolic, mathematical, or computational domain as plans, differential equations, CAD schematics, or algorithmic simulations (e.g., aerodynamic simulation models, state-space representations).
- • **Hardware-in-the-Loop (HIL) Simulation:** Physical microcontrollers executing control loops against a simulated mathematical plant in real time.
- • **Digital Twin:** A continuous, synchronized, one-to-one real-time mapping between an operating physical material system and its symbolic computational counterpart.

Axis 3: Static vs. Dynamic Systems

- • **Static Systems:** Systems predominantly composed of structural components whose states do not change, or change negligibly, over the observation interval (e.g., a civil bridge girder, a steel turbine tower).
- • **Dynamic Systems:** Systems characterized by frequent, continuous, or discrete state transitions over time and space, driven by operating and flow components (e.g., an electrical generator coupled to aerodynamic wind variations).

Dynamic systems exhibit rich mathematical subclasses: linear vs. non-linear; continuous-time vs. discrete-time; periodic vs. event-driven; deterministic vs. stochastic vs. adaptive; single-input single-output (SISO) vs. multiple-input multiple-output (MIMO); and stable vs. unstable vs. chaotic.

Axis 4: Closed vs. Open Systems

- • **Closed Systems:** Systems that have zero or negligible exchange of matter, energy, or information with their external environment. While no material system is strictly closed in nature, the closed-system abstraction is vital for deriving fundamental thermodynamic laws and theoretical bounds (e.g., Carnot efficiency, Maxwell's Demon).
- • **Open Systems:** Systems that continuously exchange energy, mass, and information across their demarcated boundaries. All operational cyber-physical systems are open systems.

2.3 Comprehensive Case Study: Demarcation and Classification of a Wind Turbine

To demonstrate how the demarcation framework and the four classification axes apply to an engineering CPS, consider a grid-connected commercial wind turbine:

- • **Particular System (PS) Components:** Structural (tower, nacelle housing, foundation, rotor hub); Operating (variable-pitch composite blades, planetary gearbox, doubly-fed induction generator, AC-DC-AC power converter); Flow (atmospheric wind kinetic energy, electrical current, dissipated friction heat); Measuring (blade strain gauges, shaft encoders, anemometers, bus voltage/current sensors); Computing (programmable logic controllers, embedded DSP pitch controllers); Communication (fiber-optic fieldbus, SCADA network).
- • **Peculiar Function (PF):** Convert aerodynamic kinetic energy from incoming wind into standardized 50 Hz AC electrical energy synchronized with the utility grid.
- • **Conditions of Production (C1):** Thermodynamic and aerodynamic laws (Betz limit: maximum theoretical aerodynamic power extraction efficiency is 59.3%) and electromagnetic induction laws.
- • **Conditions of Reproduction (C2):** Automated predictive maintenance schedules, continuous lubricating oil filtration, pitch actuator calibration, and overspeed aerodynamic braking interlocks.
- • **External Conditions (C3):** Atmospheric wind velocity profiles, utility grid voltage stability, wholesale feed-in electricity tariffs, and environmental temperature extremes.

Classification Axis	System Class	Engineering Justification
Natural vs. Human-Made	Human-Made	Synthesized and manufactured through human engineering.
Material vs. Conceptual	Hybrid / All Stages	Conceptual in CAD/aerodynamic simulation; HIL during bench testing; material when installed; hybrid as a Digital Twin.

Static vs. Dynamic	Nonlinear Stochastic Dynamic	Rotor rotational speed and electrical power output fluctuate continuously with turbulent wind variations.
Closed vs. Open	Open System	Absorbs wind kinetic energy, exports electrical energy, dissipates thermal heat, and exchanges SCADA telemetry.

3. Uncertainty and Probability Theory (§3.1)

3.1 Foundational Concepts & Taxonomy of Uncertainty

Uncertainty characterizes aspects of an object, process, or system that are wholly or partly unknown. In CPS engineering, uncertainty is classified into four distinct categories:

- **1. Chance (Aleatory) Uncertainty:** Inherent physical randomness in environmental phenomena or hardware measurement noise (e.g., thermal noise in radio receivers, turbulent gusts hitting an aircraft, manufacturing tolerances in mechanical gears). Quantified using probability density and mass functions.
- **2. Strategic / Operational Uncertainty:** Uncertainty emerging from interactions among autonomous, intelligent, or competitive decision-makers (e.g., traffic route choices by autonomous vehicles, multi-agent bidding in automated electricity markets). Modeled using game theory.
- **3. Combined Uncertainty:** Systems featuring both stochastic initial states and strategic multi-turn interactions (e.g., dominoes, card games, distributed robotic search tasks).
- **4. Fundamental / Epistemic Uncertainty:** Inherent unpredictability arising from incomplete knowledge, unmodeled high-order dynamics, or extreme sensitivity to initial conditions (e.g., quantum indeterminacy, non-linear deterministic chaos).

