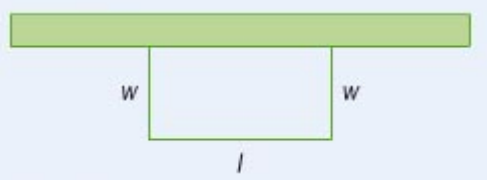
Optimization

The product of two positive numbers is 48. Find the two numbers so that the sum of the first number plus three times the second number is a minimum.

A rectangular plot of farmland is enclosed by 180 m of fencing material on three sides. The fourth side of the plot is bordered by a stone wall. Find the dimensions of the plot that enclose the maximum area. Find the maximum area.



a. Create an area model (function) in terms of w.

b. Use your understanding of ***quadratics*** to find the value of w that leads to the greatest area.

c. Use your understanding of ***calculus*** to find the value of w that leads to the greatest area.

You have 20 meters of fencing available. You will use it to build an enclosure for your hedgehogs. What dimensions will maximize the area ?

The cost C of ordering and storing x units of a product is $C(x) = x + \frac{10\,000}{x}$. A delivery truck can deliver at most 200 units per order. Find the order size that will minimize the cost.

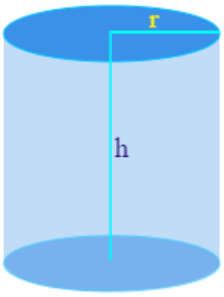
Find the dimensions of an open box with a square base and surface area of 192 square centimetres that has a maximum volume.

Guided Problem 1:

Soft drink cans are often packaged in cylindrical cans made from aluminium. The standard volume of such a can is 330 mL. Companies are always interested in minimizing costs of production in order to maximize profit. Are companies using the minimum amount of aluminium in the production of these cans?

<u>Our goal</u>: Minimize the amount of surface area of the can while keeping volume at 330 mL.

a) Add the cylinder volume and surface area formulas below.



Volume =

Surface Area =

b) Equate your volume equation to 330 since the volume must be 330 mL.

c) Currently, the area formula is written *in terms of* two variables, r and h .
Since we have a system of two equations, we can substitute one into the other in order to reduce the number of variables. To do this:

i) Solve the volume equation from b) for h.

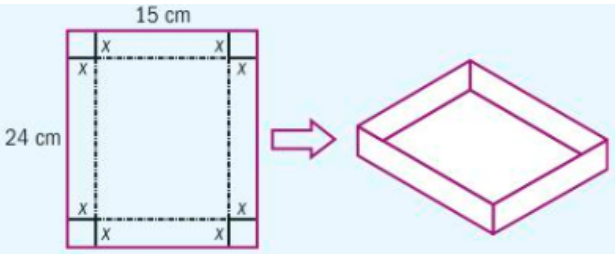
ii) Then, substitute into the area formula.

d) Since we are trying to minimize the surface area, we should differentiate A, in terms of r.

e) Now, use your derivative function for A and to find the minimum area.

f) What is the radius that minimizes the surface area? What is the height?

A cardboard box manufacturer makes open boxes by cutting equal squares of side length x cm from the corners of a rectangular piece of cardboard measuring 15 cm by 24 cm. The sides are then folded up, as shown in the diagram. Find x so that the volume of the box is maximized, and find the maximum volume of the box. Check your answers graphically.



A small company manufactures and sells fishing poles. The cost of manufacturing fishing poles can be modelled by the function $c(x) = 7x + 3$, where x is the number of batches (each containing 1000 poles) manufactured. Revenue is modelled by $r(x) = x^3 - 10x^2 + 20x$. The company has enough workers to produce a maximum of 1200 fishing poles.

a State the domain of both the cost and revenue functions.

b Find the number of fishing poles that the company should manufacture in order to maximize its profits.

The company introduces a new process which allows them to produce 6000 poles.

c Find the number of poles that would cause the company to minimize profits (or maximizes losses).

d By graphing the cost and revenue functions, find the number of poles which should be manufactured if the company is to just break even (that is, the production level at which the costs and revenues are equal).

e It would not maximize the company's profits to produce as many poles as workers are capable of producing. Use your graph to explain why.