3.2 Probability Theory vs. Statistics in CPS

- • **Probability Theory:** A purely mathematical, axiomatic framework founded by Andrey Kolmogorov (1933). It operates deductively upon idealized mathematical abstractions, sample spaces, and formal probability measures.
- • **Statistics:** An empirical, inductive discipline that analyzes real-world experimental data, sensor observations, and parameter estimations. Provides the theoretical foundations for statistical machine learning, Kalman filtering, and anomaly detection in CPS.

3.3 Mathematical Formalism of Uncertainty (Kolmogorov Axioms)

Consider an observation process defined on a system. Let S denote the sample space containing all possible mutually exclusive outcomes, and let E denote an event of interest (a subset $E \subseteq S$). A function $P: 2^S \rightarrow [0, 1]$ is a valid probability measure if and only if it satisfies Kolmogorov's Three Axioms:

$$\textbf{Axiom 1 (Non-negativity):}\quad P(E) \geq 0,\quad \forall\, E \subseteq S$$
$$\textbf{Axiom 2 (Normalization):}\quad P(S) = 1$$
$$\textbf{Axiom 3 (Additivity):}\quad \text{For mutually disjoint events } E_1 \cap E_2 = \emptyset,\quad P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

(Eq. 3.1)

3.4 Random Variables and Probability Distributions

A random variable X(a) is a mathematical function that maps every observable attribute a ∈ A into a real number: X: A → ℝ.

- • **Discrete Random Variable:** Takes values in a countable set S_X = {x_1, x_2, ...}. Its probability mass function (PMF) satisfies p_X(x) = P(X = x) with ∑ p_X(x) = 1.
- • **Continuous Random Variable:** Takes values across an uncountable continuum (e.g., ℝ). Characterized by its Cumulative Distribution Function (CDF) F_X(x) = P(X ≤ x) and Probability Density Function (PDF) f_X(x) = dF_X(x)/dx.

The statistical properties of a random variable are quantified by its statistical moments:

> **Expected Value (Mean):** $E[X] = \sum x \cdot p_X(x)$ [Discrete] or $E[X] = \int x \cdot f_X(x)\, dx$ [Continuous]
> **Variance:** $Var(X) = E[X^2] - (E[X])^2$ *(Eq. 3.2)*

3.5 Random Processes (Stochastic Processes) in CPS

A stochastic process is an indexed collection of random variables {X(t) : t ∈ T} or {X[k] : k ∈ ℕ}, where the index represents discrete time steps, timestamps, or spatial locations. Stochastic processes in CPS are categorized based on their temporal memory:

- **1. Memoryless Processes:** Future outcomes are statistically independent of past realizations: P(X[n] = x_n | X[n-1], ..., X[0]) = P(X[n] = x_n).
- **2. Markov Processes:** The future state depends exclusively on the present state, rendering the past history irrelevant: P(X[n] = x_n | X[n-1], X[n-2], ..., X[0]) = P(X[n] = x_n | X[n-1]).
- **3. Path- / History-Dependent Processes:** The probability distribution of future states is a function of the entire historical trajectory of the system (e.g., non-Markovian degradation in mechanical bearings).
- **4. Processes with Dynamic Sample Spaces:** Processes where the cardinality or composition of the state space dynamically evolves based on prior outcomes (e.g., domino draws without tile replacement).

4. Information and Its Different Forms (§4.3)

4.1 Data vs. Information

In systems engineering, data and information are distinct concepts:

- • **Data:** An unbounded set D of raw symbols, signals, bits, strings, or numbers acquired by sensors. Data may be unstructured, noisy, uninterpretable, or meaningless.
- • **Information:** A strictly qualified subset of data (I ⊂ D) that is syntactically structured, semantically interpretable, and provides a trustworthy resolution of uncertainty regarding an observable attribute of a system.

4.2 Typology of Data in CPS Architectures

- **1. Primary Data:** Directly accessible numeric values (e.g., current reading from an ammeter, raw binary string from an ADC).
- **2. Secondary Data:** Information inferred from the absence of data (e.g., complete absence of acoustic vibration signals indicating an unpowered motor).
- **3. Metadata:** Data describing other data (e.g., sensor sampling frequency, hardware calibration timestamp, geographic GPS tag).
- **4. Operational Data:** Data describing the operating health or communication performance of the CPS itself (e.g., packet retransmission counts, battery voltage of an IoT node).
- **5. Derivative Data:** High-level information computed indirectly from primary data (e.g., calculating industrial plant productivity or human occupancy based on aggregate smart meter electrical power profiles).

4.3 Shannon Information Entropy and Mathematical Information

Claude Shannon (1948) established that the mathematical quantification of information corresponds to the reduction of uncertainty regarding which message is selected from an alphabet of possible messages, deliberately discarding semantic meaning.

For a discrete random variable X with alphabet S_X, the average information content (Shannon Entropy) measured in bits is defined as:

$$H(X) = -\sum [\, P(X = x_i) \cdot \log_2 P(X = x_i) \,] \quad [\text{bits}] \qquad \textit{(Eq. 4.1)}$$

Shannon entropy measures the minimum average number of binary (yes/no) questions required to uniquely resolve the unknown state of X. For N possible outcomes, entropy is bounded:

$$0 \leq H(X) \leq H_{max} = \log_2(N) \qquad \textit{(Eq. 4.2)}$$

H(X) = 0 indicates absolute determinism (zero uncertainty, zero information yielded upon observation). Maximum entropy H_max occurs under a uniform distribution (P(x_i) = 1/N, representing maximum ignorance).

For two random variables X and Y, the Joint Entropy H(X,Y), Conditional Entropy H(X|Y), and Mutual Information I(X;Y) satisfy:

$$H(X|Y) = -\sum\sum P(x, y) \cdot \log_2 P(x|y) \qquad \textit{(Eq. 4.3)}$$
$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)$$

Mutual Information I(X;Y) quantifies the information shared between X and Y—that is, the reduction in uncertainty about X achieved by observing Y.

4.4 The Four Different Forms of Information

- **1. Mathematical Information:** Shannon's probabilistic formulation where messages are modeled as mathematical variables. Focuses strictly on transmission limits, channel capacity, and lossless compression bounds, treating semantic meaning as irrelevant.

- **2. Semantic Information:** Information defined by its meaning, context, and functional intent. Subdivided into: (a) Instructional (action-guiding rules/code); (b) Factual (verifiable truth claims; differentiated into true information, unintentional misinformation, and malicious disinformation); and (c) Environmental (physical correlations where state changes in one process inform about another, e.g., thermal expansion in mercury columns).
- **3. Biological Information:** Information stored, transmitted, and processed via biochemical and genetic mechanisms (DNA/RNA nucleotide sequences, cellular signaling cascades, hormonal pathways) essential for the reproduction and evolution of living organisms.
- **4. Physical Information:** Reflects the fundamental physical law that "Information is Physical" (Rolf Landauer, 1961). The erasure or irreversible manipulation of one bit of logical information in any physical medium dissipates a minimum thermodynamic heat into the environment:

$$E_{erasure} \geq k_B \cdot T \cdot \ln(2) \quad [\text{Joules/bit}] \qquad \textit{(Eq. 4.4)}$$

where $k_B = 1.3806 \times 10^{-23}$ J/K is the Boltzmann constant and T is absolute temperature in Kelvin. This principle establishes the ultimate energy efficiency limit for all data centers, computing microchips, and cyber-physical controllers.

4.5 Physical Reality (Level 0) vs. Symbolic Reality (Level 1+)

Following Proposition 4.4, CPS operates across two distinct realities:

- • **Physical Reality (Level 0 Processes):** Continuous physical phenomena, thermodynamics, and kinematics governed by inviolable natural laws.
- • **Symbolic / Cyber Reality (Level 1+ Processes):** Discrete data structures, network packets, digital states, and algorithmic models. Bounded by energy constraints (Landauer) and communication capacity (Shannon).
- • **Hierarchy of Levels:** Level 1 processes operate on data acquired directly from Level 0 physical reality (e.g., digital ADC readings). Level 2 processes operate on Level 1 data (e.g., calculating spatial averages across sensor arrays). Level N processes operate recursively on Level N-1 representations.

5. Network Types and Topologies (§5.2)

5.1 Graph-Theoretic Formalism & Structural Metrics

A network is abstracted mathematically as a graph G = (V, E), where $V = \{v_1, v_2, ..., v_N\}$ represents the set of N nodes (vertices) and $E = \{e_1, e_2, ..., e_L\}$ represents the set of L links (edges) connecting pairs of nodes.

The fundamental mathematical metrics that govern network structure are:

- • **Adjacency Matrix A = (a_ij):** An N × N matrix where a_ij = 1 if an edge connects node i to node j, and a_ij = 0 otherwise. For undirected networks, A is symmetric (a_ij = a_ji). In weighted networks, entries take real values w_ij representing connection strength, distance, or link capacity.
- • **Node Degree (k_i):** The total number of links directly connected to node i. For directed networks, degree splits into out-degree (k_i^out) and in-degree (k_i^in):

$$k_i = \sum [a_{ij}] - a_{ii} \quad \text{[Undirected]}$$
$$k_i^{out} = \sum [a_{ij}], \quad k_i^{in} = \sum [a_{ji}] \quad \text{[Directed]}$$

(Eq. 5.1)

- • **Total Link Count (L):** For an undirected network, summing all node degrees counts every link twice:

$$L = (1/2) \cdot \sum k_i = (1/2) \cdot \sum\sum a_{ij}$$

(Eq. 5.2)

- • **Path Length, Distance, and Diameter:** The sequence of links traversed to travel from node i to node j. The Distance $d_{i \to j}$ is the length of the shortest path between them. The Network Diameter is the maximum distance between any pair of nodes in the graph: $Diam(G) = \max d_{i \to j}$.
- • **Clustering Coefficient (C_i):** Quantifies the local link density among the x_i neighbors of node i. If L_i represents the actual number of links connecting those x_i neighbors, the clustering coefficient is:

$$C_i = 2 \cdot L_i / [x_i \cdot (x_i - 1)]$$

(Eq. 5.3)

$C_i = 1$ indicates that node i and its neighbors form a fully connected clique; $C_i = 0$ indicates that none of the neighbors are linked to each other.

5.2 Deterministic Network Topologies

- **1. Peer-to-Peer (P2P):** Consists of isolated pairs of nodes directly connected to each other (N = 2, L = 1, k = 1). The baseline model for point-to-point wireless transmitter-receiver channels.
- **2. Star / One-to-Many / Many-to-One:** A single central master node is connected to N-1 peripheral leaf nodes ($k_{hub} = N-1$, $k_{leaf} = 1$, L = N-1). Common in master-slave industrial architectures. The central node provides a minimum network diameter of 2, but represents a catastrophic single point of failure.
- **3. Line Topology:** Nodes are connected in a linear chain. End nodes have k = 1; interior nodes have k = 2. Total links L = N-1, clustering coefficient C = 0, and diameter Diam = N-1. Latency scales linearly with network size.
- **4. Ring Topology:** A closed-loop line where each node connects to exactly two neighbors ($k_i = 2, \forall\ i$). Total links L = N. For N > 3, $C_i = 0$. Diameter is $\lfloor N/2 \rfloor$. Offers alternate clockwise/counter-clockwise routing paths.
- **5. Complete (Fully Connected) Network:** Every node is directly connected to every other node in the network. Degree $k_i = N-1$, clustering coefficient $C_i = 1$. The total number of links represents the maximum possible connectivity in a graph:

$$L_{complete} = N \cdot (N - 1) / 2$$

(Eq. 5.4)

5.3 Random and Complex Network Topologies

Erdős–Rényi (ER) Random Networks

Introduced by Paul Erdős and Alfréd Rényi (1959). An ER network G(N, p) consists of a fixed number of N nodes where each possible pair of nodes is connected by an edge independently with probability p. The degree distribution follows a Binomial distribution, which converges to a Poisson distribution for large N:

$$P(k) = \binom{N-1}{k} \cdot p^k \cdot (1-p)^{N-1-k} \approx \left[\frac{\langle k \rangle^k \cdot e^{-\langle k \rangle}}{k!} \right]$$
$$\langle k \rangle = p \cdot (N-1), \quad \mathrm{Var}(k) = p \cdot (1-p) \cdot (N-1)$$

(Eq. 5.5)

When p = 0, the graph is a set of isolated vertices; when p = 1, it becomes a complete graph. A critical percolation transition occurs when ⟨k⟩ = 1, below which the network consists of small disconnected components, and above which a giant connected cluster spanning the network emerges.

Watts–Strogatz (WS) Small-World Networks

Proposed by Duncan Watts and Steven Strogatz (1998) to model the small-world phenomenon observed in social and infrastructural systems. WS networks interpolate between a regular ring lattice and a random network:

- • **Step 1 (Regular Lattice):** Start with a ring lattice of N vertices where each node is connected to its m nearest neighbors (m is even, $m \geq 2$). Total edges = $N \cdot m / 2$.
- • **Step 2 (Random Rewiring):** Move clockwise around the ring and rewire each edge with probability p to a uniformly chosen random node (avoiding self-loops and duplicate edges).
- • **Structural Emergence:** For p = 0, the graph is completely regular with high clustering but large diameter. For p = 1, all edges are randomized, producing an ER-like graph with low clustering. For intermediate values ($0.01 < p < 0.1$), the network enters the Small-World Regime: it retains high clustering ($C \gg C_{random}$) while the average shortest path length drops precipitously to near-random levels ($L \approx L_{random}$) due to the formation of topological shortcuts.

Barabási–Albert (BA) Scale-Free Networks

Albert-László Barabási and Réka Albert (1999) discovered that many real-world technological networks (the World Wide Web, the Internet router backbone, airline transit networks) do not have a characteristic bell-shaped degree distribution. Instead, they exhibit heavy-tailed Power-Law Degree Distributions:

$$P(k) \sim k^{-\gamma}, \quad \text{where } 2 < \gamma < 3$$

(Eq. 5.6)

BA scale-free networks are generated via two continuous mechanisms:

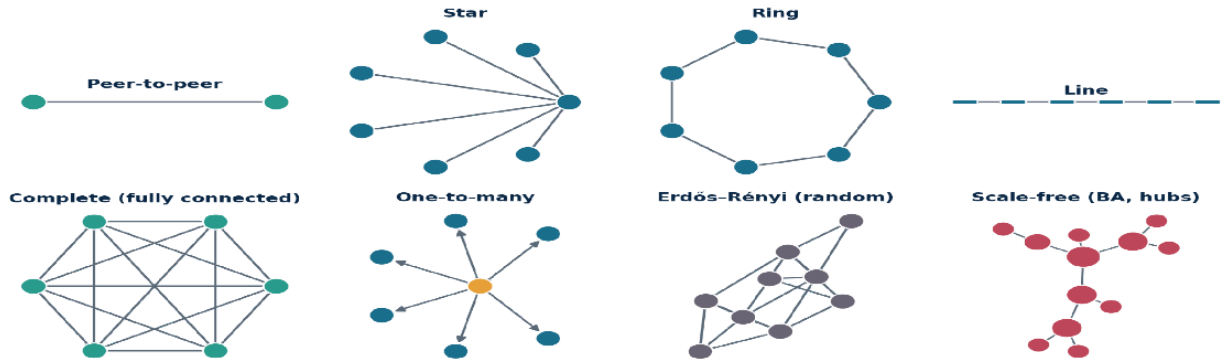
- **1. Continuous Growth:** Starting from a small seed network, new nodes are continuously added at every time step.
- **2. Preferential Attachment ("Rich Gets Richer"):** A new node connects to m existing nodes with a probability $\Pi(k_i)$ proportional to the existing node's current degree:

$$\Pi(k_i) = k_i / \left[\sum k_j \right]$$

(Eq. 5.7)

This process results in the emergence of highly connected nodes known as Hubs. Scale-free networks possess the property of Scale Invariance: $f(a \cdot x) = a^{-\gamma} \cdot f(x)$, meaning no single characteristic node degree defines the system.

Network Topology	Average Path Length	Clustering Coeff.	Robustness Properties
Star	Short (Diam = 2)	Zero (C = 0)	Fragile; single point of failure at center hub.
Ring (Regular)	Long (Diam = N/2)	Zero (for m = 2)	High path redundancy; slow information spreading.
Erdős–Rényi (Random)	Short (~ ln N / ln ⟨k⟩)	Low (C ≈ p)	Homogeneous; uniform tolerance to random failures.
Watts–Strogatz	Short (~ ln N)	High (C ≈ C_regular)	Small-world behavior; rapid decentralized routing.
Barabási–Albert	Ultra-Short (~ ln ln N)	Moderate	Robust against random node failures; highly fragile to targeted hub attacks.



6. Processes on Networks and Applications (§5.3)

6.1 Communication Systems: Robustness, Resilience, and Percolation

Using the communication scenario between two actors (Amanda and Bob), Nardelli illustrates how physical topology dictates operational performance:

- • **PSTN (Public Switched Telephone Network):** A dedicated, circuit-switched hierarchical star architecture. Once established, the link provides guaranteed bandwidth and low latency, but is critically vulnerable to outages at central switching nodes.
- • **VoIP (Voice over IP):** A packet-switched architecture running across the decentralized Internet mesh. Messages are sliced into independent IP packets that traverse diverse routes. It lacks deterministic transmission guarantees, but provides high resilience through dynamic rerouting around failed links.
- • **Robustness:** The ability of an individual link to withstand physical failure.
- • **Resilience:** The capacity of the network as a whole to sustain its end-to-end communication function through alternative pathways when components fail.
- • **Percolation:** The critical condition under which an interconnected cluster spans the entire network. If too many links fail, the network falls below its percolation threshold, fragmenting into isolated subgraphs.

6.2 Transportation Networks & Urban Dynamics

- • **Euler's Seven Bridges of Königsberg (1736):** Euler proved that a closed walk traversing every bridge exactly once is possible if and only if every vertex has an even degree, or exactly two vertices have odd degrees. Because all four land masses in Königsberg had odd degrees (3, 3, 3, 5), the path was impossible, laying the foundations of graph topology.
- • **Traveling Salesman Problem (TSP):** Determining the shortest closed tour that visits N cities exactly once. Modeled as a complete graph with edge weights representing geographical distances. It is NP-hard, requiring heuristic optimization in industrial CPS logistics.
- • **Berlin S-Bahn Railway Network:** Features outer radial lines (star topology) integrated with a central orbital ring. Radial lines funnel commuters into the urban core, while the ring disperses transfer traffic across multiple interchange hubs, preventing network congestion collapse.

6.3 Epidemic Spreading & The Basic Reproduction Number (R_0)

Pathogen propagation across human populations—as well as malware propagation across networked industrial CPS nodes—is modeled using compartmental dynamical systems:

- **1. Susceptible-Infected (SI) Model:** No recovery mechanism. All nodes eventually transition into the infected state: $dI/dt = \beta \cdot S \cdot I$.
- **2. Susceptible-Infected-Susceptible (SIS) Model:** Nodes recover at rate γ but acquire no immunity, returning to the susceptible state. The system reaches an endemic equilibrium (for $R_0 > 1$) or a disease-free state (for $R_0 < 1$): $dI/dt = \beta \cdot S \cdot I - \gamma \cdot I$.
- **3. Susceptible-Infected-Recovered (SIR) Model:** Infected nodes recover at rate γ and acquire permanent immunity (or are permanently patched): $dS/dt = -\beta \cdot S \cdot I$, $dI/dt = \beta \cdot S \cdot I - \gamma \cdot I$, $dR/dt = \gamma \cdot I$.

The long-term fate of an epidemic is determined by the Basic Reproduction Number R_0:

$$R_0 = \beta \cdot \langle k \rangle / \gamma \qquad \textit{(Eq. 6.1)}$$

where β is transmission probability per contact, $\langle k \rangle$ is the average contact degree, and γ is recovery rate. On heterogeneous scale-free contact networks, Pastor-Satorras and Vespignani (2001) proved that the epidemic threshold vanishes as the variance of the degree distribution diverges ($\langle k^2 \rangle \to \infty$). Consequently, viruses or malware can spread even under vanishingly small transmission rates ($\beta \to 0$) due to super-spreader hubs.

7. Decisions and Actions (§6.1, §6.2)

7.1 Decision-Makers vs. Agents (Definition 6.1)

A decision-making process is an informational mediator that takes processed data as inputs and selects one or more actions from a set of alternatives. System entities are classified into:

- • **Decision-Makers:** Computational algorithms, software controllers, or human operators that evaluate choices based on logical and mathematical relations.

- • **Agents:** Physical hardware components (actuators, servomotors, valves, breakers) or human personnel that execute physical operations. Agents are controlled symbolically via data commands, not by direct physical force.
- • **Overlap:** An entity can be exclusively a decision-maker (e.g., an optimization algorithm running in a remote server), exclusively an agent (e.g., a solenoid valve), or both simultaneously (e.g., an autonomous mobile robot).

7.2 Levels of Actions and Decisions (Proposition 6.1)

Actions are physical processes that modify physical state, and thus sit at Level 0. Decision-making is a symbolic process sitting at Level 1 or higher:

- • **Level 1 Decision-Makers:** Directly issue commands to physical actuators (e.g., a local microcontroller setting motor PWM).
- • **Level 2 Decision-Makers:** Define the decision rules, objective functions, or operating constraints governing Level 1 controllers (e.g., a supervisory SCADA optimization engine).

Irrespective of the decision level, all decision choices are grounded in and bounded by the physical capabilities of Level 0 actuators. Autonomy in cyber reality is always limited by physical reality.

7.3 Topologies of Decision-Making

- **1. Centralized Decision-Making:** A single central decision-maker commands all physical agents across a one-to-many topology. Agents possess zero autonomy. Achieves global optimization, but creates a single point of failure and communication bottlenecks.
- **2. Distributed Decision-Making:** Every node is both a decision-maker and an agent across a peer-to-peer or mesh network. Nodes possess local autonomy and coordinate via consensus, voting, or game-theoretic interaction. Highly fault-tolerant, but consensus convergence may be slow.
- **3. Decentralized Decision-Making:** A multi-tiered hierarchy featuring several regional decision-makers governing local clusters of agents. Combines localized responsiveness with hierarchical policy coordination.

7.4 Illustrative Case Study: Dressing Children for a Event (Example 6.1)

To illustrate the three forms of decision-making, consider a family with two parents and four children:

- • **Centralized:** One parent unilaterally selects clothes for all four children. The parent is the sole decision-maker; the children act strictly as agents.
- • **Distributed:** The children select their own clothing autonomously through discussion and consensus (e.g., agreeing on matching colors). Parents do not intervene in the choice.
- • **Decentralized:** Parents and children jointly deliberate, or parents assign clothing categories while children pick colors. Decision authority is distributed across multiple tiers.

In all three cases, autonomy is constrained by the wardrobe options available. Furthermore, the decision to attend the party was imposed beforehand as an external boundary condition, illustrating that autonomy is always bounded.

8. Rule-Based Decisions (§6.5)

8.1 Concept and Industrial Generalization

Every decision-making process is governed by rules. Mathematical optimization follows the rule: “always select the parameter vector x* that minimizes $f_0(x)$ subject to constraints.” Game theory follows the rule: “always select the strategy that maximizes individual expected payoff given competitors' choices.” Rule-based decisions generalize decision-making to arbitrary logical input-output mappings (IF-THEN statements, logic circuits, lookup tables, and heuristic rules).

8.2 Case Study: Automated Urban Weather Alarm System (Example 6.6)

Consider a municipal monitoring system with two weather stations measuring ambient temperature (T) and wind velocity (W). Experts define three dangerous events:

- • **Event A:** T > 30 °C (Heatwave risk)
- • **Event B:** T < -20 °C (Severe freezing risk)
- • **Event C:** W > 70 km/h (Gale force wind risk)

The most dangerous physical condition occurs when severe freezing and gale-force winds coincide (Event B AND Event C, causing rapid structural icing and wind chill). The system evaluates risk using logic gates:

- • **Local Risk Level 2:** Evaluated as (Event A OR Event B) AND (NOT Event C).
- • **Local Risk Level 3:** Evaluated by a logical AND gate: Event B AND Event C.
- • **City-Wide Alarm:** A central municipal supervisory station collects status from both stations via an OR gate (Level 2 if either station reports Level 3) and an AND gate (City Highest Alarm if both stations report Level 3 simultaneously).

8.3 Inherent Limitations of Decision-Making in CPS (§6.6)

- **1. Normative and Contingent:** Decision rules are normative engineering conventions, not immutable physical laws. A controller cannot adapt to scenarios not encoded in its rule base.
- **2. Vulnerability to Data Corruption:** Informed decisions depend entirely on accurate, structured data. Noise, transmission packet dropouts, or sensor tampering directly corrupt control actions.
- **3. Dynamic Feedback Invalidation:** When a predictive model's output triggers physical actions, it alters the physical process it was designed to predict, creating closed-loop feedback instabilities.
- **4. Reflexive Undecidability:** Higher-order reflexive decisions (“deciding which rule to use to select a rule”) can become self-referential and mathematically undecidable, analogous to Turing’s Halting Problem.

9. Worked Numerical Problems with Step-by-Step Solutions

Problem 1: Shannon Information Entropy of an Industrial Vibration Sensor

A vibration monitoring sensor mounted on an industrial robotic arm outputs four diagnostic operational states: Normal (N), Warning (W), Degraded (D), and Critical (C). Over 10,000 hours of operation, empirical telemetry yields the probability distribution: P(N) = 0.50, P(W) = 0.25, P(D) = 0.125, and P(C) = 0.125.

1. Calculate the Shannon information entropy H(X) of the sensor diagnostic stream in bits.

2. Compute the maximum theoretical uncertainty H_max and determine the information gained from the prior distribution.

Step-by-Step Solution:

Check normalization: $\sum P(x_i) = 0.50 + 0.25 + 0.125 + 0.125 = 1.00$. Using Eq. 4.1:

$H(X) = - [P(N) \log_2 P(N) + P(W) \log_2 P(W) + P(D) \log_2 P(D) + P(C) \log_2 P(C)]$

Note that: $\log_2(0.50) = -1$, $\log_2(0.25) = -2$, $\log_2(0.125) = -3$.

$H(X) = - [0.50(-1) + 0.25(-2) + 0.125(-3) + 0.125(-3)]$

$H(X) = - [-0.50 - 0.50 - 0.375 - 0.375] = - [-1.75] = 1.75$ bits/symbol.

Maximum entropy occurs when all $N = 4$ states are equiprobable ($P = 0.25$):

$H_max = \log_2(4) = 2.00$ bits/symbol.

Information gained (uncertainty resolution) from historical calibration:

$\Delta I = H_max - H(X) = 2.00 - 1.75 = 0.25$ bits.

Problem 2: Network Topology Analysis and Graph Metrics

1. A safety-critical fly-by-wire flight control CPS connects $N = 6$ avionics computers in a complete (fully connected) network topology. Determine the node degree k_i of each avionics node and the total number of physical communication links L.

2. An industrial IoT wireless sensor network consists of $N = 100$ environmental monitoring nodes modeled as an Erdős–Rényi random graph with link probability $p = 0.04$. Calculate the average degree $\langle k \rangle$ and the total expected link count $\langle L \rangle$.

Step-by-Step Solution:

1. Complete Network ($N = 6$):

Degree of every node: $k_i = N - 1 = 6 - 1 = 5$ connections.

Total physical links: $L = N(N - 1)/2 = (6 \times 5)/2 = 15$ links.

2. Erdős–Rényi Network ($N = 100$, $p = 0.04$):

Average node degree: $\langle k \rangle = p \cdot (N - 1) = 0.04 \times (100 - 1) = 0.04 \times 99 = 3.96$ links/node.

Total expected links: $\langle L \rangle = (1/2) \cdot N \cdot \langle k \rangle = (1/2) \times 100 \times 3.96 = 198$ links.

Problem 3: Epidemiological Malware Propagation on an Industrial Sensor Mesh

A malicious worm infects field devices in an automated water utility network. The network comprises interconnected nodes with an average node contact degree $\langle k \rangle = 6$. The malware has an infection transmission probability per contact $\beta = 0.20$ per hour. The local intrusion response system patches infected firmware at a recovery rate $\gamma = 0.40$ per hour.

1. Calculate the Basic Reproduction Number R_0 for this cyber-physical network.

2. State whether the malware will cause a persistent epidemic outbreak or die out naturally.

3. Determine the critical recovery rate γ_crit required to eradicate the malware without altering network topology.

Step-by-Step Solution:

1. Calculation of R_0 using Eq. 6.1:

$R_0 = (\beta \cdot \langle k \rangle) / \gamma = (0.20 \times 6) / 0.40 = 1.20 / 0.40 = 3.00$.

2. Epidemic Outcome Evaluation:

Since $R_0 = 3.00 > 1$, the pathogen exceeds the epidemic threshold. The malware will spread exponentially in the early stages and persist across the utility network in an endemic state.

3. Critical Recovery Rate:

To ensure eradication, R_0 must be strictly less than 1:

$R_0 = (\beta \cdot \langle k \rangle) / \gamma_crit < 1 \quad \Rightarrow \quad \gamma_crit > \beta \cdot \langle k \rangle = 0.20 \times 6 = 1.20$ per hour.

The firmware patching mechanism must operate at least 3 times faster (increasing from 0.40 to > 1.20 hr^{-1}) to prevent persistent infection.